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THE TORSIONAL STRENGTH OF RECTANGULAR REINFORCED  
CONCRETE BEAMS SUBJECTED TO COMBINED LOADING

by



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A THESIS

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FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend  
to the Faculty of Graduate Studies for acceptance, a thesis entitled  
THE TORSIONAL STRENGTH OF RECTANGULAR REINFORCED CONCRETE BEAMS SUBJECTED  
TO COMBINED LOADING submitted by Arthur Edward McMullen in partial ful-  
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## ABSTRACT

The primary objective of this investigation was to study the behavior of reinforced concrete beams subjected to combined loading with particular regard to their behavior at failure.

The experimental phase of this investigation consisted of testing 34 reinforced concrete beams. Twenty two beams were subjected to various combinations of bending and torsion and twelve were subjected to various combinations of bending, torsion and shear. All beams had a nominal cross section of 6" x 12" and a nominal concrete strength of 5000 psi. Both longitudinal and transverse reinforcement in various combinations was provided in all beams.

The testing equipment that was designed and fabricated for this investigation permitted independent application of the twisting and transverse loads. The ratio between the twisting moment and the bending moment could be changed at any time during a test.

All beams were tested to failure by applying the load in a series of increments. Each increment consisted of increasing to a pre-determined level the transverse load or the twisting load or both, depending on the type of test. In the cases where both types of load were applied in the same increment, the transverse load was applied first. Twenty nine beams were subjected to loads such that for any one test the ratio of twisting moment to bending moment at the end of each increment



was a constant. For the other five beams this ratio was different at the end of each increment and four of these beams were subjected to various sequences of load.

Based on the observations made in the experimental phase of this investigation three idealized failure surfaces have been presented, two of which were first suggested by Lessig. Equations for the ultimate torsional strength of a beam based on each of the three failure surfaces have been developed and their method of solution has been presented. These equations have also been simplified and an interaction diagram consisting of three straight lines has been presented along with the applicable equations.

The correlation between the experimental results of 109 beams and the theoretical results obtained from both the simplified analysis and the more comprehensive analysis has been given. Of these beams, 34 are the beams tested in this investigation and 75 are beams that have been tested by other investigators.

The observations and test results indicate that reinforced concrete beams that are subjected to bending, torsion and moderate amounts of transverse shear can fail by three different modes. These modes of failure are characterized by the formation of a hinge adjacent to one face of the beam and yielding of the reinforcement adjacent to the face opposite to the hinge. The modes of failure predicted by the analysis agree with the observed modes of failure.



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Messrs. H. Panse and T. Cunningham assisted in fabrication and testing of the specimens.

Mr. K. Sorensen helped in fabrication of the specimens, reduction of the test results and preparation of the figures.

Miss H. Wozniuk typed the final manuscript.

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## LIST OF SYMBOLS

### Dimensions and Areas

b = beam width

h = beam depth

$L_1$  = length of compression zone for a mode 1 failure

$L_2$  = length of compression zone for a mode 2 failure

$L_3$  = length of compression zone for a mode 3 failure

$c_1$  = length of mode 1 failure surface projected on the longitudinal axis of the beam

$c_2$  = length of mode 2 failure surface projected on the longitudinal axis of the beam

$c_3$  = length of mode 3 failure surface projected on the longitudinal axis of the beam

c =  $c_1$ ,  $c_2$  or  $c_3$  for a predicted mode 1, 2 or 3 failure respectively

$c_{m1}$  = maximum length of mode 1 failure surface projected on the longitudinal axis of the beam

$c_{m2}$  = maximum length of mode 2 failure surface projected on the longitudinal axis of the beam

$c_{m3}$  = maximum length of mode 3 failure surface projected on the longitudinal axis of the beam

$c_m$  =  $c_{m1}$ ,  $c_{m2}$  or  $c_{m3}$  for a predicted mode 1, 2 or 3 failure respectively

S = spacing of ties



## LIST OF SYMBOLS (continued)

$a_1$  = distance from bottom face of beam to centroid of  $A_{s1}$   
 $a_2$  = distance from vertical face of beam to centroid of  $A_{s2}$   
 $a_3$  = distance from top face of the beam to centroid of  $A_{s3}$   
 $a_{1t}$  = distance from bottom face of beam to centroid of bottom leg of tie  
 $a_{2t}$  = distance from vertical face of beam to centroid of vertical leg  
           of tie  
 $a_{3t}$  = distance from top face of beam to centroid of top leg of tie  
 $x_1$  = distance from top face of beam to the neutral axis  
 $x_2$  = distance from vertical face of beam on which the compression  
           zone is located to the neutral axis  
 $x_3$  = distance from bottom face of beam to the neutral axis  
 $x$  =  $x_1$ ,  $x_2$  or  $x_3$  for a predicted mode 1, 2 or 3 failure respectively  
 $z_1$  =  $h_{01} - k_1 x_1 / 2$   
 $z_2$  =  $b_{02} - k_1 x_2 / 2$   
 $z_3$  =  $h_{03} - k_1 x_3 / 2$   
 $h_{01}$  =  $h - a_1$   
 $h_{03}$  =  $h - a_3$   
 $b_{02}$  =  $b - a_2$   
 $y_1$  =  $k_{01}(h - a_{1t} - k_1 x_1 / 2) + (1 - k_{01})(1 - k_{01} - 4a_{2t}/b) b/4$   
 $y_2$  =  $k_{02}(b - a_{2t} - k_1 x_2 / 2) + (1 - k_{02})(1 - k_{02} - 4a_{1t}/h) h/4$   
 $y_3$  =  $k_{03}(h - a_{3t} - k_1 x_3 / 2) + (1 - k_{03})(1 - k_{03} - 4a_{2t}/b) b/4$   
 $\alpha_1$  =  $A_{s1} S / a_v$



## LIST OF SYMBOLS (continued)

|            |   |
|------------|---|
| $\alpha_2$ | $= A_{s2} S/a_v$  |
| $\alpha_3$ | $= A_{s3} S/a_v$  |
| $\beta_1$  | $= 2h + b$  |
| $\beta_2$  | $= 2b + h$  |
| $a_v$      | = area of one leg of a tie  |
| $A_{s1}$   | = area of longitudinal reinforcement at the bottom face of the beam   |
| $A_{s2}$   | = area of longitudinal reinforcement at the vertical face of the beam opposite to the face on which the compression zone is located |
| $A_{s3}$   | = area of longitudinal reinforcement at the top face of the beam  |

Moments and Forces

|           |   |
|-----------|---|
| $M_{t1}$  | = theoretical ultimate twisting moment of a beam for a mode 1 failure                                     |
| $M_{t2}$  | = theoretical ultimate twisting moment of a beam for a mode 2 failure                                     |
| $M_{t3}$  | = theoretical ultimate twisting moment of a beam for a mode 3 failure                                     |
| $M_t$     | = theoretical ultimate twisting moment of a beam  |
| $M_{b1}$  | = theoretical ultimate bending moment of a beam for a mode 1 failure                                      |
| $M_{b2}$  | = theoretical ultimate bending moment of a beam for a mode 2 failure                                      |
| $M_{b3}$  | = theoretical ultimate bending moment of a beam for a mode 3 failure                                      |
| $M_{tt}$  | = test value of ultimate twisting moment of a beam  |
| $M_{bt}$  | = test value of ultimate bending moment of a beam   |
| $M_{tr}$  | = theoretical ultimate twisting moment of an over reinforced beam   |
| $M_{t1a}$ | = theoretical ultimate twisting moment of a beam for a mode 1 failure<br>at $\emptyset$ equal to $\infty$ |



## LIST OF SYMBOLS (continued)

$M_{t2a}$  = theoretical ultimate twisting moment of a beam for a mode 2 failure at  $\phi$  equal to  $\infty$

$M_{t3a}$  = theoretical ultimate twisting moment of a beam for a mode 3 failure at  $\phi$  equal to  $\infty$

$M_{ta}$  = the smallest of the values  $M_{t1a}$ ,  $M_{t2a}$  and  $M_{t3a}$

$M_{t1b}$  = theoretical ultimate twisting moment of a beam for a mode 1 failure at  $\phi$  equal to 1.0

$M_{t2b}$  = theoretical ultimate twisting moment of a beam for a mode 2 failure at  $\phi$  equal to 1.0

$M_{t3b}$  = theoretical ultimate twisting moment of a beam for a mode 3 failure at  $\phi$  equal to 1.0

$M_{tb}$  = the smallest of the values  $M_{t1b}$ ,  $M_{t2b}$  and  $M_{t3b}$

$M_{bb}$  = theoretical ultimate bending moment of a beam at  $\phi$  equal to 1.0

$M_{t1c}$  = theoretical ultimate twisting moment of a beam for a mode 1 failure at  $\phi$  equal to 0.25

$M_{t2c}$  = theoretical ultimate twisting moment of a beam for a mode 2 failure at  $\phi$  equal to 0.25

$M_{t3c}$  = theoretical ultimate twisting moment of a beam for a mode 3 failure at  $\phi$  equal to 0.25

$M_{tc}$  = the smallest of the values  $M_{t1c}$ ,  $M_{t2c}$  and  $M_{t3c}$

$M_{bc}$  = theoretical ultimate bending moment of a beam at  $\phi$  equal to 0.25

$M_{bu}$  = theoretical ultimate bending moment of a beam subjected to pure bending

$V$  = theoretical ultimate transverse shear force of a beam

$V_t$  = test value of the ultimate transverse shear force of a beam



## LIST OF SYMBOLS (continued)

$V_u$  = ultimate transverse shear force of a beam subjected only to bending and transverse shear as defined in ACI Standard 318-63

Stresses

$f_{yt}$  = yield stress of ties

$f_{y1}$  = yield stress of the longitudinal reinforcement at the bottom face of the beam

$f_{y2}$  = yield stress of the longitudinal reinforcement at the vertical face of the beam opposite to the face on which the compression zone is located

$f_{y3}$  = yield stress of the longitudinal reinforcement at the top face of the beam

$f'_c$  = compressive strength of concrete determined by tests on 6 x 12 inch concrete cylinders

Dimensionless Factors

$k_1$  = a coefficient used to determine the depth of the equivalent compression zone

$k_e$  = a coefficient related to the effective length of the compression zone and the strength of the concrete in the compression zone

$k_{01}$  =  $b/(2h + b)$

$k_{02}$  =  $h/(2b + h)$

$k_{03}$  =  $k_{01}$

$k_{y1}$  =  $f_{y1}/f_{yt}$



## LIST OF SYMBOLS (continued)

$$k_{y2} = f_{y2}/f_{yt}$$

$$k_{y3} = f_{y3}/f_{yt}$$

$$p_1 = f_{yt} a_v h/f_{y1} A_{s1} S$$

$$p_2 = f_{yt} a_v b/f_{y2} A_{s2} S$$

$$p_3 = f_{yt} a_v h/f_{y3} A_{s3} S$$

$\emptyset$  = ratio of torsional moment to bending moment

$$\delta = \frac{b}{2} \left[ \frac{\text{transverse shear force}}{\text{twisting moment}} \right]$$

$$m_1 = p_1 b/h$$

$$m_2 = p_2 h/b$$

$$m_3 = p_3 b/h$$

$m_{01}$  = optimum value of  $m_1$

$m_{02}$  = optimum value of  $m_2$

$m_{03}$  = optimum value of  $m_3$

$m$  =  $m_1$ ,  $m_2$  or  $m_3$  for a predicted mode 1, 2 or 3 failure respectively

$m_o$  =  $m_{01}$ ,  $m_{02}$  or  $m_{03}$  for a predicted mode 1, 2 or 3 failure respectively

Miscellaneous

$\psi$  = ratio of transverse shear force to bending moment



## CHAPTER I

### INTRODUCTION

#### 1-1 General Remarks

The members of a reinforced concrete structure are subjected to transverse shear forces, axial forces, bending moments and torsional moments in various combinations. The behavior and ultimate strength of reinforced concrete members subjected to various combinations of transverse shear force, axial force and bending moment have been investigated rather extensively. However, the behavior and ultimate strength of reinforced concrete members subjected to a torsional moment combined with transverse shear, axial load or bending moment have received less attention. The reasons for this are:

- (a) Torsion is usually a secondary effect in concrete structures
- (b) It is usually possible to arrange the members in a structure so that they are subjected to only very small twisting moments
- (c) Torsion tests require special testing equipment which is not readily available.

Torsion is seldom taken into consideration in the design of concrete structures. Current practice assumes that if the members are arranged so that they are subjected to only very small twisting moments the factor of safety will adequately provide for the effects of torsion.



However, since knowledge of the behavior of reinforced concrete has increased considerably in recent years and since design procedures are continually being refined it is essential that the effects of torsion be understood so that provision can be made for them in design. In addition, having to arrange the members so that they are subjected to only small twisting moments imposes an undesirable restriction on the designer.

Nylander (1945) tested beams that were reinforced with longitudinal reinforcement only since he considered this to be the type of beam most often encountered in actual structures. He found that for low stress in the reinforcement, the bending moment exerted a favorable influence on the torsional strength of a beam. However, since current practice is to provide most reinforced concrete beams with at least nominal transverse reinforcement, it is doubtful if Nylander's beams can be considered typical.

Cowan (1953) found that subjecting a beam to a small bending moment increases its torsional strength. He suggested that the torsional strength of a beam was equal to the sum of the torsional strength of the plain concrete section and the contribution of the reinforcement. However, since his equations are based on elastic theory they can not be used to determine the ultimate strength of a beam.

Lessig (1959) suggested two possible modes of failure and derived equations for the torsional strength of a beam. These modes of failure involve the formation of an inclined hinge adjacent to one of



the faces of the beam. In the equations the torsional strength of the plain concrete section is not considered. Furthermore, the equations indicate that the addition of a bending moment can only reduce the torsional strength of a beam.

Yudin (1962) and Gesund, Schuette, Buchanan and Gray (1964) developed equations for the torsional strength of a beam that are based on the formation of a hinge parallel to the axis of the beam. These equations indicate that addition of a bending moment can only reduce the torsional strength of a beam.

Pandit (1965), similarly to Cowan (1953), assumed that the torsional strength of a beam was equal to the sum of the torsional strength of the plain concrete section and the contribution of the reinforcement. He stated that the beams he tested showed no evidence of the formation of hinges. The beams tested by Pandit were first subjected to bending and were then twisted to failure.

Goode and Helmy (1965) performed tests in which the beams were first twisted and then bent to failure. They stated that these beams sustained an ultimate bending moment at least equal to the ultimate bending moment that would be expected in pure flexure.

It can easily be seen that there is disagreement with regard to the behavior of beams subjected to combined loading. Before a design procedure can be formulated it is essential to obtain a better understanding of this behavior. It is hoped that this investigation will make a contribution to that end.



## 1-2 Object

The objectives of this investigation are:

- (1) To observe the behavior of reinforced concrete beams subjected to combined loading with particular regard to their behavior at failure.
- (2) To study the effect of bending moment and transverse shear on the torsional strength of beams.
- (3) To determine the effect of sequence of loading on the ultimate strength of beams.
- (4) To study the effect of various combinations of longitudinal and transverse reinforcement on the behavior and strength of beams.
- (5) To develop a method for predicting the strength of rectangular, reinforced concrete beams subjected to combined loading.

## 1-3 Scope

In this investigation a total of 34 beams were subjected to various combinations of load and sequences of load. All beams had a nominal cross section of 6 inches by 12 inches, a nominal concrete strength of 5000 psi and were reinforced with both longitudinal and transverse reinforcement.

Three beams were subjected to pure torsion and nineteen beams were subjected to various combinations of bending and torsion. These beams were divided into four groups. Twelve beams divided into three groups were subjected to various combinations of bending, torsion and shear.



All beams were tested to failure by applying the load in a series of increments. Each increment consisted of increasing to a pre-determined level the transverse load or the twisting load or both, depending on the type of test. In those cases where both types of load were applied in the same increment, the transverse load was applied first. For 29 beams the ratio of twisting moment to bending moment at the end of each increment was a constant in any one test. For the other five beams this ratio was different at the end of each increment and four of these beams were subjected to various sequences of load.

An analysis is presented which is based on observations made during testing. This analysis was performed with the aid of an IBM 7040 computer and the results obtained are compared with the experimental results. In addition, the experimental results of 75 beams tested by other investigators are compared with the theoretical results obtained from the analysis. The analysis is then simplified and the results from the simplified analysis are compared to the experimental results.



## CHAPTER II

### REVIEW OF PREVIOUS RESEARCH

#### 2-1 Introduction

Although torsion in reinforced concrete beams has been investigated for many years, most investigations dealing with combined bending and torsion have been performed within the last decade. In general, the behavior and ultimate strength of beams subjected to pure bending is well understood but the behavior and ultimate strength of beams subjected to either pure torsion or combined bending and torsion is not. Since both the limiting cases of pure bending and pure torsion have been adequately reviewed previously, they will not be reviewed here. This chapter presents a review of previous research in which the specimens were subjected to various combinations of bending, torsion and transverse shear.

#### 2-2 Previous Research

Nylander (1945) tested forty four beams of square, rectangular, and T shaped cross section. Thirty four were tested in combined bending and torsion and ten were tested in combined bending, torsion and shear. None of the beams was provided with transverse reinforcement. He stated that if the stress in the reinforcement due to bending was low, the bending moment exerted a favorable effect on the torsional strength. In addition, he concluded that the ultimate capacity of a member subjected to bending,



torsion and shear could be obtained by equating the sum of the torsional and transverse shearing stresses to the ultimate tensile strength of the concrete.

Cowan (1953) proposed equations to predict the ultimate strength of reinforced concrete beams under combined loading and classified all failures as either primary torsion or primary bending failures. He stated that primary bending failure was governed by crushing of the compressed concrete whereas primary torsion failure was governed by the development of  $45^{\circ}$  helical cracks. The failure theory that he used is a combination of Rankine's maximum principal stress theory and Coulomb's internal friction theory. His equations indicate that the addition of bending increases the torsional strength whereas the addition of torsion reduces the bending capacity only slightly. Hence, he recommended designing a reinforced concrete section independently for bending and torsion without reduction of the maximum permissible stresses. This reduces the shape of the bending-torsion interaction diagram to a rectangle. Furthermore, he recommended adding the torsional resistance due to the reinforcement to that of a plain concrete section in order to obtain the torsional resisting moment of the reinforced concrete section.

Cowan and Armstrong (1955) tested seven rectangular reinforced concrete beams. The results of these tests were used to substantiate the theory outlined above.

Lessig (1959) defined two possible modes of failure for rectangular reinforced concrete beams subjected to combined loading. These failure



modes are based on the assumption that at failure a plastic hinge that is inclined to the longitudinal axis of the beam forms on either the top or the side of the beam. By making a number of assumptions and using two equations of equilibrium, one for moments and the other for forces, an equation for the ultimate torsional strength of a beam subjected to combined loading is derived for each of the modes of failure. These equations indicate that a bending moment can only reduce the ultimate torsional moment of a beam. A significant assumption that is made is that at failure the tensile strength of the concrete does not contribute to the torsional moment. This is in direct contrast with Cowan's assumption that the torsional resisting moment of a reinforced concrete beam is composed of the torsional resistance of the plain concrete section plus the contribution of the reinforcement.

Chinenkov (1959) tested a total of 36 beams in combined bending and torsion. The results of these tests were used to substantiate the theory proposed by Lessig. Twenty three of these beams were reinforced with two plane welded skeletons of reinforcement placed adjacent to the vertical faces of the beam. Failure of these beams was generally by splitting of the concrete on the unreinforced sides of the beam. The remaining 13 beams were reinforced with both longitudinal bars and closed ties. Chinenkov stated that at failure these beams developed a plastic hinge adjacent to their top face. It was noted that the appearance of cracks on the flexural compression face did not lead to failure; failure occurred only when the flexural tension reinforcement yielded. The ratio



between the experimental and the theoretical ultimate twisting moments for these beams was generally greater than 1.0. This was interpreted as evidence that the uncracked portion of concrete was resisting tension. However, the theoretical ultimate twisting moments were computed by simplified versions of Lessig's original equations. The effect of the simplifications on the theoretical ultimate twisting moments was not stated.

All thirteen of the beams with closed ties were tested at a low ratio of twisting moment to bending moment. This ratio varied between 0.1 and 0.4. Consequently, no information was obtained regarding the effect of small bending moments on the torsional strength of the beams.

Lyalin (1959) tested thirty six beams in combined bending, torsion and shear and compared the experimental failure moments to the theoretical failure moments computed using Lessig's simplified equations. The ratio of twisting moment to bending moment at which thirty of these beams were tested was between 0.1 and 0.5. Of the remaining six, two were tested at a ratio of 1.0 and four were tested in pure torsion.

Lyalin concluded from the test results that full redistribution of stress between longitudinal and transverse reinforcement is obtained over a large range of the ratio between these two types of reinforcement. He also concluded that the amount of reinforcement necessary to cause a compression failure in a beam subjected to bending and torsion is less than the amount required in a beam that is subjected to pure flexure and that this amount varies with the ratio of twisting moment to bending moment.



Lessig (1961) reported tests on forty two beams. Six of these were tested in pure torsion, six were tested in combined bending and torsion, and the remainder were tested in combined bending, torsion and shear. Except for the beams tested in pure torsion, they were all tested at a ratio of twisting moment to bending moment of 0.8 or less. On the basis of the test results, equations were proposed for determining the minimum amount of reinforcement necessary to cause a compression failure. In addition, limits on the permissible variation of the ratio between the longitudinal and the transverse reinforcement were proposed.

Yudin (1962) stated that the equations proposed by Lessig indicated that a deficiency in longitudinal reinforcement could be compensated for by an excess of transverse reinforcement and vice versa. He claimed that the most rational procedure was not to take moments about an inclined axis but to use equilibrium of internal and external moments about two axes to establish two equations. One of these axes should be parallel to the longitudinal axis of the beam and the other should be perpendicular to it. Furthermore, he claimed that three equilibrium equations instead of two should be used for each possible mode of failure, that the failure surface could be defined with sufficient accuracy by assuming that on three faces of the beam it makes an angle of  $45^{\circ}$  with the longitudinal axis, and that since the depth of the compression zone was small it could be neglected. On the basis of these premises he derived equations for determining the ultimate strength of a reinforced concrete beam subjected to combined loading.



Yudin (1964) reported the results of tests on twenty two beams. He claimed that the results substantiated the validity of his equations. These beams were tested at a ratio of twisting moment to bending moment that was between zero and 0.2.

Gesund and Boston (1964) reported results of tests on ten beams. Eight of these beams were square and two were rectangular. All beams were reinforced with longitudinal reinforcement only. Two beams were subjected to pure torsion whereas the remainder were subjected to combined bending and torsion. It was assumed that after cracking had defined a failure surface the torsional moment was resisted by the combined action of the uncracked portion of the concrete and the dowel action of the reinforcement. It was not found possible to write an expression for the first of these two quantities but an expression was derived for the torsional resistance due to the dowel action. The ultimate torsional moment computed using this expression was lower than the experimental moment in all cases. They concluded that beams subjected to bending and torsion may fail in one of several different torsional modes, or in bending and that if there is no transverse reinforcement, the dowel action of the longitudinal reinforcement is of paramount importance in resisting the torsion. They also stated that the bending resistance of a beam may be severely reduced by the torsion if the failure hinge forms on one side and thus greatly reduces the lever arm.

Gesund, Schuette, Buchanan and Gray (1964) reported tests on twelve beams subjected to combined bending and torsion. Eight of these



beams were square, four were rectangular and all were provided with both longitudinal and transverse reinforcement. A theoretical failure surface was established utilizing a hinge that was located on the top surface of the beam and was parallel to the longitudinal axis of the beam. On the basis of this failure surface and some further assumptions, equations were derived for the ultimate torsional moment and the ultimate flexural moment of a reinforced concrete beam. Several of these equations are similar to Yudin's equations and indicate that a bending moment of any magnitude will not increase the torsional strength of a beam. Two equations for torsional resistance were proposed. The first was based on the assumption that the entire torsional resistance was provided by the tensile forces in the ties and the second on the assumption that the total torsional resistance was provided by the dowel action of the ties plus the dowel action of the longitudinal reinforcement. The ultimate torque was taken as the greater of the two values given by these two equations.

The investigators classed all failures as either predominantly bending or predominantly torsion. They concluded that even a small amount of transverse reinforcement was sufficient to produce a bending rather than a torsional failure for a ratio of twisting moment to bending moment equal to or less than 0.5. Other conclusions stated were that the transverse reinforcement transforms torque on a reinforced concrete beam into additional bending moment, that if sufficient transverse reinforcement is provided to prevent torsional failure, the stresses in this reinforcement



have no effect on the strength of the beam, and that the dowel action of the longitudinal reinforcement frequently provides greater torsional resistance than the transverse reinforcement.

Collins, Walsh and Hall (1965) stated that the results of a series of beams that they tested indicated that a bending moment could increase the torsional strength of some beams. They reported that one beam which exhibited this increased torsional strength failed by rotating about a hinge that was inclined to the axis of the beam and was located on the bottom face.

Goode and Helmy (1965) stated that in their opinion the contribution of the concrete to the torsional resistance could not be ignored. They also stated that beams which they tested by first applying a twisting moment and then applying a bending moment until failure occurred exhibited an ultimate bending moment that was at least as great as the ultimate bending moment of the beam in pure flexure.

Evans and Sarkar (1965) reported tests on eighteen hollow reinforced concrete beams tested in combined bending and torsion. The failure surface that they used is defined by cracks on the bottom and the two vertical faces of the beam. The angle that the crack on the bottom face of the beam makes with the longitudinal axis is determined by the direction of the maximum principal stress. This crack is assumed to continue up the vertical faces of the beam at the same angle to the longitudinal axis for a vertical distance equal to 0.6 times the beam depth. From this level to the top surface of the beam the cracks are assumed to be



at  $45^{\circ}$  to the longitudinal axis of the beam. The failure surface is defined on the top face of the beam by a rectangular compression zone making an angle of  $45^{\circ}$  with the longitudinal axis. The failure surface is discontinuous at the top corners of the beam. An equation for the ultimate torsional moment is derived by using equilibrium of internal and external moments about the "compression fulcrum" which is located at mid depth of the compression zone.

Pandit (1965) reported tests on thirty six plain and reinforced rectangular concrete beams. In these tests the bending load was applied first and then twisting moment was applied until failure occurred. Nine of the beams were plain concrete and twenty seven were reinforced with both longitudinal and transverse reinforcement. One of the plain concrete beams was tested in pure bending, two in pure torsion, three in bending and torsion and three in bending, torsion and shear. Seven of the reinforced concrete beams were tested in pure torsion, one in pure bending, ten in combined bending and torsion and nine in combined bending, torsion and shear.

All failures were classified as either cleavage or hybrid and it is stated that there was no evidence of the formation of plastic hinges on the top or the sides of the beams. The test results indicated that a small bending moment could in some cases increase the torsional strength of the member.

An analysis for the ultimate strength of reinforced concrete beams subjected to combined loading was proposed. In this analysis it



was assumed that the torsional strength of a reinforced concrete beam was the sum of the torsional strength of the plain concrete beam and the contribution due to the reinforcement. The torsional strength of the plain concrete beam was calculated by using a modification of the sand heap analogy which considered the beam to be composed of a region of compressed concrete and another region of uncompressed concrete. The unit torsional strength of the compressed concrete was defined by Cowan's theory of failure and the unit torsional strength of the uncompressed concrete was established empirically from the test results. It was assumed that the flexural compression was uniformly distributed over an area equal to one-fourth of the area of the cross section of the beam. It was also assumed that the splitting of the concrete on the top face of the beam caused by the twisting moment reduced the lever arm of the internal flexural moment. The lever arm was assumed to vary linearly with respect to bending moment over the entire range from pure flexure to pure torsion.

The number of ties participating in resisting the torsional moment was taken as the smallest number of ties intersected by a potential crack making an angle of  $45^{\circ}$  with the axis of the beam. The ties intersected by this potential failure crack were assumed to yield at failure provided there was adequate top and bottom longitudinal reinforcement present to provide for the longitudinal component of the diagonal tension. The contribution of the reinforcement to the torsional strength was assumed to be governed by the yield of top, bottom or transverse reinforcement,



whichever occurred first. When the beam was subjected to a bending moment in addition to a torsional moment the longitudinal reinforcement was required to be sufficient to resist both the bending moment and the longitudinal component of the diagonal tension. As a further contribution of the reinforcement to the torsional strength, the longitudinal bars were considered to exhibit dowel strength proportional to the plastic moments they could develop.



## CHAPTER III

### TEST SPECIMENS, EQUIPMENT AND PROCEDURE

#### 3-1 Test Specimens

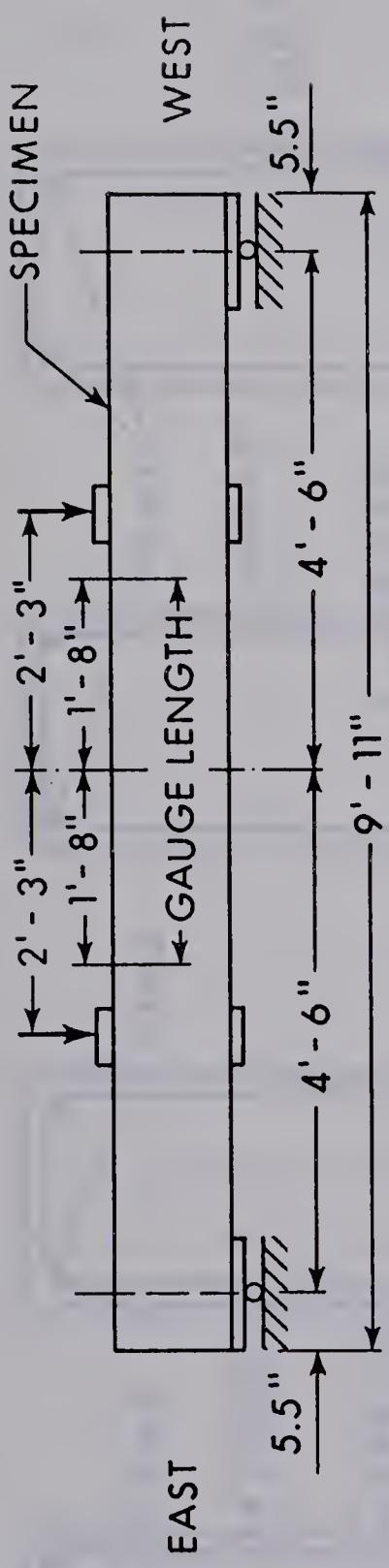
The thirty-four beams reported in this investigation were all provided with both longitudinal and transverse reinforcement. They were divided into seven groups as outlined in TABLE 3-1. The beams of Groups 1 to 4 inclusive were designed to be tested in combined bending and torsion, whereas those of Groups 5 to 7 were designed to be tested in combined bending, torsion and shear. All beams had a nominal cross section of 6 x 12 inches and their overall length was 9'-11". The nominal compressive strength of the concrete was 5000 psi, the concrete mix being the same for all beams. The compressive and tensile strengths of the concrete were determined by means of compression and splitting tensile tests respectively on 6 x 12 inch control cylinders. These are given in TABLE 4-1 of CHAPTER IV. The method of fabrication of the specimens and the properties of the sand, coarse aggregate and reinforcing steel are given in Appendix A. The transverse reinforcement provided in these beams was obtained from the manufacturer in two batches which were designated as lot number one and two. All beams of Group 3 and Beams 1-1 to 1-5 of Group 1 were reinforced with transverse reinforcement from lot number 1, whereas all other beams were fabricated with transverse reinforcement from lot number two. Extra reinforcement, both longitudinal and transverse, was provided outside



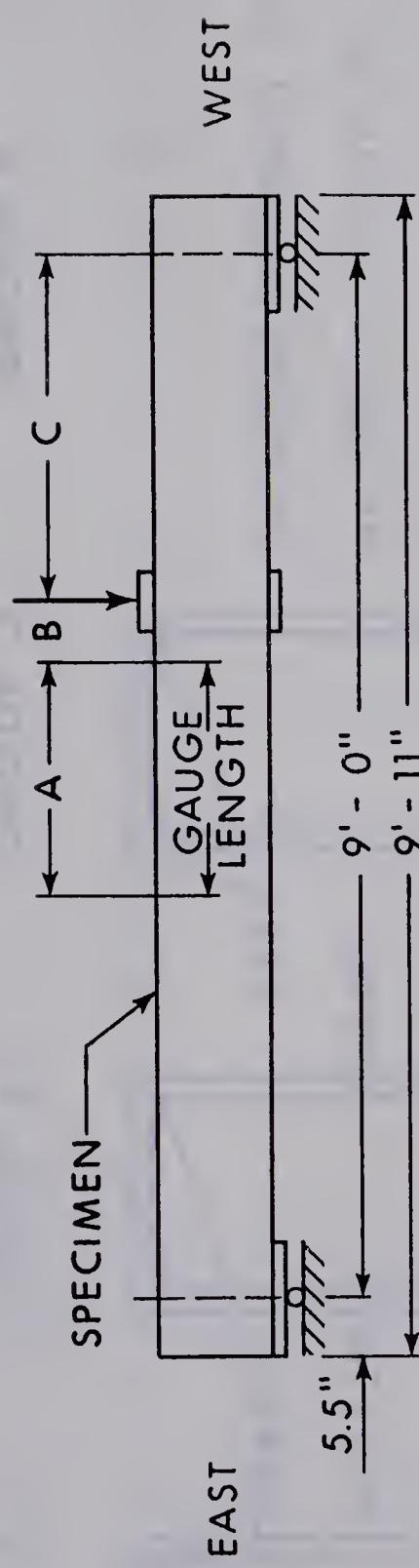
of the gauge length to ensure that failure occurred within the gauge length. The gauge lengths of the beams and their position in the beams are shown in FIGURE 3-1.

The details of the reinforcing steel are shown in FIGURE 3-2 and TABLE 3-1. The reinforcement for the beams of Group 1 was designed such that the top longitudinal reinforcement was less than the bottom. The reinforcement for the beams of Group 2 was identical to that of Group 1 except that the top longitudinal reinforcement was equal to the bottom. The beams of Group 3 were reinforced identically to those of Group C tested by Pandit (1965). Group 4 beams were reinforced such that for Beams 4-1 to 4-4 inclusive the longitudinal reinforcement remained constant whereas the spacing of the transverse reinforcement was progressively increased. The reinforcement was designed such that the test results of Beam 3-4 could be utilized in conjunction with these beams. Beams 4-5 and 4-6 were designed so they could be utilized in conjunction with Beam 1-5. Both the bottom longitudinal and the transverse reinforcement were progressively reduced from that of Beam 1-5. The reinforcement for all beams of Groups 5 and 6 was identical to that of Group 1. The beams of Group 7 were provided with larger bottom reinforcing bars than the beams of Groups 1, 5 and 6, and the spacing of the transverse reinforcement was increased. This was done to obtain beams that would be stronger in flexure but weaker in transverse shear.





(a) COMBINED BENDING AND TORSION



| GROUP | BEAM NO.       | A       | B  | C        |
|-------|----------------|---------|----|----------|
| 5     | 5 - 1          | 2' - 2' | 7" | 2' - 10" |
| 5     | 5 - 2 TO 5 - 4 | 2' - 2' | 6" | 2' - 10" |
| 6     | 6 - 1 TO 6 - 4 | 2' - 2' | 6" | 5' - 8"  |
| 7     | 7 - 1 TO 7 - 4 | 2' - 2' | 6" | 5' - 8"  |

(b) COMBINED BENDING TORSION AND SHEAR

FIGURE 3 - 1 TEST ARRANGEMENT FOR COMBINED LOADING



FIGURE 3 - 2 REINFORCING STEEL DETAILS

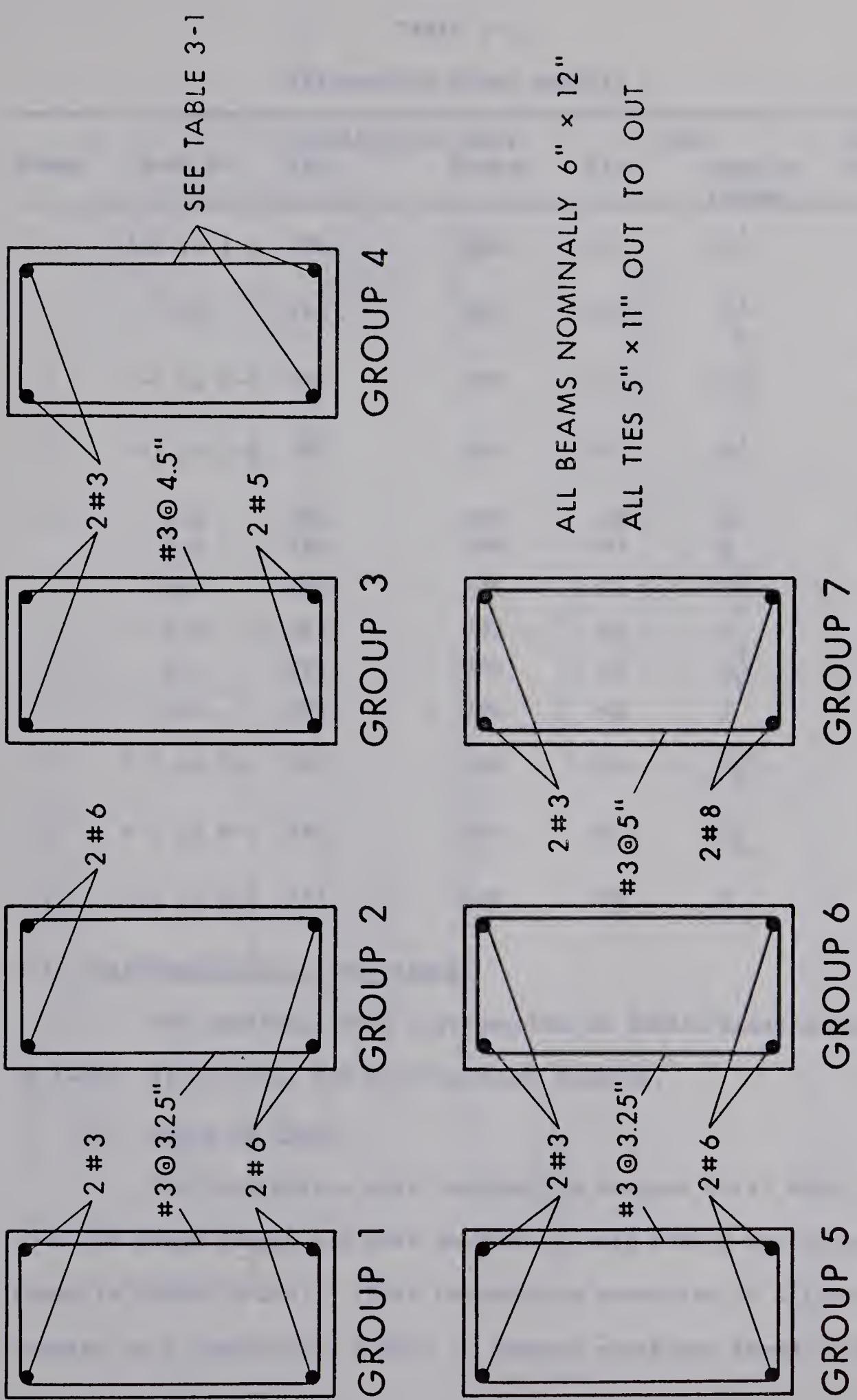




TABLE 3-1  
REINFORCING STEEL DETAILS

| Group | Beam No.   | Longitudinal Bars |        | Ties | Lot No.           |                |
|-------|------------|-------------------|--------|------|-------------------|----------------|
|       |            | Top               | Bottom | Size | Spacing<br>inches | of Ties        |
| 1     | 1-1 to 1-5 | 2#3               |        | 2#6  | #3                | $3\frac{1}{4}$ |
|       | 1-6        |                   | 2#3    | 2#6  | #3                | $3\frac{1}{4}$ |
| 2     | 2-1 to 2-5 | 2#6               |        | 2#6  | #3                | $3\frac{1}{4}$ |
| 3     | 3-1 to 3-5 | 2#3               |        | 2#5  | #3                | $4\frac{1}{2}$ |
| 4     | 4-1        | 2#3               |        | 2#5  | #3                | 3              |
|       | 4-2        | 2#3               |        | 2#5  | #3                | 6              |
|       | 4-3        | 2#3               |        | 2#5  | #3                | $7\frac{1}{2}$ |
|       | 4-4        | 2#3               |        | 2#5  | #3                | 9              |
|       | 4-5        | 2#3               |        | 2#5  | #3                | $4\frac{3}{8}$ |
|       | 4-6        | 2#3               |        | 2#4  | #3                | 7              |
| 5     | 5-1 to 5-4 | 2#3               |        | 2#6  | #3                | $3\frac{1}{4}$ |
| 6     | 6-1 to 6-4 | 2#3               |        | 2#6  | #3                | $3\frac{1}{4}$ |
| 7     | 7-1 to 7-4 | 2#3               |        | 2#8  | #3                | 5              |

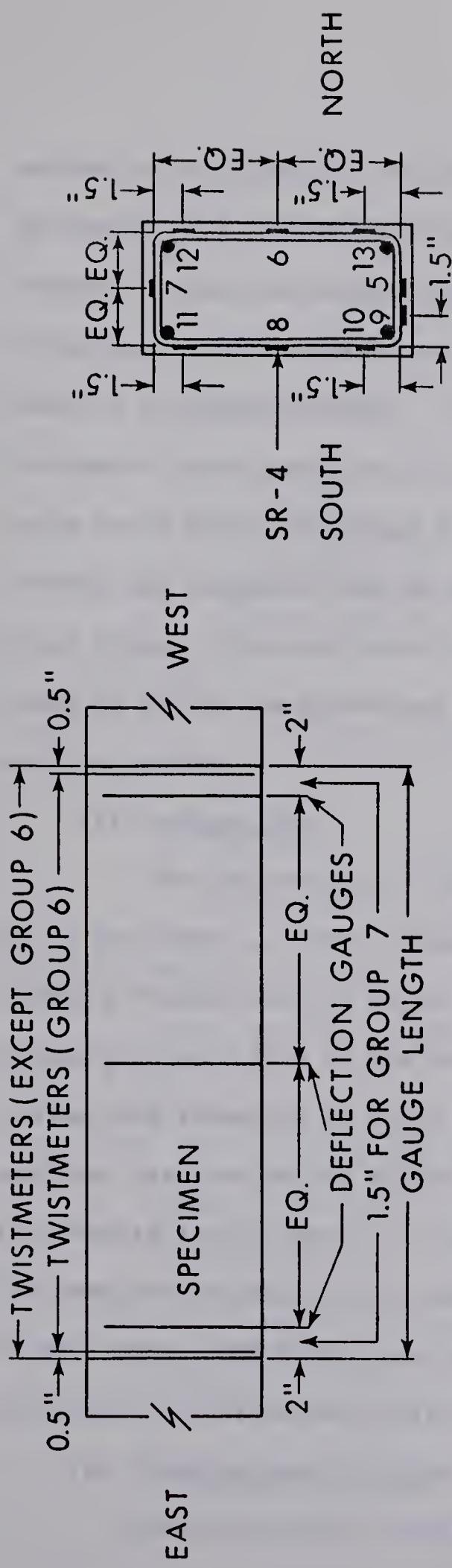
### 3-2 Instrumentation of Specimens

The specimens were instrumented to enable measurement of angle of twist, deflections and reinforcement strains.

#### (a) Angle of Twist

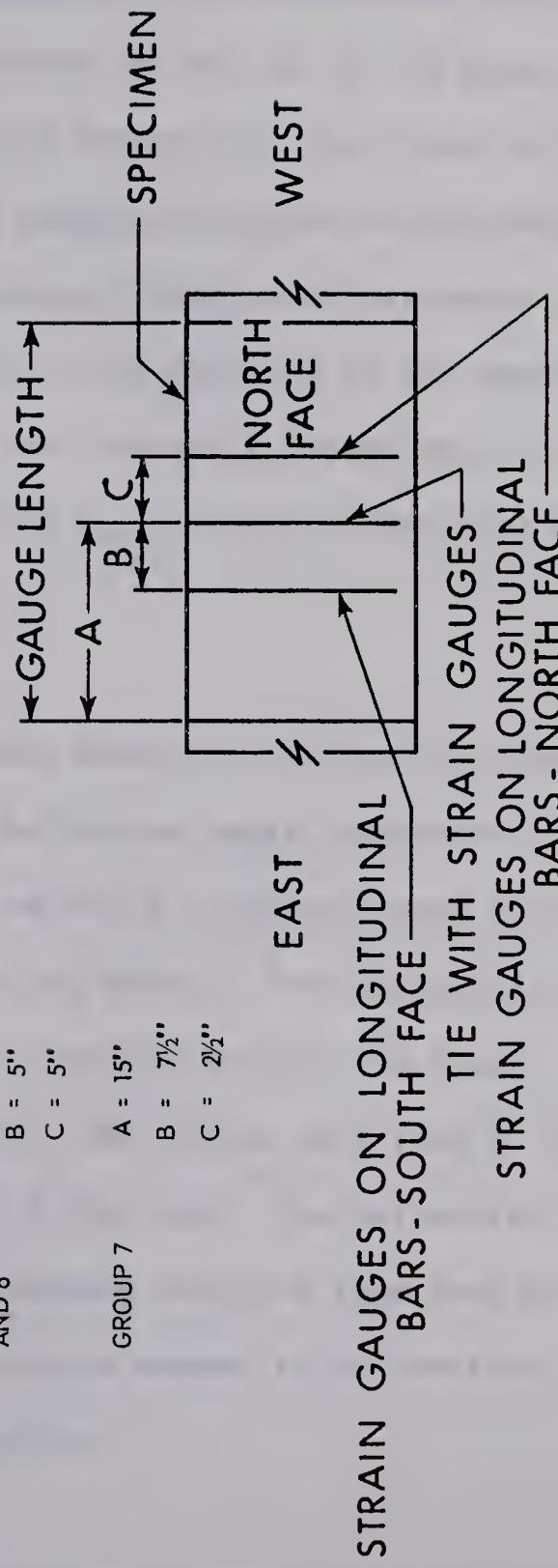
The twistmeters were designed to measure total angle of twist over the gauge length and were mounted at each end of the gauge length as shown in FIGURE 3-3(a). These twistmeters consisted of a level bubble mounted on a length of 1 1/2" x 1" channel which was fixed to a base at





(a) TWISTMETERS AND DEFLECTION GAUGES

(b) BEAM 3 - 2



(c) LOCATION OF STRAIN GAUGES

FIGURE 3 - 3 INSTRUMENTATION OF SPECIMENS



one end by a pin joint. The other end of the channel was supported by the needle of a micrometer screw that was mounted on the base. Close contact between the channel and the micrometer screw needle was maintained by springs. This assembly was then attached to the top of the beam by means of a clamping bracket. The distance between the pin joint and the micrometer screw needle was 15" and the smallest division on the micrometer was 0.001". The angle of twist through which each twistmeter rotated was computed from the difference in the readings of the micrometer screw. The total angle of twist over the gauge length was computed by taking the difference between the angles of twist obtained from each twistmeter.

(b) Deflections

The deflections of the beam were measured at three locations which are shown in FIGURE 3-3(a). The deflection gauges consisted of a clamping bracket mounted on the beam, from which a rod projected horizontally on each side of the beam and at mid depth. From these rods, scales were suspended at equal distances from the face of the beam. The smallest division on the scales was 0.01". The scales were read by using two precise levels located to each side of the beam. The deflection of the beam was obtained by averaging the readings obtained from each side of the gauge. The deflection obtained in this manner is the vertical deflection of the center of the cross section.

(c) Reinforcement Strains

SR-4 electrical resistance strain gauges were used to measure



strains in the reinforcement. In general, one gauge was mounted on each of the longitudinal bars and four gauges were mounted on one of the ties. In order to facilitate reference to these gauges, the following numbering system was established.

Longitudinal Bars

| <u>Location of Bar</u>  | <u>No. of Gauge</u> |
|-------------------------|---------------------|
| Top face, north side    | 1                   |
| Top face, south side    | 2                   |
| Bottom face, north side | 3                   |
| Bottom face, south side | 4                   |

Tie

| <u>Location of Gauge</u>        | <u>No. of Gauge</u> |
|---------------------------------|---------------------|
| Bottom horizontal leg<br>of tie | 5                   |
| North vertical leg of tie       | 6                   |
| Top horizontal leg of tie       | 7                   |
| South vertical leg of tie       | 8                   |

The locations of gauges 5 to 8 are illustrated in FIGURE 3-3(b).

Beams 3-2 and 3-5 were provided with additional gauges as discussed below.

(i) Beam 3-2: Five additional gauges were mounted on the tie. These gauges were numbered 9 through 13. The locations are detailed in FIGURE 3-3(b).

(ii) Beam 3-5: Twelve additional gauges were mounted on the longitudinal bars and on other ties. These gauges were numbered as follows:



Longitudinal Bars

| <u>Location of Bar</u>  | <u>No. of Gauge</u> |
|-------------------------|---------------------|
| Top face, north side    | 9                   |
| Top face, south side    | 10, 17              |
| Bottom face, north side | 11, 18              |
| Bottom face, south side | 12                  |

Tie

| <u>Location of Gauge</u>        | <u>No. of Gauge</u> |
|---------------------------------|---------------------|
| Bottom horizontal leg<br>of tie | 13, 19              |
| North vertical leg of tie       | 14, 20              |
| Top horizontal leg of tie       | 15                  |
| South vertical leg of tie       | 16                  |

Gauges 13 to 16 were all mounted on the same tie, whereas gauges 19 and 20 were mounted on another tie.

For the beams of Groups 5, 6 and 7, the location of the gauges on the longitudinal reinforcement and the location of the instrumented tie are shown in FIGURE 3-3(c).

3-3 Test Equipment

The arrangement of the equipment used for testing the beams is illustrated by FIGURES 3-4, 3-5 and 3-6.

The equipment for applying the twisting moment was completely independent of the equipment used to apply the bending moment. The transverse load was applied by a 100 kip Amsler jack. This load was first transferred to a distributing beam that was supported at one end by a ball and at the other end by a roller. This ball and roller each rested on a roller assembly which in turn rested on a pipe collar. The specimen



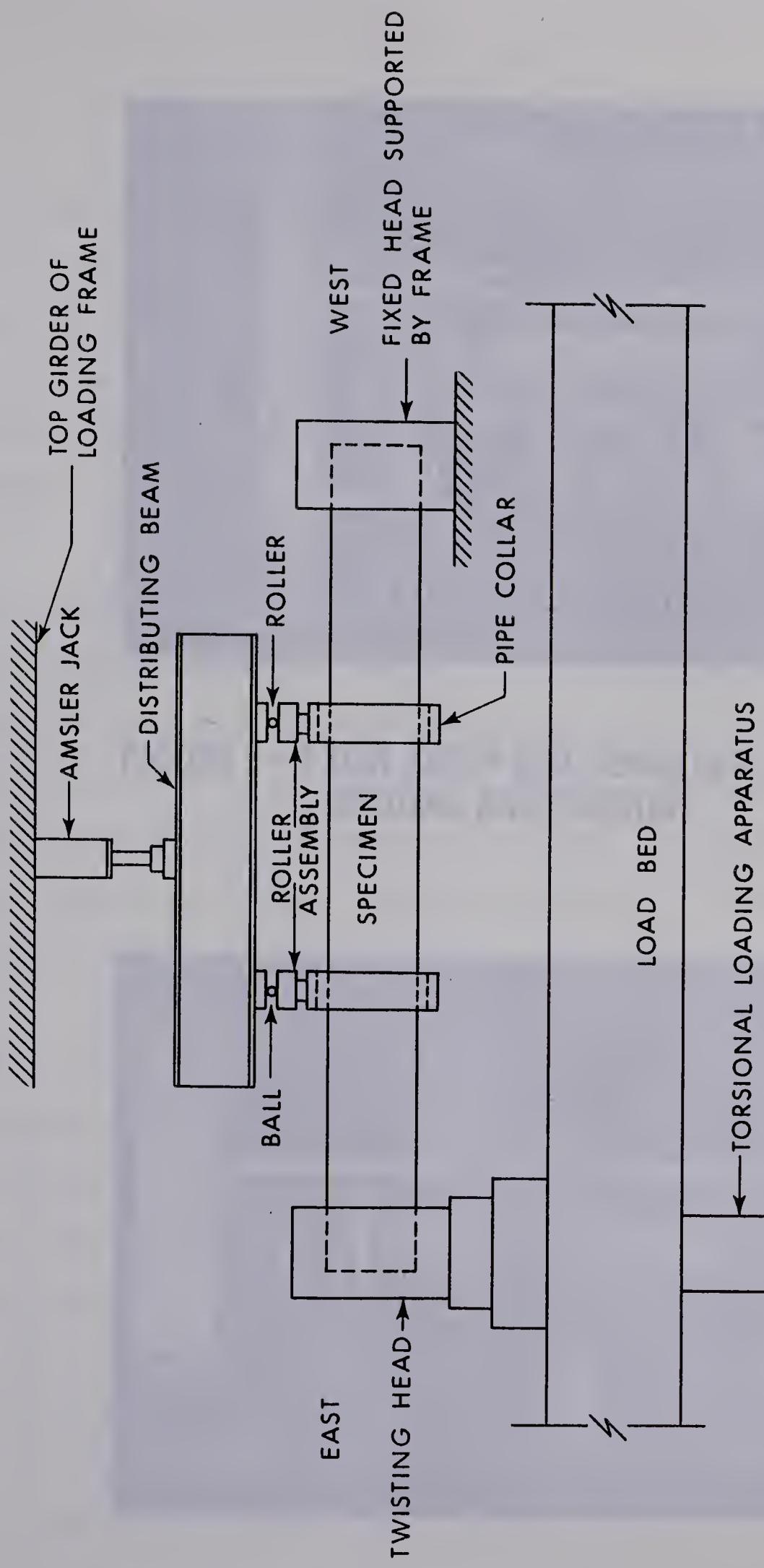


FIGURE 3 - 4 EQUIPMENT ARRANGEMENT FOR COMBINED BENDING AND TORSION



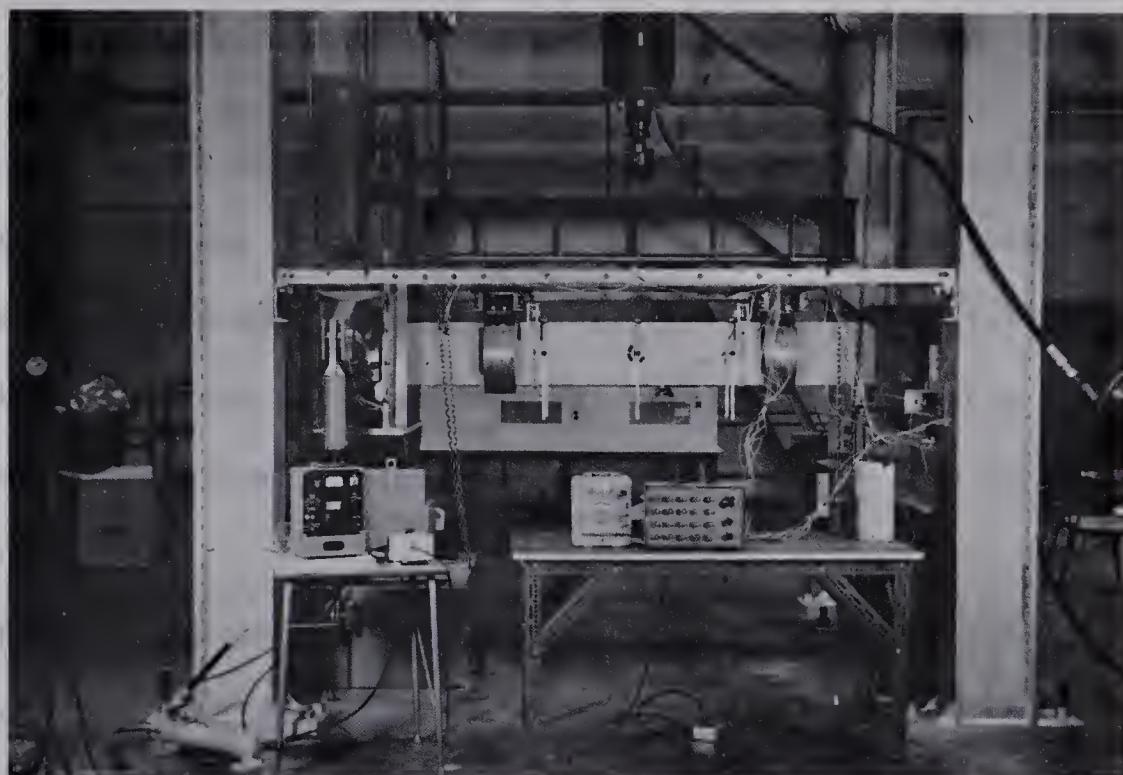


FIGURE 3 - 5 TEST SETUP FOR COMBINED  
BENDING AND TORSION

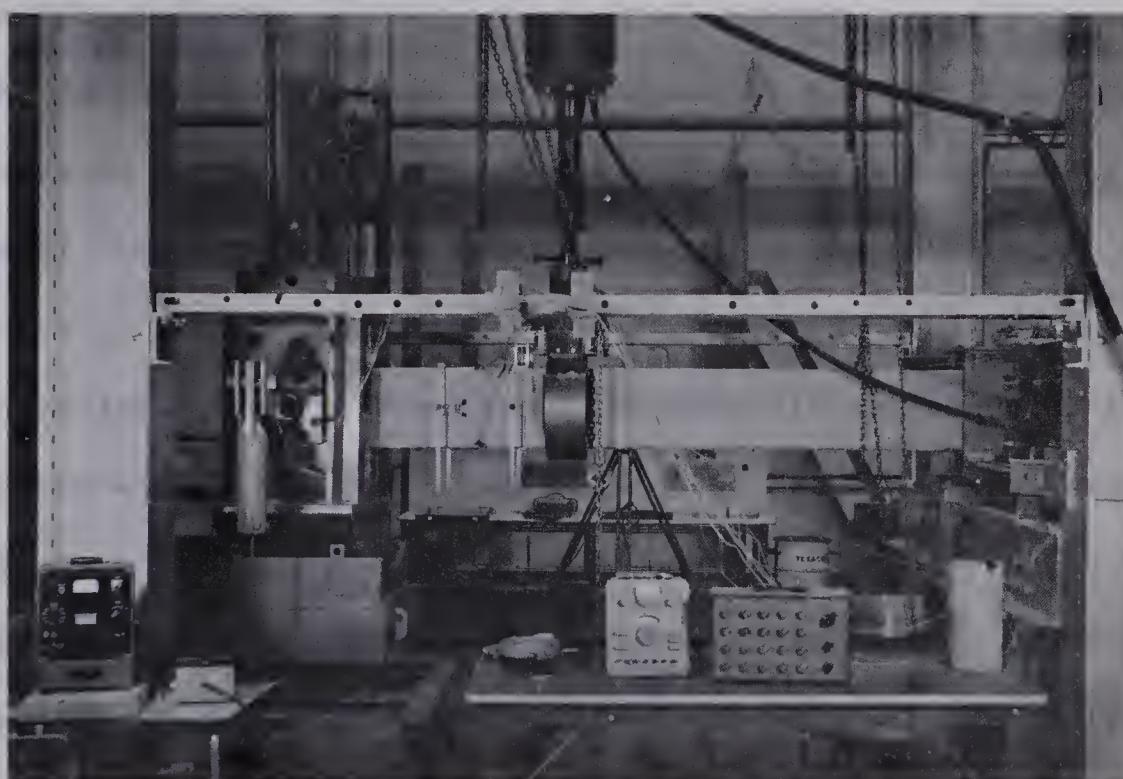


FIGURE 3 - 6 TEST SETUP FOR COMBINED  
BENDING, TORSION AND SHEAR



was clamped inside the pipe collars. They permitted the twisting moment to be transmitted freely from one end of the specimen to the other. For those specimens subjected to a single transverse load, the distributing beam was not used; the transverse load was transferred directly to the roller assembly. The position of the single transverse load is shown in FIGURE 3-1.

The east end of the specimen was supported by the twisting head through which the torsional moment was applied. The twisting head is illustrated by FIGURES 3-7, 3-8 and 3-9. It permitted the specimen to rotate about its longitudinal axis and about both a horizontal and vertical axis perpendicular to the axis of the specimen. The forces in the cables which were attached to the arms of the twisting head were measured by load cells and an indicator. These had been calibrated using a 200 kip universal testing machine.

The forces in the cables were produced by the torsional loading system illustrated in FIGURES 3-10 and 3-11. The cables attached to the torsion arms of the twisting head were connected to the ends of the cross head by adjustable connecting devices. Each end of the cross head was provided with a roller system to ensure that binding did not occur as the cross head moved within the guides provided. The cross head was loaded at its center by two 20 kip hydraulic jacks coupled in series. The hydraulic pressure was generated by a manual pump. The purpose of coupling the jacks in series was to increase the stroke to 16", which corresponds to a rotation of the twisting head of approximately  $26^{\circ}$ . This is slightly



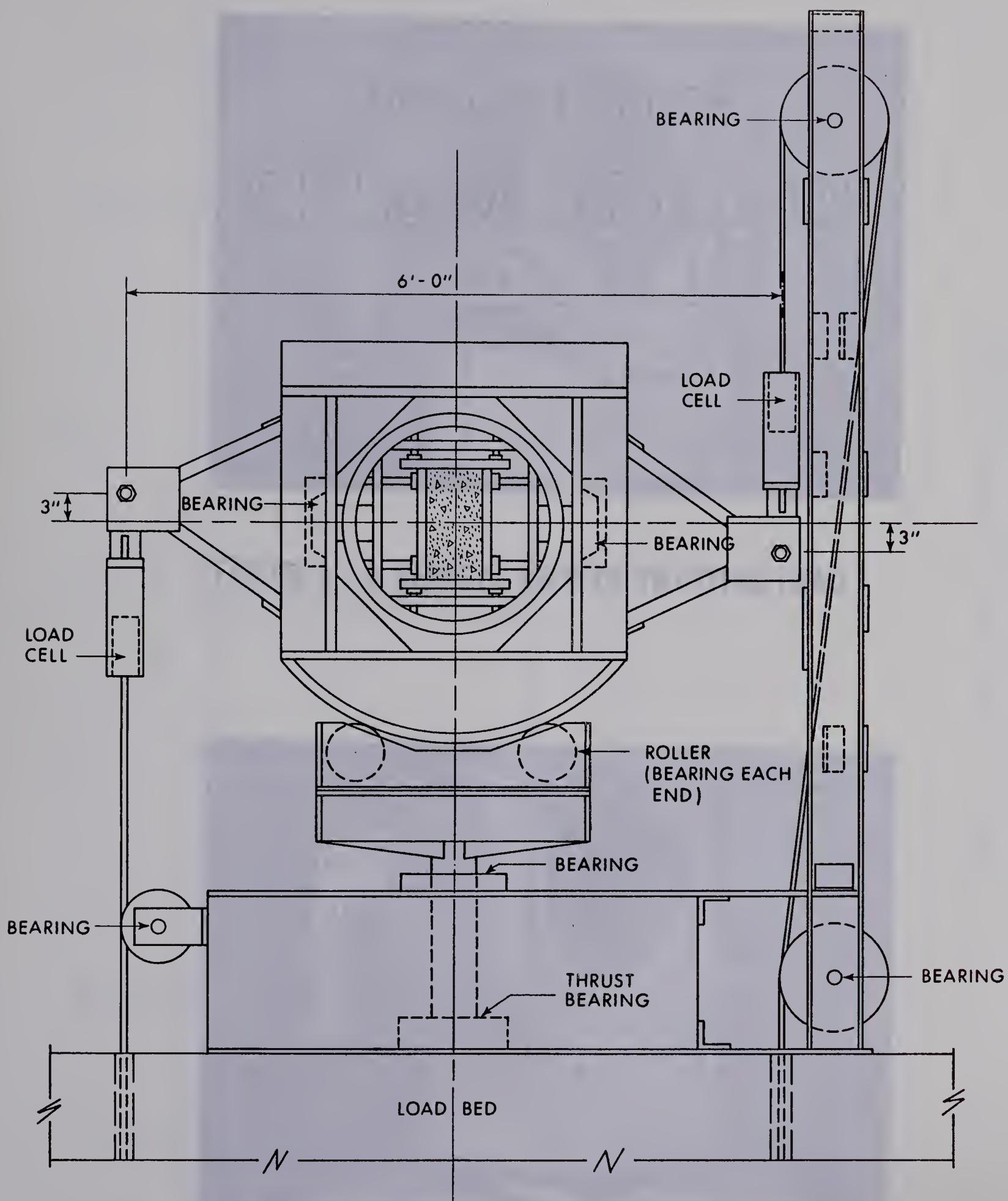


FIGURE 3 - 7 TWISTING HEAD

FIGURE 3 - 8 VARIABLE TWISTING HEAD SHOWN  
ROTATION ABOUT THE SPANNING





FIGURE 3 - 8 GENERAL VIEW OF TWISTING HEAD

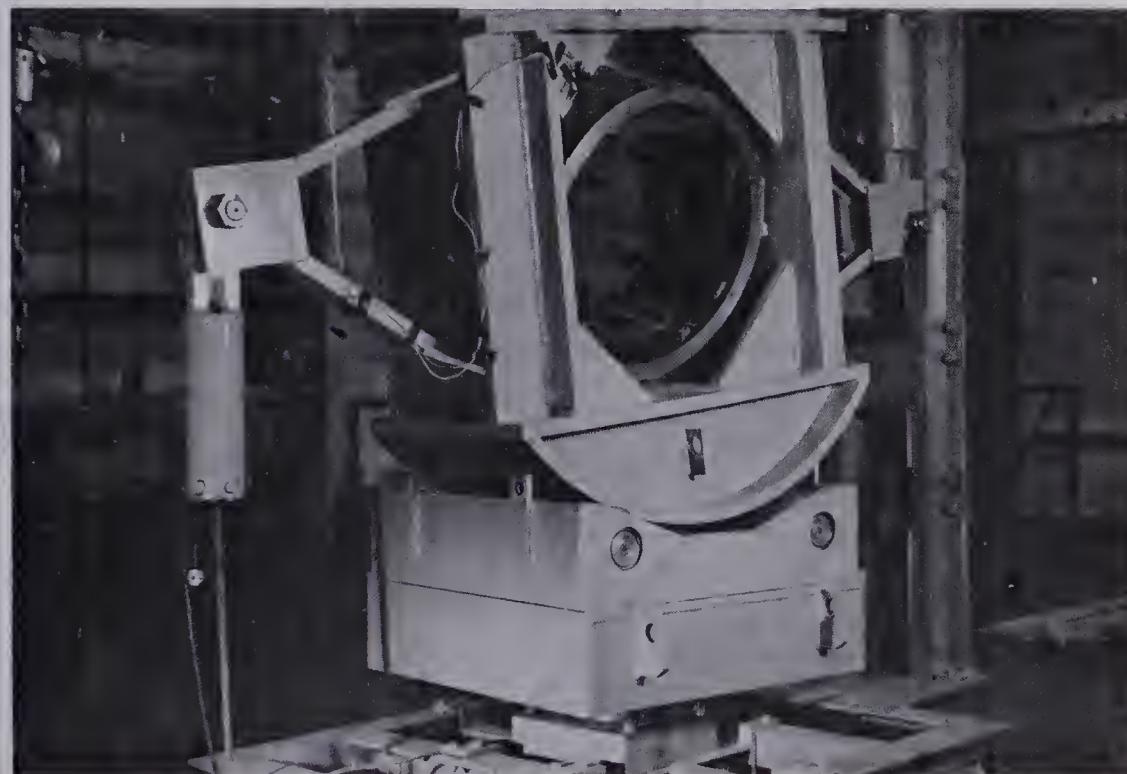


FIGURE 3 - 9 VIEW OF TWISTING HEAD SHOWING  
ROTATION ABOUT THREE AXES



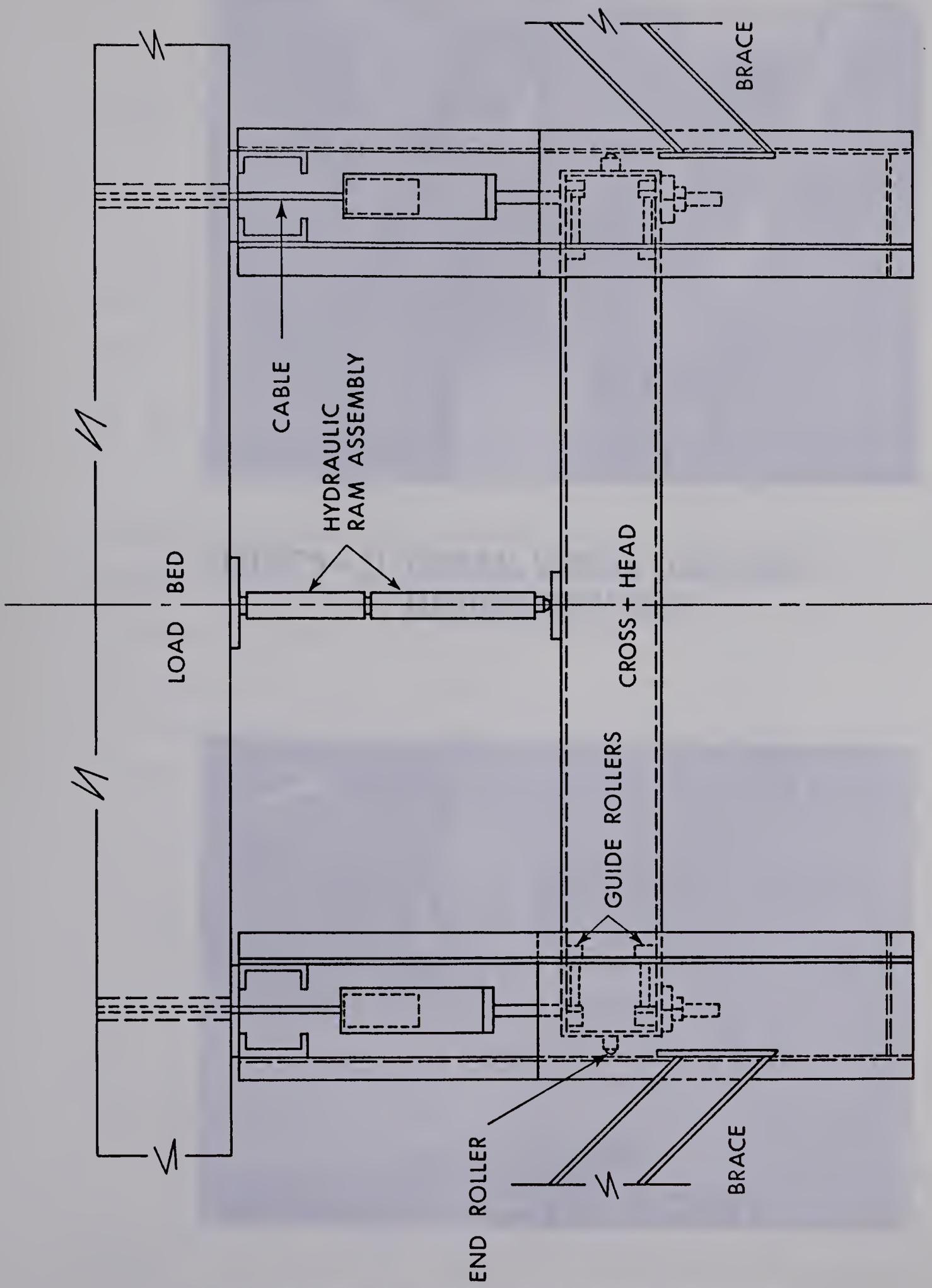


FIGURE 3 - 10 TORSIONAL LOADING EQUIPMENT





FIGURE 3 - 11 GENERAL VIEW OF TORSIONAL  
LOADING EQUIPMENT

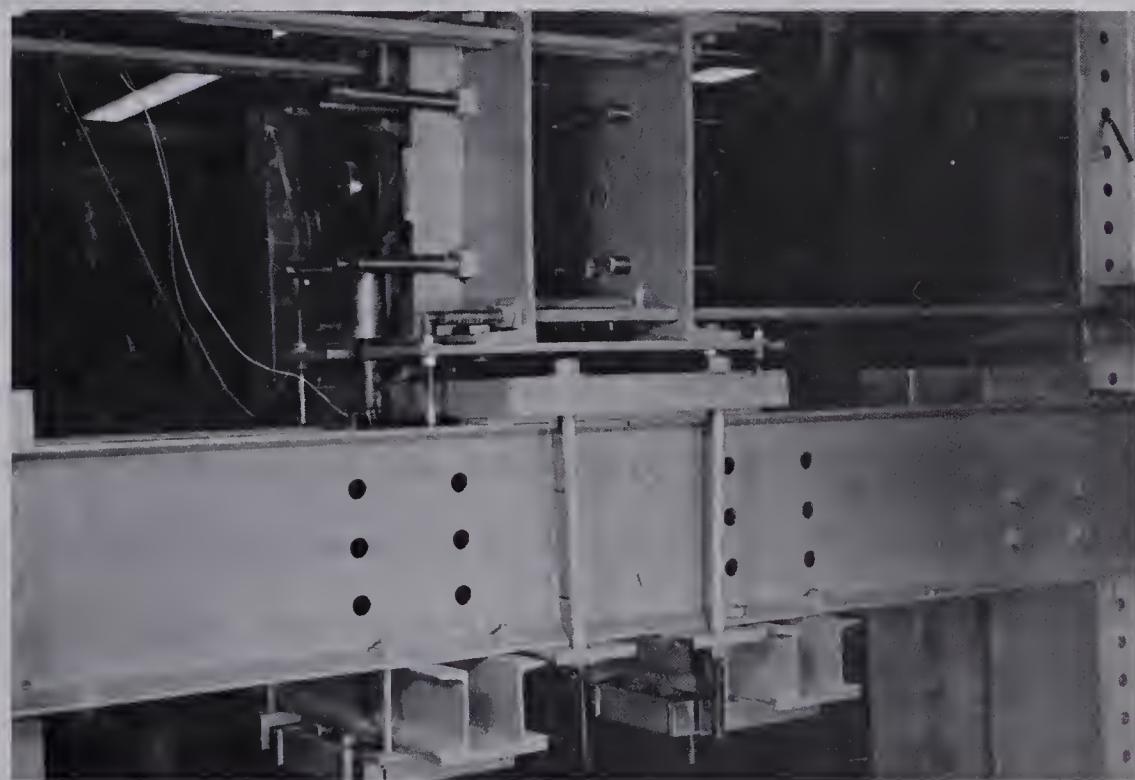


FIGURE 3 - 12 GENERAL VIEW OF FIXED HEAD



greater than the maximum rotation of which the twisting head is capable.

The west end of the specimen was supported by the fixed head which is illustrated by FIGURES 3-12 and 3-13. This fixed head permitted the specimen to translate in a direction parallel to its longitudinal axis, and to rotate about a horizontal axis perpendicular to its longitudinal axis.

#### 3-4 Testing Procedure

Each specimen was tested to failure by applying the load in a series of increments. Each increment consisted of increasing to a predetermined level the transverse load or the twisting load or both, depending on the type of test. In those cases where both types of load were applied in the same increment, the transverse load was applied first. In the advanced stages of a test, the loads were generally applied in fractions of an increment. If failure did not occur during or after the application of a fraction of an increment, a further fraction was applied until failure occurred or a full increment was reached. After each full increment of load was reached, the transverse and twisting loads were held constant and recorded. Readings of the electrical resistance strain gauges, the twistmeters and the deflection gauges were also recorded and the crack patterns were marked. In all instances load application was continued well beyond the stage at which peak values were observed.

The first transverse load applied to a specimen was the load due to the testing equipment, i.e. the distributing beam and roller assemblies. Included in this load for the computation of the bending moment



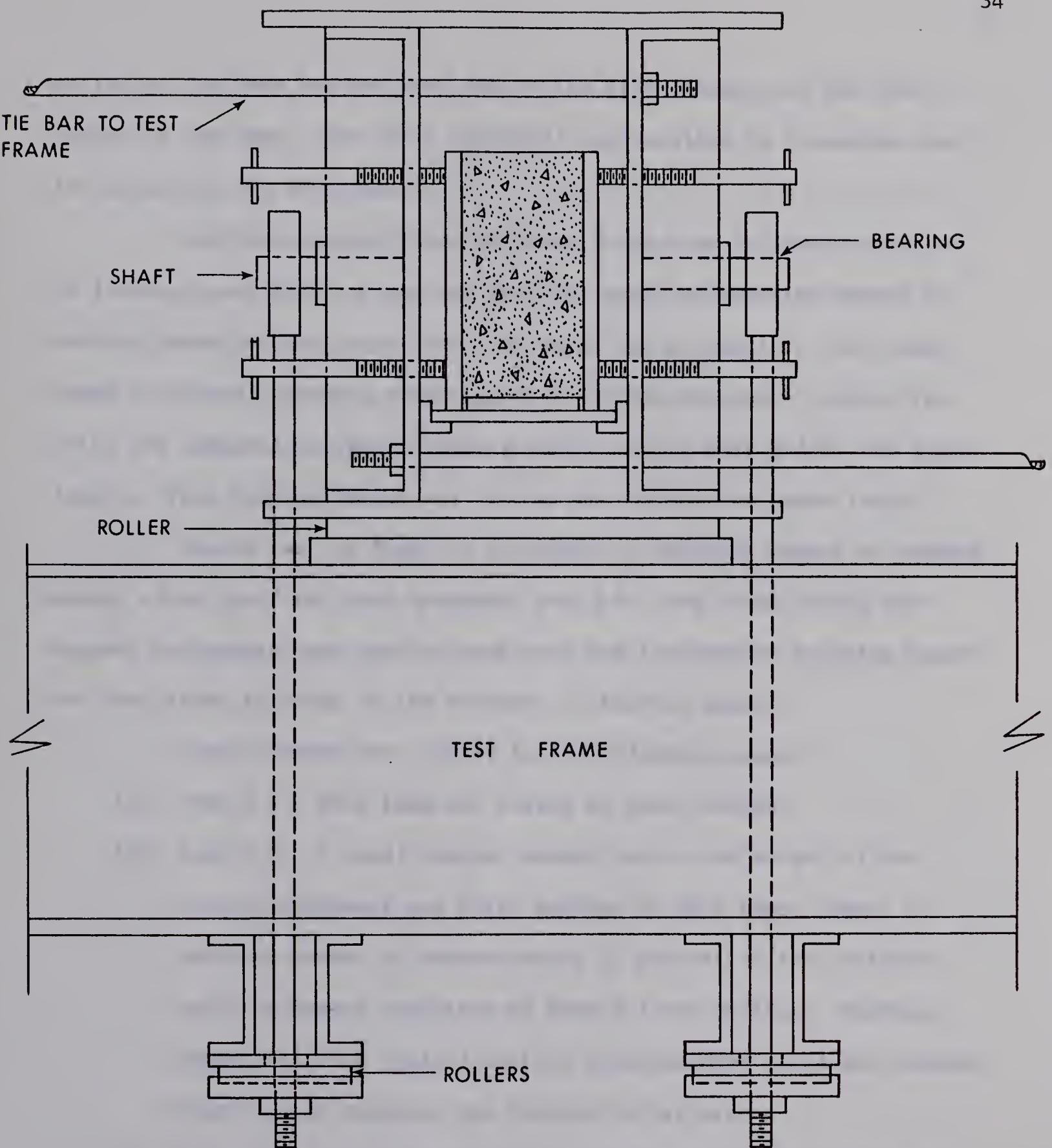


FIGURE 3 - 13 FIXED HEAD



acting on the beam was the load due to the pipe collars and the dead weight of the beam. The first torsional load applied to a specimen was the weight of the cross head.

All beams except those of Group 3 and Beam 1-2 were subjected to loadings such that for any one test the ratio of twisting moment to bending moment at the end of each increment was a constant. For those beams which had a bending moment gradient within the gauge length, this ratio was computed using the bending moment at the west end of the gauge length. This bending moment was the largest within the gauge length.

In the test of Beam 1-2 the ratio of twisting moment to bending moment after the first load increment was 2.0. Due to an error, subsequent increments were applied such that the increase in twisting moment was four times as large as the increase in bending moment.

Group 3 beams were tested in the following manner:

- (a) Beam 3-1: This beam was tested in pure torsion.
- (b) Beam 3-2: A small bending moment due to the weight of the testing equipment was first applied to this beam. Next, a twisting moment of approximately 75 percent of the ultimate twisting moment exhibited by Beam 3-1 was applied. Bending moment was then applied until a predetermined level was reached. Finally, the specimen was twisted to failure.
- (c) Beam 3-3: This beam was first subjected to transverse load until the bending moment was equal to the maximum bending moment sustained by Beam 3-2. It was then twisted to failure.



(d) Beam 3-4: The bending moment due to the weight of the testing equipment was first applied. Next, a twisting moment of approximately 75 percent of the ultimate twisting moment exhibited by Beam 3-1 was applied. Then transverse load was applied until the bending moment was equal to approximately one-half of the ultimate bending moment of Beams 3-2 and 3-3. The twisting moment was then increased to the ultimate value of twisting moment exhibited by Beam 3-1. Finally, the specimen was bent to failure.

(e) Beam 3-5: The bending moment due to the weight of the testing equipment was first applied. The beam was then twisted to a predetermined level. Finally, the specimen was bent to failure.



## CHAPTER IV

### TEST RESULTS

#### 4-1 Introduction

The principal test results and the general behavior of the beams are presented in this chapter. More detailed results, which include readings at the end of each load increment and plots of twisting moment versus angle of twist, deflection, and reinforcement stress for each beam, are presented in Appendix B.

#### 4-2 Principal Test Results

The principal test results, which include the material and geometrical properties of the beams and the maximum moments and forces sustained by them, are presented in TABLE 4-1. The maximum recorded twist is generally the angle of twist recorded at the end of the load increment immediately preceding failure. For those beams tested in combined bending, torsion and shear (Groups 5, 6, and 7) the bending moment listed is the maximum bending moment within the gauge length and occurred at the west end of the gauge length. The ratio of twisting moment to bending moment at which the tests were conducted is denoted as  $\emptyset$  , and is discussed in SECTION 4-3(d). The value of  $\emptyset$  listed in TABLE 4-1 for any beam is the value which existed at failure. For those beams which failed during application of an increment of load, the value of  $\emptyset$  listed in TABLE 4-1



TABLE 4-1

## PRINCIPAL TEST RESULTS

| Beam | Concrete Strength<br>Compressive*<br>p.s.i. | Concrete Strength<br>Tensile*<br>p.s.i. | Beam<br>Dimensions<br>Width<br>in. | Beam<br>Dimensions<br>Depth<br>in. | Max<br>Bending<br>Moment<br>in.kips | Max<br>Twisting<br>Moment<br>in.kips | Max<br>Recorded<br>Twist<br>Radians/in.<br>$\times 10^6$ | $\theta = \frac{\text{Max. Tw. Mom.}}{\text{Max. B. Mom.}}$ | Max<br>Shear<br>Force<br>kips |
|------|---|---|------------------------------------|------------------------------------|-------------------------------------|--------------------------------------|--|---|-------------------------------|
| 1-1  | 5190  | 495                                     | 6.19                               | 12.13                              | 0                                   | 122                                  | 1233   | $\infty$  | --                            |
| 1-2  | 4440  | 398                                     | 6.38                               | 12.13                              | 47                                  | 138                                  | 1435   | 2.94  | --                            |
| 1-3  | 5060  | 416                                     | 6.00                               | 12.13                              | 70                                  | 140                                  | 1730   | 2.00  | --                            |
| 1-4  | 4970  | 440                                     | 6.38                               | 12.13                              | 159                                 | 159                                  | 1258   | 1.00  | --                            |
| 1-5  | 4820  | 433                                     | 6.00                               | 12.13                              | 267                                 | 131                                  | 1210   | 0.49  | --                            |
| 1-6  | 5540  | 522                                     | 6.00                               | 12.13                              | 362                                 | 90                                   | 315  | 0.25  | --                            |
| 2-1  | 5750  | 460                                     | 6.38                               | 12.13                              | 0                                   | 181                                  | 1483   | $\infty$  | --                            |
| 2-2  | 5020  | 327                                     | 6.38                               | 12.13                              | 88                                  | 172                                  | 1593   | 1.96  | --                            |
| 2-3  | 5500  | 480                                     | 6.00                               | 12.13                              | 166                                 | 166                                  | 1850   | 1.00  | --                            |
| 2-4  | 5220  | 495                                     | 6.25                               | 12.13                              | 267                                 | 134                                  | 875  | 0.50  | --                            |
| 2-5  | 5310  | 433                                     | 6.38                               | 12.13                              | 362                                 | 90                                   | 335  | 0.25  | --                            |
| 3-1  | 5330  | 462                                     | 6.25                               | 12.13                              | 0                                   | 115                                  | 1440   | $\infty$  | --                            |
| 3-2  | 5330  | 515                                     | 6.38                               | 12.13                              | 111                                 | 117                                  | 1733   | 1.05  | --                            |
| 3-3  | 5460**                                      | -                                       | 6.38                               | 12.13                              | 111                                 | 120                                  | 1430   | 1.08  | --                            |
| 3-4  | 5460**                                      | -                                       | 6.25                               | 12.13                              | 188                                 | 115                                  | 1253   | 0.61  | --                            |
| 3-5  | 5820  | 497                                     | 6.38                               | 12.13                              | 265                                 | 73                                   | 738  | 0.28  | --                            |
| 4-1  | 5380  | 504                                     | 6.25                               | 12.13                              | 200                                 | 120                                  | 818  | 0.60  | --                            |
| 4-2  | 5700  | 442                                     | 6.38                               | 12.13                              | 166                                 | 101                                  | 935  | 0.61  | --                            |
| 4-3  | 5850  | 513                                     | 6.00                               | 12.13                              | 153                                 | 93                                   | 1094   | 0.61  | --                            |
| 4-4  | 5670  | 475                                     | 6.25                               | 12.13                              | 146                                 | 85                                   | 1019   | 0.58  | --                            |
| 4-5  | 5570  | 433                                     | 6.38                               | 12.13                              | 212                                 | 103                                  | 717  | 0.49  | --                            |
| 4-6  | 6280  | 477                                     | 6.00                               | 12.13                              | 132                                 | 66                                   | 567  | 0.50  | --                            |
| 5-1  | 5710  | 446                                     | 6.25                               | 12.13                              | 65                                  | 128                                  | 1528   | 1.97  | 0.76                          |
| 5-2  | 6370  | 428                                     | 6.38                               | 12.13                              | 143                                 | 141                                  | 1746   | 0.99  | 1.89                          |
| 5-3  | 6060  | 418                                     | 6.00                               | 12.13                              | 278                                 | 130                                  | 1390   | 0.47  | 3.87                          |
| 5-4  | 5800  | 409                                     | 6.25                               | 12.13                              | 389                                 | 99                                   | 1815   | 0.25  | 5.51                          |
| 6-1  | 5860  | 459                                     | 6.25                               | 12.13                              | 65                                  | 129                                  | 1397   | 1.98  | 1.81                          |
| 6-2  | 5930  | 424                                     | 5.94                               | 12.13                              | 149                                 | 145                                  | 1821   | 0.97  | 4.27                          |
| 6-3  | 5700  | 413                                     | 6.25                               | 12.13                              | 264                                 | 132                                  | 1459   | 0.50  | 7.67                          |
| 6-4  | 5720  | 504                                     | 6.38                               | 12.13                              | 427                                 | 107                                  | 1006   | 0.25  | 12.45                         |
| 7-1  | 6080  | 482                                     | 5.94                               | 12.13                              | 56                                  | 112                                  | 1592   | 2.00  | 1.56                          |
| 7-2  | 5210  | 438                                     | 6.25                               | 12.13                              | 115                                 | 115                                  | 955  | 1.00  | 3.32                          |
| 7-3  | 5700  | 382                                     | 6.38                               | 12.13                              | 275                                 | 132                                  | 1504   | 0.48  | 8.11                          |
| 7-4  | 5340  | 447                                     | 5.94                               | 12.13                              | 505                                 | 125                                  | 870  | 0.25  | 14.97                         |

\*Average value from two 6 x 12 inch cylinders

\*\*Compressive strength of Beams 3-3 and 3-4 is based on one 6 x 12 inch cylinder



is lower than that discussed in SECTION 4-3(d).

#### 4-3 Behavior

##### (a) General Behavior Prior to Failure

The twisting moment versus angle of twist relationship for each beam is presented in Appendix B. For those beams tested in pure torsion, the initial stage of this relationship was linear. However, after the level of twisting moment at which cracks in the concrete were observed was reached, this relationship became curved. The length of the linear portion of this relationship decreased for those beams tested in combined bending and torsion and the complete relationship was curved for those beams tested with a value of  $\phi = 0.25$ .

The twisting moment-deflection relationship for each beam is presented in Appendix B. This relationship varied considerably and was found to be dependent on the value of  $\phi$  and on the amount of top longitudinal reinforcement relative to the amount of bottom longitudinal reinforcement. Those beams which had equal top and bottom reinforcement and which were tested with a high value of  $\phi$  exhibited small deflections. For purposes of this discussion a high value of  $\phi$  is regarded as 2.0 or greater and a low value as 0.5 or less. As the value of  $\phi$  at which these beams were tested was decreased, the deflection of the beams at any particular stage of the test became progressively larger. The beams that were tested at a high value of  $\phi$  and provided with less top longitudinal reinforcement than bottom exhibited negative or upward deflections in the latter stages of the test. As the value of  $\phi$  at which these beams were tested was de-



creased the deflection at any particular stage of the test became less negative and subsequently became positive. At  $\emptyset$  equal to 0.25 this type of beam exhibited a positive deflection of the same order of magnitude as exhibited by the type of beam which had equal top and bottom reinforcement and which was tested at  $\emptyset$  equal to 0.25. The beams that exhibited negative deflection during the latter stages of the test usually had zero or a small positive deflection up to the level of twisting moment at which cracking occurred. After cracking occurred the deflection became negative.

Neither the quantity nor the distribution of the reinforcement affected the appearance and initial development of cracks. However, the value of  $\emptyset$  at which the test was conducted affected them considerably. For those beams tested at a high value of  $\emptyset$ , the first cracks appeared at mid depth of the vertical faces and were inclined at approximately  $45^{\circ}$  to the axis of the beam. Further loading extended these cracks to the top and bottom faces. These cracks, where they first appeared on the top and bottom faces, made an angle of approximately  $90^{\circ}$  with the axis of the beam. However, as they progressed they became inclined to the axis of the beam at approximately  $45^{\circ}$ . All faces exhibited numerous cracks at twisting moments considerably less than the ultimate values. The effect of reducing the value of  $\emptyset$  was to delay the appearance of cracks on the top face and to cause the cracks on the vertical faces to progress downward more rapidly than upward. Nevertheless, cracks generally appeared on the top face within one or two increments of first cracking and at twisting moments considerably less than the ultimate.



First cracking of the beams tested at a low value of  $\emptyset$  generally occurred on the bottom face and the lower portion of the vertical faces. On the bottom face these cracks were nearly perpendicular to the axis of the beam and on the vertical faces they were nearly vertical. As further loading was applied to the beam, these cracks at first progressed vertically up the vertical faces but subsequently the extensions became inclined. At the same time, inclined cracks formed on the vertical faces. They were either new cracks or cracks which started at and progressed downward from the vertical cracks. Inclined cracks were also observed on the bottom face at this time. All faces of the beams were cracked at twisting moments less than the ultimate values.

FIGURE 4-1 illustrates the effect of the amount of reinforcement in a beam on the crack pattern. Beams with large amounts of reinforcement exhibited more numerous but narrower cracks than those with small amounts of reinforcement.

The sequence of development of cracks on several beams tested with a single transverse load and at a low value of  $\emptyset$  was somewhat different than that described above and is illustrated by FIGURES 4-2 and 4-3. The first cracks observed on these beams were inclined cracks that occurred on the south face, i.e. the face on which the diagonal tension stresses due to transverse shear and torsional shear were additive. Further loading caused new inclined cracks to appear on the south face and the existing ones to extend toward the top and bottom faces. Inclined cracks on the top and bottom faces also appeared at this time. The north face was either



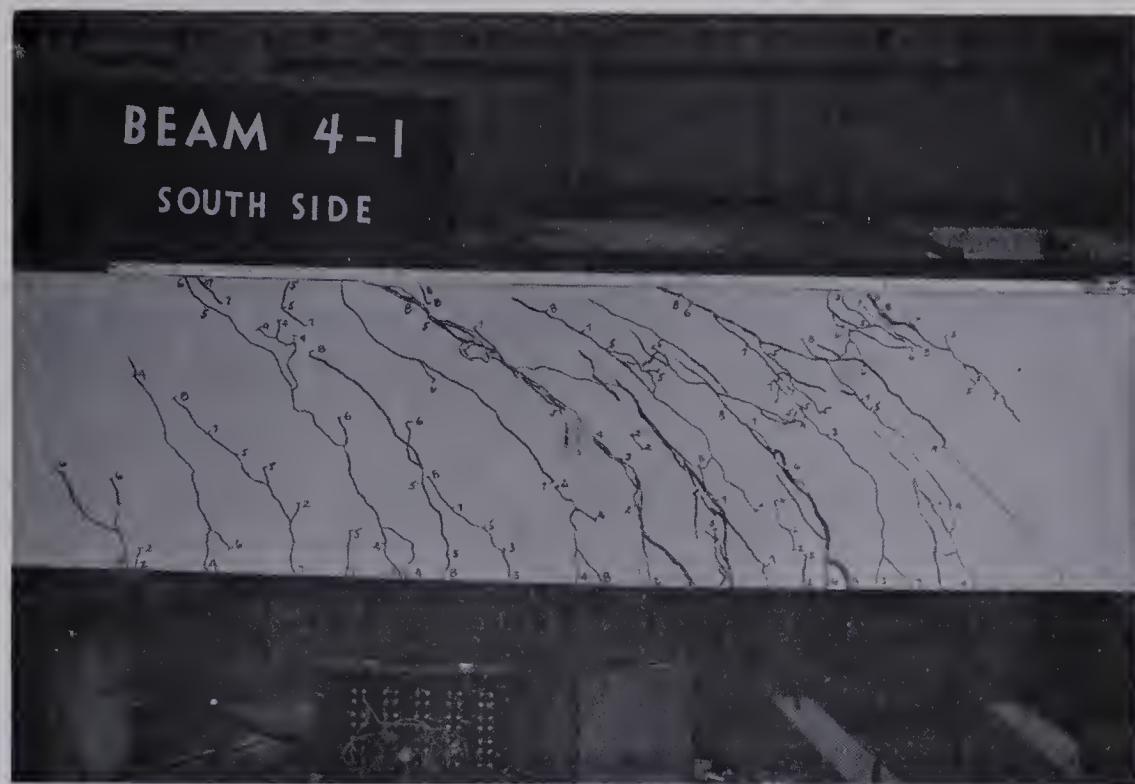


FIGURE 4 - 1 CRACK PATTERN OF BEAMS 4 - 1 AND 4 - 4



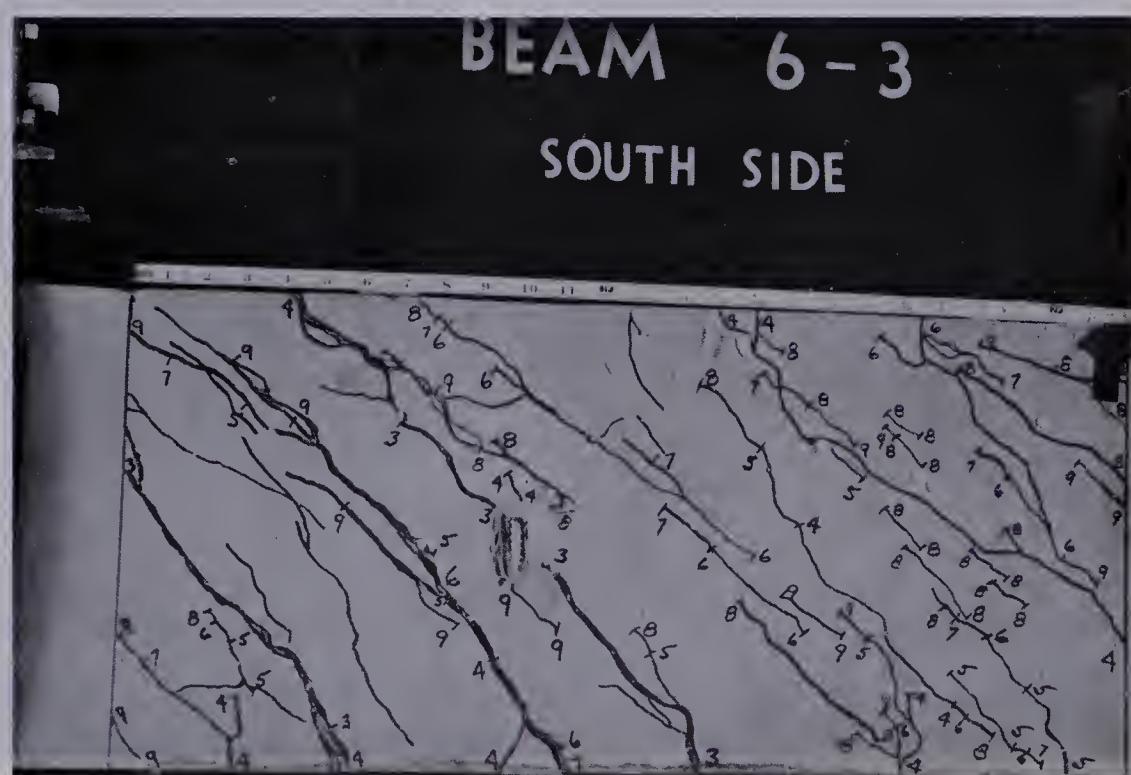


FIGURE 4 - 2 CRACK PATTERN OF BEAM 6 - 3



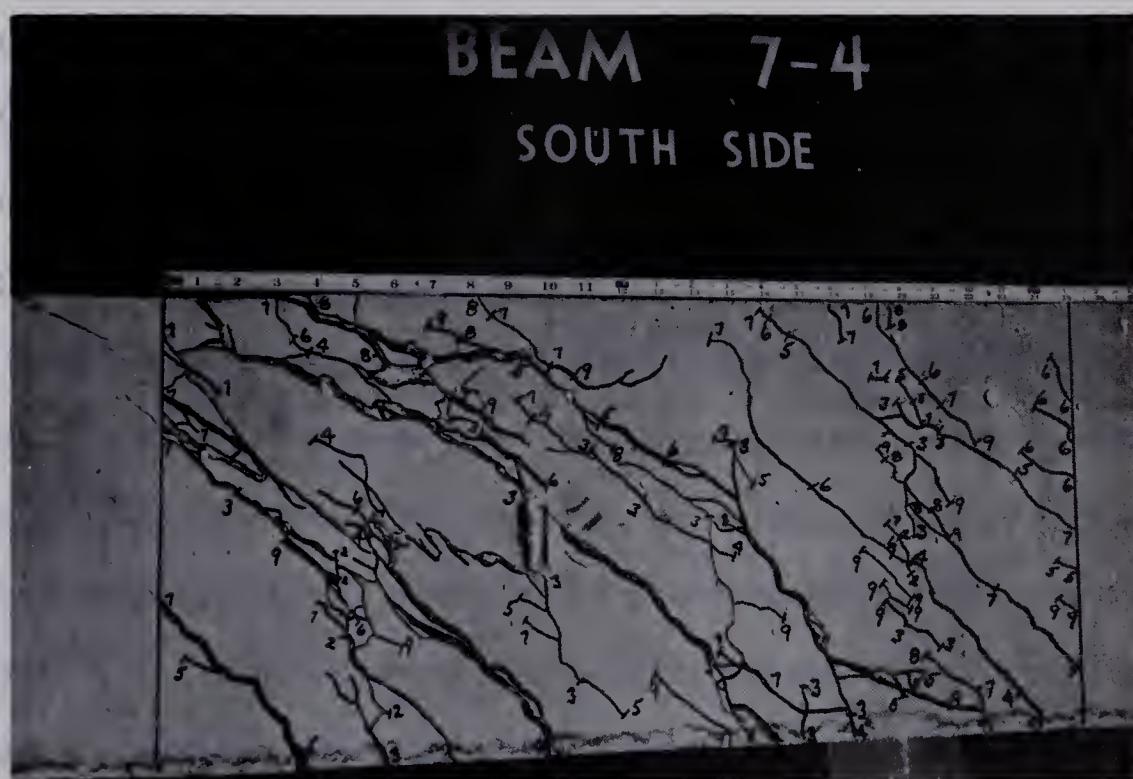
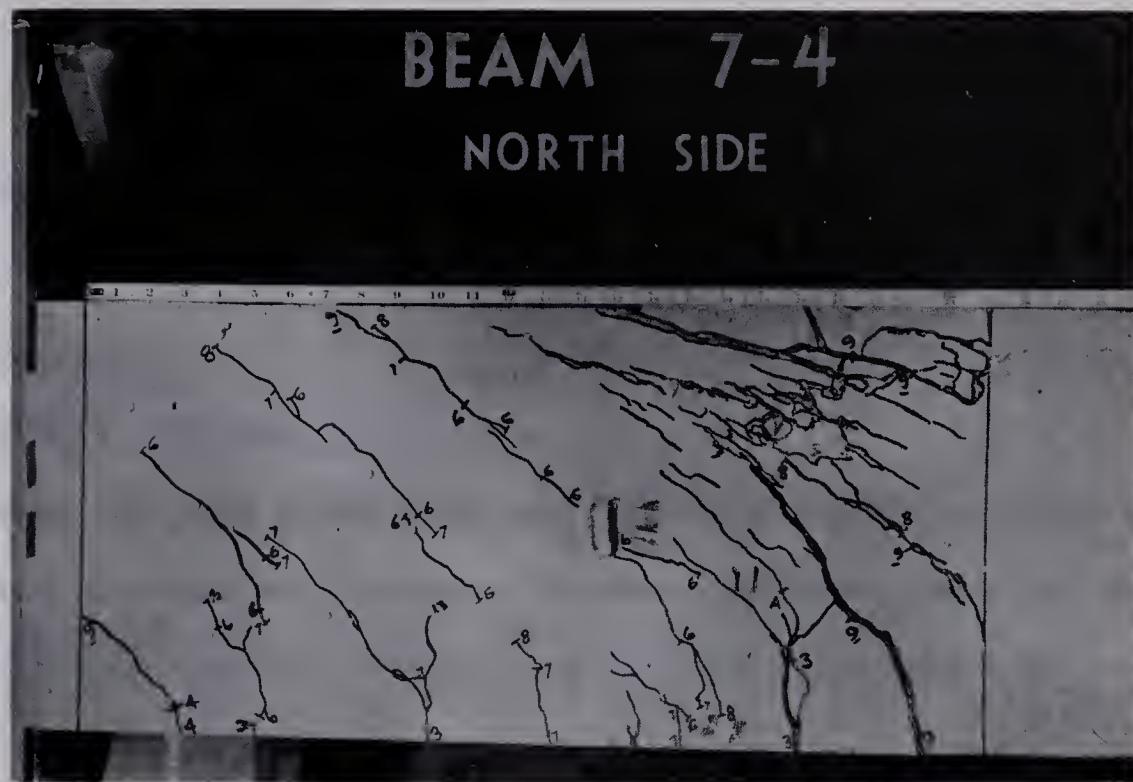


FIGURE 4 - 3 CRACK PATTERN OF BEAM 7 - 4



uncracked or had a few small vertical or inclined cracks. Subsequent increments of load produced inclined cracks on the north face as well as the other faces.

(b) General Behavior at Failure

When the beam could no longer sustain the loads to which it was subjected or could not sustain an increase in these loads, it was considered to have failed. The forces acting on it at this time were considered to be the failure forces.

In all tests, application of load was carried well beyond observance of peak values of twisting load and transverse load. Most beams failed while the twisting moment portion of the load increment was being applied. However, several failed during application of the bending portion and several failed after application of a full load increment and while readings were being taken.

At failure there was a large increase in the angle of twist and, if the level of transverse load was high, in the vertical deflection. At this time the transverse load and the twisting load decreased noticeably. Due to the load maintaining feature of the transverse loading equipment, the previous level of transverse load was quickly regained, but further application of twist in an effort to reach the previous level of twisting load resulted only in further rotation of the beam with little or no corresponding increase in load. However, when twisting was stopped and the beam was allowed to stabilize, the level of twisting load was found to be 80 to 90 percent of the peak value, with the transverse load at its peak value.



However, the magnitude of the twisting load slowly decreased with time. Further application of twist in an effort to increase the twisting load resulted in a temporary increase in twisting load accompanied by a large rotation of the beam. All beams tested exhibited appreciable ductility.

The large rotation which was observed at failure and also upon unloading of the specimen took place about an axis located in different regions of the beam cross section in different beams. The beams that were subjected to a low value of  $\theta$  at failure rotated about an axis in the vicinity of the top reinforcement. The beams that were subjected to high values of  $\theta$  and were provided with less top longitudinal reinforcement than bottom rotated about an axis located in the vicinity of the bottom reinforcement. The location of the axis of rotation in beams tested with an intermediate value of  $\theta$  was less obvious but could be deduced from the shape and width of the failure cracks. Several specimens appeared to have two failure planes, each involving an axis of rotation on a different face of the beam. This is discussed further in SECTION 4-3(d).

Failure of a beam was accompanied by a large rotation and a widening of cracks on the faces of the beam other than the one adjacent to the axis of rotation. In most cases the cracks which widened at failure and defined the failure plane were cracks which had first appeared at moments considerably lower than the ultimate moments. These cracks were essentially a continuous crack which spiralled around the three faces of the beam. However, failure of some specimens was accompanied by the formation of new cracks that joined two sets of previously existing spiral



cracks. In addition, some beams appeared to have a failure plane defined by parts of two sets of spiral cracks that, on the surface of the beam, were not joined. FIGURE 4-4 shows the crack pattern of Beam 5-3 and illustrates this type of failure plane. It appeared that the failure plane in this beam was defined on the north face of the beam by a crack from one set of spiral cracks, on the south face of the beam by a crack from another set, and on the bottom face by a combination of two cracks. This type of failure surface appeared most often in those beams tested with a bending moment gradient within the gauge length. The failure surface appeared to be crowded to one end of the gauge length.

Many of the beams that were tested with low or high values of  $\phi$  and had an axis of rotation adjacent to the top or bottom surface of the beam developed a "Z" shaped crack at failure. This crack occurred on the face adjacent to the axis of rotation and connected the failure cracks appearing on the vertical faces. It generally did not form on those specimens tested with an intermediate value of  $\phi$ , or on those specimens in which the axis of rotation appeared to be located adjacent to one of the vertical faces.

(c) Reinforcement Strains

Strains in the reinforcement were small until cracking of the concrete occurred. After cracking occurred, the strains increased at a greater rate and were observed to be dependent on the amount of top longitudinal reinforcement and on the value of  $\phi$ . The beam that was tested in pure torsion and was provided with equal top and bottom reinforcement exhibited tensile strains in the longitudinal reinforcement that



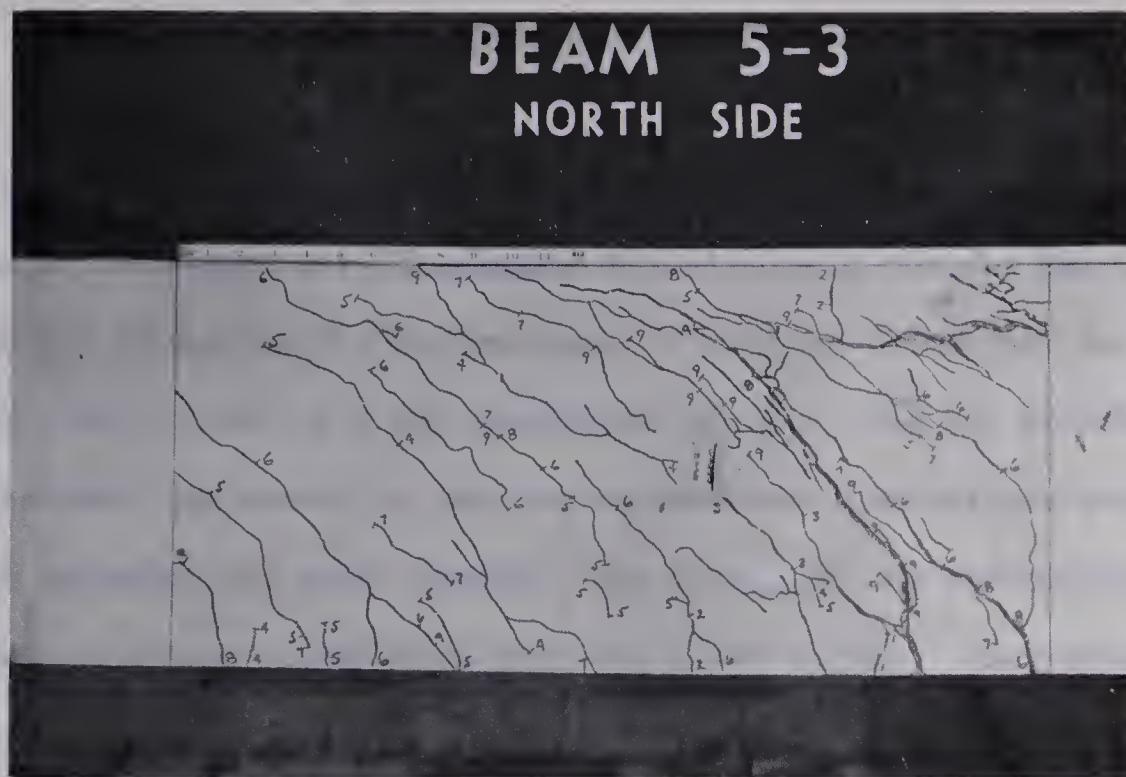


FIGURE 4 - 4 CRACK PATTERN OF BEAM 5 - 3



were nearly the same in all bars. The effect of decreasing the value of  $\phi$  for beams of this type was to decrease the strain in the top bars and to increase the strain in the bottom bars. For those beams that were tested with a high value of  $\phi$  and were provided with less top reinforcement than bottom, the strain in the top reinforcement at failure was tensile and exceeded the yield strain. The strain in the bottom reinforcement was tensile but was less than the yield strain. The effect of decreasing the value of  $\phi$  was to increase the tensile strain that existed in the bottom reinforcement when the beam failed. When beams of this type were tested at low values of  $\phi$ , the strain in the top reinforcement at failure was compressive whereas the strain in the bottom reinforcement was tensile and exceeded the yield strain.

In many cases the failure plane did not cross the tie which was instrumented and when it did cross this tie, it intersected only one leg of it. Consequently, it was difficult to establish if the ties intersecting the failure plane yielded at failure. One further difficulty was that the equipment used for reading the strain was not of the continuous recording type. As a result, no strain readings could be obtained when failure occurred during application of the load. This was not a serious disadvantage with regard to the longitudinal reinforcement since either the top or bottom reinforcement usually yielded several load increments before failure occurred. However, this was not so for the ties, particularly with regard to the top and bottom legs. In order to obtain further information, one gauge was constantly observed during load application in the latter stages of the



test. It was noted that if the failure plane crossed the leg of the tie which was being observed, this gauge indicated a strain larger than the yield strain at the time the moments were at their peak values. In these cases the strain in this particular leg of the tie at the end of the load increment immediately preceding failure was often considerably less than the yield strain but immediately before failure occurred, the strain began to increase rapidly. In an attempt to obtain additional information, readings were taken in many of the tests after failure had occurred and while the beam was still supporting loads less than the ultimate values. The strains in the ties are discussed further in SECTION 4-3(d).

(d) Detailed Observations

(i) Group 1

The beams of this group had less longitudinal reinforcement at the top than at the bottom. Except for Beam 1-2 they were tested such that for each beam the value of  $\phi$  at the end of each full increment of the load was a constant. Within the gauge length the twisting moment and the bending moment were constant.

Beam 1-1: This beam was tested in pure torsion ( $\phi = \infty$ ). At failure, rotation occurred about an axis near the bottom face. The failure plane, which was well defined, did not cross the tie which was instrumented. The negative deflection of the beam at failure was pronounced.

Beam 1-2: At the end of the first increment of load, the twisting moment was twice the bending moment. For all subsequent load increments the increase in twisting moment applied to the beam was four times as large as



the increase in bending moment. Consequently, the value of  $\emptyset$  varied from 2.0 at the end of the first increment to 2.9 at failure. The first cracks on this specimen, which were inclined cracks on both vertical faces, were observed after the first load increment. At failure, which occurred while the twistmeters were being read, the beam rotated about an axis located near the bottom face of the beam. The failure plane did not intersect the tie which was instrumented.

Beam 1-3: The value of  $\emptyset$  at which this specimen was tested was 2.0. Failure occurred while the strain gauges were being read. The hinge, or axis of rotation at failure, was located near the bottom of the beam.

Beam 1-4: This beam was tested at  $\emptyset$  equal to 1.0 and failed immediately after the twisting portion of the last increment was applied. The development of the failure plane indicated that the hinge was located adjacent to the north face. However, the beam had a second failure plane which was fairly well developed. This second failure plane indicated that a hinge had begun to form in the vicinity of the top face. On the top face there was a fairly well defined "Z" shaped crack. Strain readings taken after failure but while the beam was still sustaining load indicated strains larger than yield strains at gauges 2, 3, 4, 5, 7 and 8. The locations of these gauges have been detailed in CHAPTER III.

Beam 1-5: Testing of this beam was carried out with  $\emptyset$  equal to 0.5. The axis of rotation at failure, which occurred while torsion was being applied, was located near the top surface of the beam. Considerable spalling of the concrete occurred in the region of the top north corner. During appli-



cation of the last increment of load, the strain indicated by gauge 5 was observed continually. While the last increment of twisting was being applied the strain steadily increased and when failure occurred the strain was increasing very rapidly. Readings taken subsequent to failure indicated strains in excess of yield strains at gauges 3, 4, 5, 6, and 8.

Beam 1-6: For this beam, which was tested at  $\emptyset$  equal to 0.25, the axis of rotation at failure was near the top face of the beam. Failure occurred during reading of the strain gauges and considerable spalling of the concrete on the top surface of the beam was observed. The peak values of moments were held throughout a considerable rotation of the beam but eventually could not be maintained as the rate of rotation and deflection began to increase rapidly. During this rotation, the strain indicated by gauge 5 was observed and was found to be increasing very rapidly. Subsequent to failure, the readings of gauges 3, 4, 5 and 6 indicated tensile strains larger than the yield strains; gauge 1 indicated compressive strain larger than the yield strain. At failure, numerous cracks that had not been evident previously appeared and joined cracks that had existed previously. The failure plane was defined by a combination of these new cracks and cracks that had existed before failure. The crack pattern of this beam is shown in FIGURE 4-5.

(ii) Group 2

This group of beams was subjected to uniform bending and twisting moments and for each beam  $\emptyset$  was a constant at the end of any increment of load. The beams were provided with top longitudinal reinforcement that



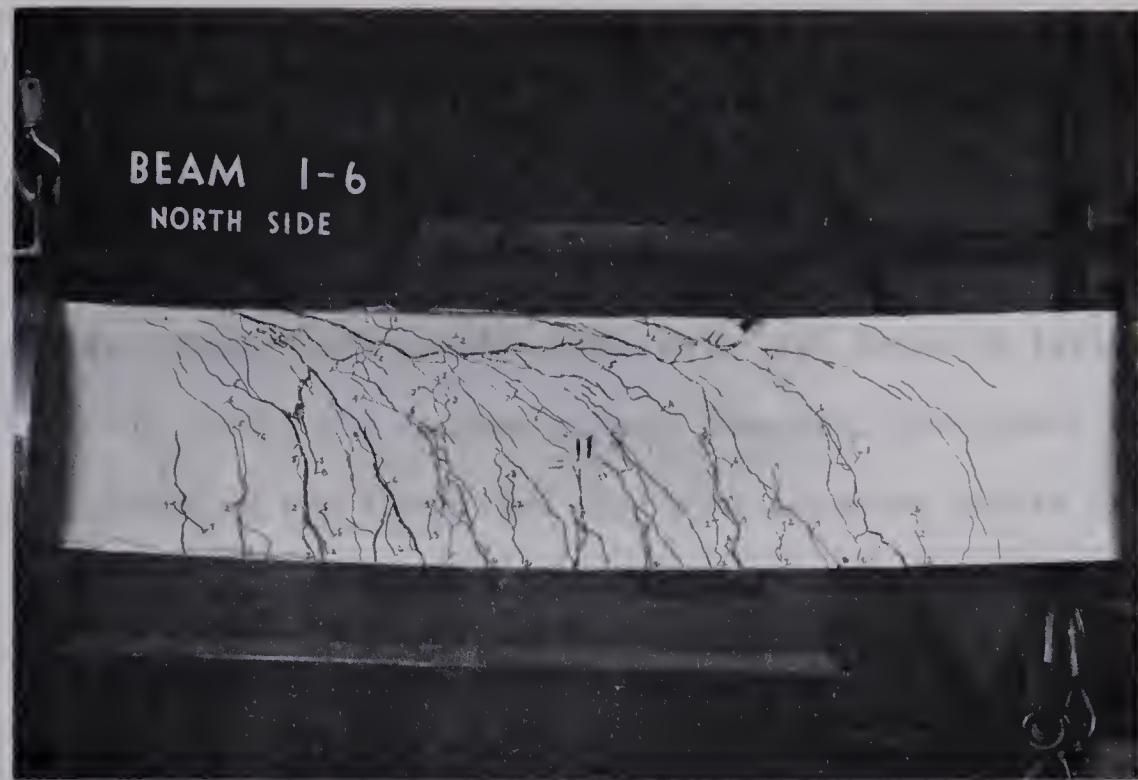


FIGURE 4 - 5 CRACK PATTERN OF BEAM 1 - 6



was equal to the bottom reinforcement.

Beam 2-1: This beam, tested in pure torsion, failed during reading of the strain gauges. Since wide cracks appeared on all faces at failure it was difficult to define the failure plane. However, the widest crack was an inclined crack on the south face that also extended across the top and bottom faces. Thus, the hinge was likely located adjacent to the north face. On the top face a "Z" shaped crack had begun to form indicating that a hinge was beginning to form on this face. Readings taken after failure indicated strains in excess of yield strains at all gauges except number 4. The crack pattern of this beam is shown in FIGURE 4-6.

Beam 2-2: Failure of this beam occurred during application of the twisting portion of the last load increment. The test was conducted at  $\emptyset$  equal to 2.0. The failure plane was difficult to define and appeared to be composed of portions of two sets of spiral cracks. However, the axis of rotation at failure was located in the vicinity of the top face of the beam. Readings taken after failure indicated strains larger than yield strains at gauges 3, 5, 6 and 8.

Beam 2-3: The value of  $\emptyset$  at which this beam was tested was 1.0. Failure, with the hinge located near the top surface, occurred immediately after application of the twisting portion of the last load increment. Strain readings taken after failure indicated strains larger than yield strains at gauges 3, 4, 5, 6 and 8.

Beam 2-4: This beam failed soon after application of the last load increment and developed a hinge in the vicinity of the top surface. The test



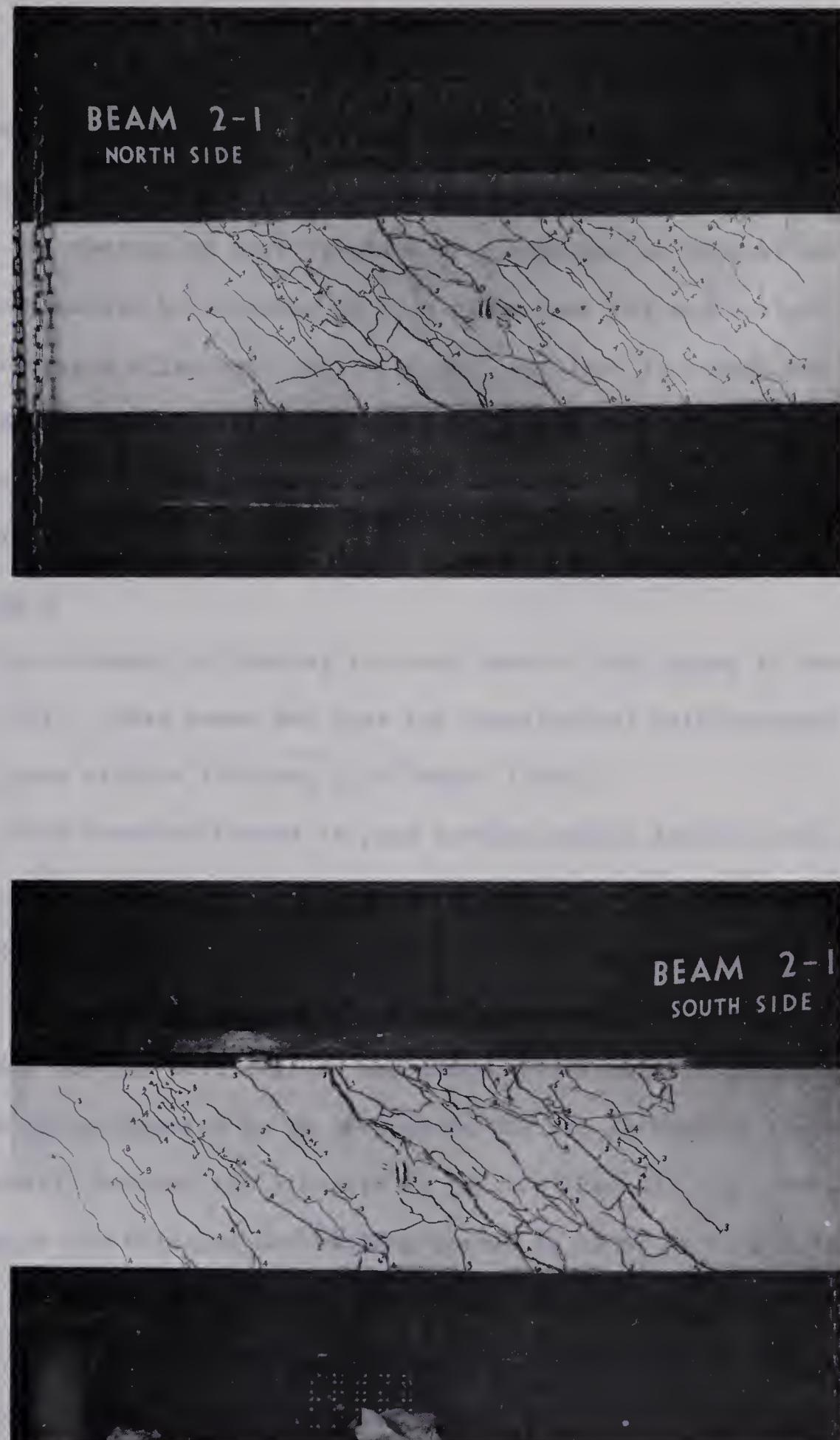


FIGURE 4 - 6 CRACK PATTERN OF BEAM 2 - 1



was performed at  $\phi$  equal to 0.5. Readings obtained after failure from strain gauges 3, 4 and 5 indicated strains larger than yield strains.

Beam 2-5: The testing of this specimen was conducted at  $\phi$  equal to 0.25. Failure, accompanied by rotation about a hinge near the top surface of the beam, occurred after application of the last load increment but peak values of moments were maintained throughout considerable rotation of the beam. Readings taken subsequent to failure indicated strains larger than yield strains at gauges 3, 4, 5 and 8.

(iii) Group 3

The sequence of loading for each beam of this group is described in CHAPTER III. These beams had less top longitudinal reinforcement than bottom and were similar to Group C of Pandit (1965).

Beam 3-1: This beam was tested in pure torsion and at failure developed a hinge in the vicinity of the bottom reinforcement. Deflection of this beam was not measured.

Beam 3-2: The value of  $\phi$  at failure of this beam was 1.05. Although the failure plane was not well defined and large cracks existed on all faces, the failure hinge appeared to be located in the top portion of the beam. The relationship between the stresses in the tie that was instrumented and the sum of the twisting and bending moment is shown in FIGURE 4-7. The sum of twisting and bending moment is used simply for convenience.

Beam 3-3: At failure, the value of  $\phi$  was 1.08 and the hinge was located adjacent to the top surface of the beam.

Beam 3-4: The value of  $\phi$  at failure of this beam was 0.61. The hinge was



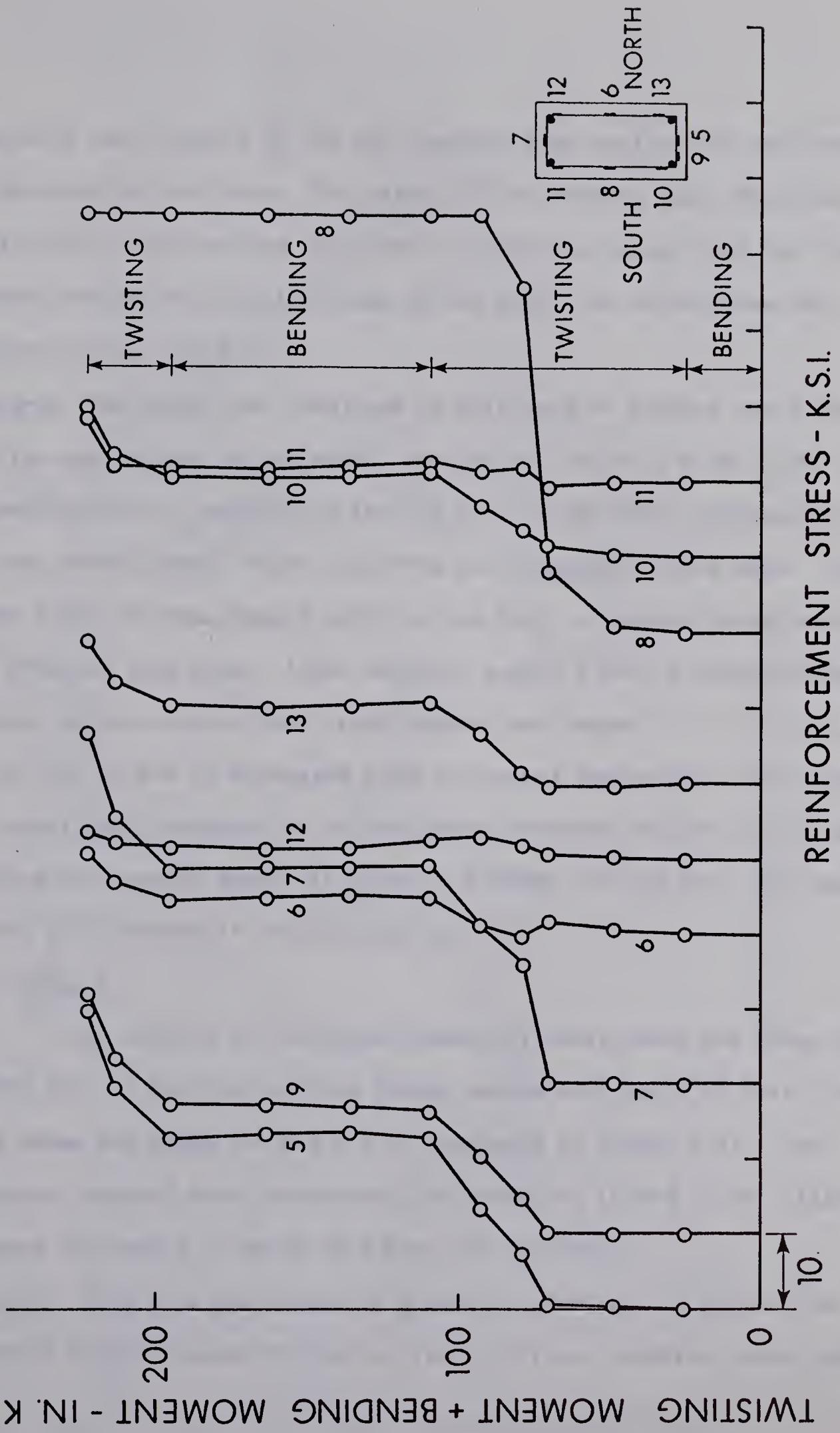


FIGURE 4-7 TIE STRESSES IN BEAM 3 - 2



located in the vicinity of the top face and some spalling of the concrete was observed on this face. The effect of the bending load, which was applied after the twisting load, was to widen the cracks that had first appeared during the twisting stage of the test. No deflections were measured during this test.

Beam 3-5: The hinge that developed in this beam at failure was located near the top surface of the beam. The failure value of  $\emptyset$  was 0.28. At failure, pieces of concrete on the top face of the beam, particularly the top, south corner, broke away from the remainder of the beam. Since gauges 1 and 18 were damaged early in the test, no useful strain readings were obtained from them. After failure, gauges 2 and 10 indicated compressive strains larger than yield strains and gauges 3, 4, 5, 6, 7, 8, 11, 12, 16, 19 and 20 indicated tensile strains larger than yield strains. The relationship between the reinforcement stresses and the sum of the twisting and bending moment is shown in FIGURES 4-8 and 4-9. The crack pattern of this beam is shown in FIGURE 4-10.

(iv) Group 4

The details of the reinforcement of these beams are given in CHAPTER III. A plot of twisting moment versus unit angle of twist for these beams and Beams 3-4 and 1-5 is presented in FIGURE 4-11. The irregular shape of this relationship for Beam 3-4 is due to the different sequence of loading to which this beam was subjected.

Beam 4-1: This beam was tested at  $\emptyset$  equal to 0.61 and at failure developed a hinge adjacent to the top face. Failure occurred during appli-



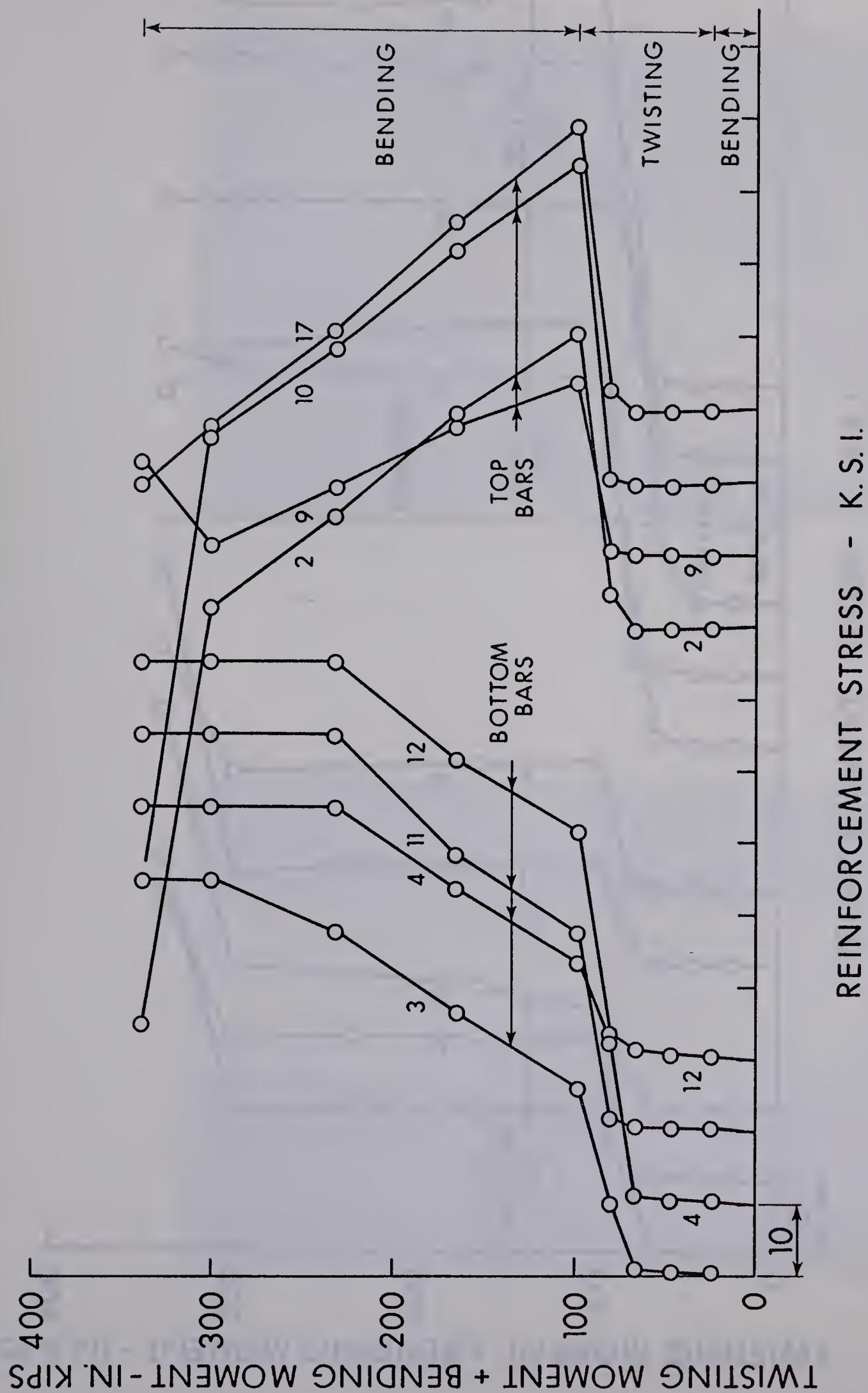


FIGURE 4 - 8 LONGITUDINAL BAR STRESSES IN BEAM 3 - 5



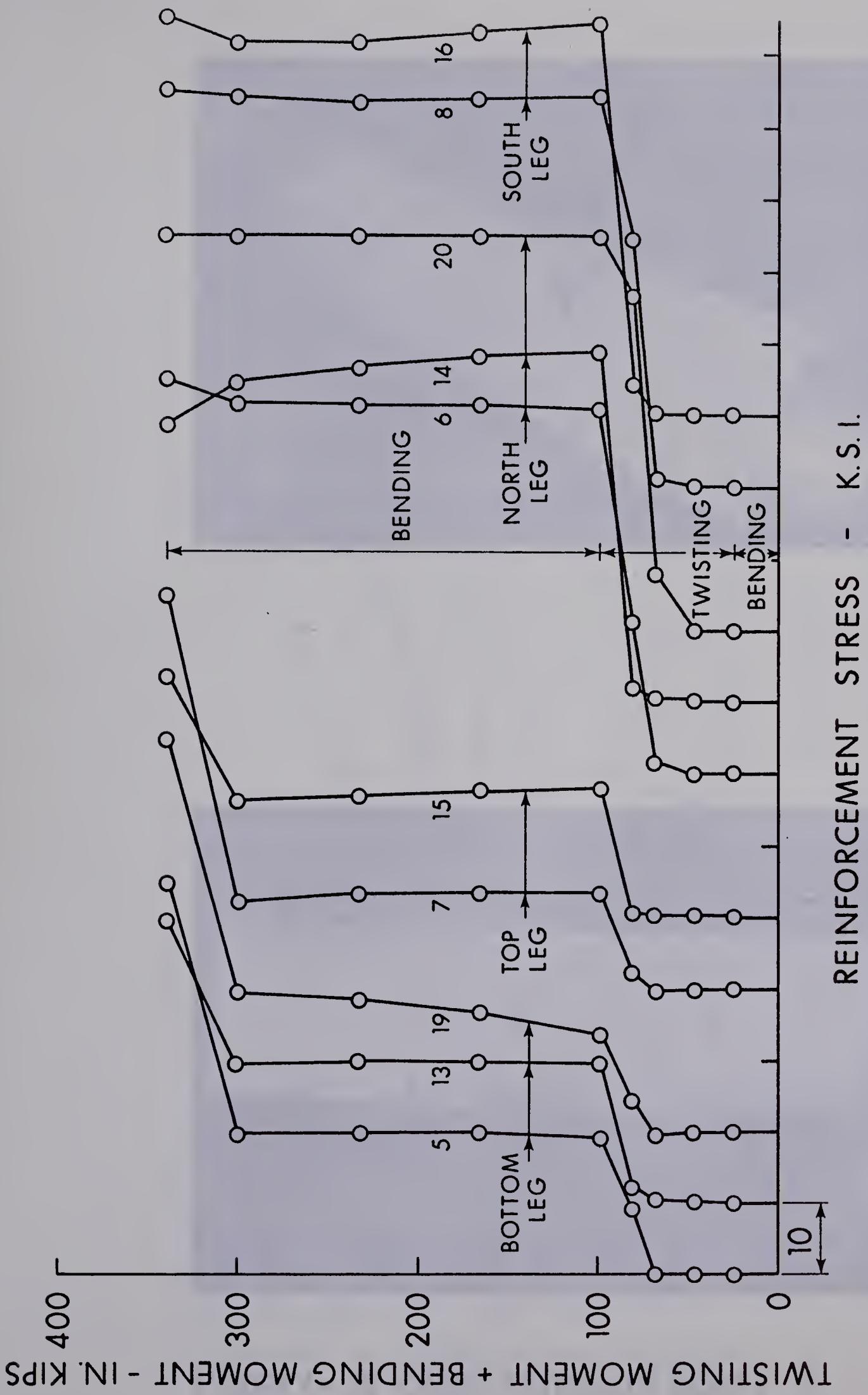


FIGURE 4 - 9 TIE STRESSES IN BEAM 3 - 5



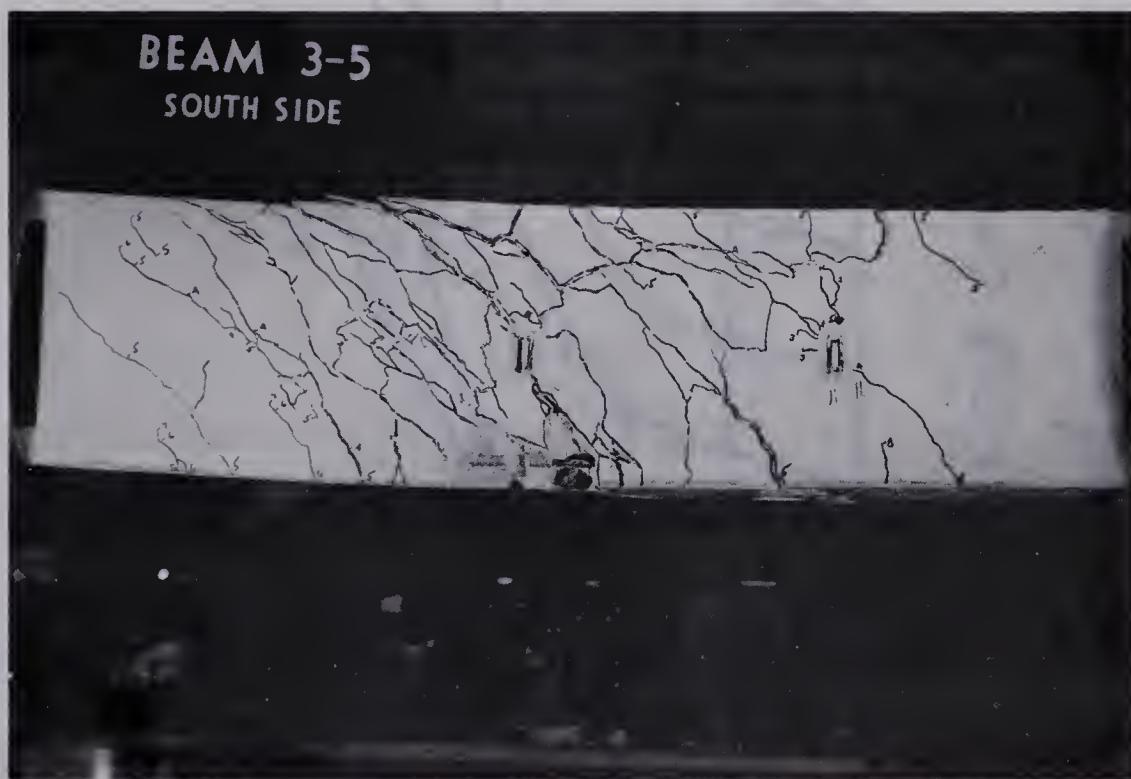
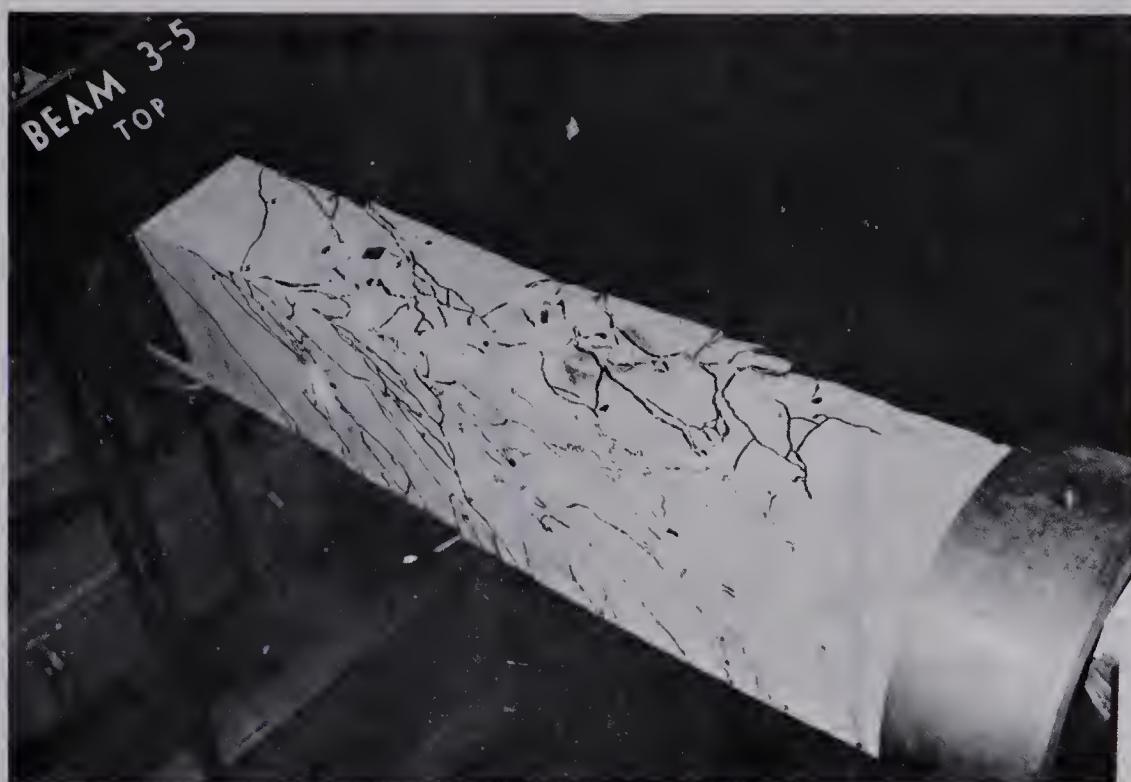


FIGURE 4 - 10 CRACK PATTERN OF BEAM 3 - 5



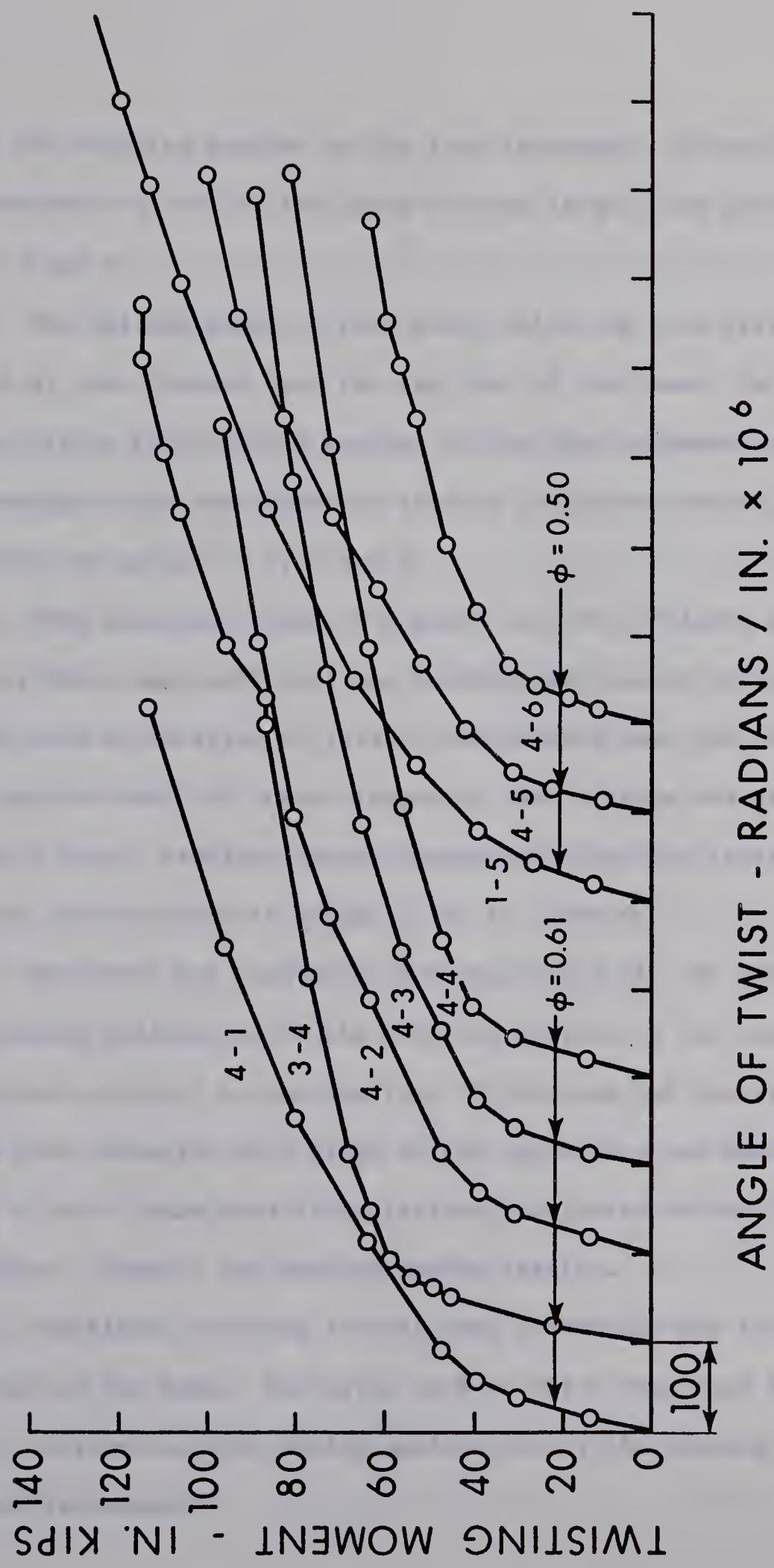


FIGURE 4 - 11 TORQUE TWIST CURVES FOR BEAMS WITH DIFFERENT REINFORCEMENT



cation of the twisting portion of the load increment. Strain readings taken subsequent to failure indicated strains larger than yield strains at gauges 3 and 4.

Beam 4-2: The failure hinge in this beam, which was also tested at  $\emptyset$  equal to 0.61, was located near the top face of the beam. Failure occurred immediately after the twisting portion of the load increment was applied. Strain readings taken subsequent to failure indicated strains larger than yield strains at gauges 3, 4, 7 and 8.

Beam 4-3: This beam was tested at  $\emptyset$  equal to 0.61. Failure occurred immediately after application of the twisting portion of the load increment. The axis of rotation at failure was located near the top face of the beam but the width of cracks suggested that a hinge was about to form on the north face. Readings taken subsequent to failure indicated strains larger than yield strains at gauges 3, 4, 6, 7 and 8.

Beam 4-4: This test was conducted at  $\emptyset$  equal to 0.61. At failure, which occurred during application of the twisting portion of the load increment, a hinge formed adjacent to the top face of the beam but the width of cracks suggested that formation of a hinge on the north face was imminent. Strain gauges 3, 5 and 8, when read after failure, indicated strains larger than yield strains. Gauge 4 was damaged during testing.

Beam 4-5: The hinge occurring in this beam at failure was located near the top face of the beam. The value of  $\emptyset$  at which this beam was tested was 0.50. Failure occurred during application of the bending portion of the load increment.



Beam 4-6: This beam also was tested at  $\phi$  equal to 0.50 and at failure a hinge formed in the vicinity of the top face. Failure occurred while the strain gauges were being read. The peak moments were maintained throughout a large rotation.

(v) Group 5

The beams of this group, which were reinforced identically to the beams of Group 1, were loaded using a single concentrated transverse load in addition to a twisting moment. Consequently, the gauge length was subjected to a constant twisting moment, a constant shear force, and a linearly varying bending moment. The maximum bending moment in the gauge length, which occurred at the west end of the gauge length, was used in establishing the value of  $\phi$  .

Beam 5-1: Failure of this beam occurred during application of twisting moment. The failure hinge was near the bottom face of the beam and the failure plane was in the east portion of the gauge length. The value of  $\phi$  at which this test was conducted was 2.0.

Beam 5-2: The value of  $\phi$  at which this test was conducted was 1.0. At failure, which occurred during application of twisting moment, a hinge formed near the north face of the beam. However, the width of cracks suggested that the formation of a hinge near the bottom face was imminent. Strain readings taken subsequent to failure indicated strains larger than yield strains at gauges 2, 3, 4, 5, 7 and 8.

Beam 5-3: This beam was tested at  $\phi$  equal to 0.50 and failed during application of twisting moment. The hinge that developed at failure was located



in the vicinity of the top surface. The failure plane was composed of parts of two sets of spiral cracks and was crowded toward the west end of the gauge length. Subsequent to failure, strain gauges 3, 4, 5 and 8 indicated strains larger than yield strains.

Beam 5-4: The value of  $\emptyset$  at which this beam was tested was 0.25. The hinge that formed at failure was located near the top face of the beam; the failure surface was in the west portion of the gauge length. Failure of this beam occurred after application of the twisting portion of the load increment and was accompanied by spalling of the concrete at the top face of the beam. The peak values of load were maintained throughout a very large rotation.

(vi) Group 6

These beams were reinforced identically to the beams of Group 5 and were also subjected to bending, torsion and transverse shear. However, the point of application of the transverse load was changed in order to increase the shear in the gauge length.

Beam 6-1: At failure, which occurred during application of twisting moment, the axis of rotation was near the bottom surface of the beam. The test was conducted at  $\emptyset$  equal to 2.0. The failure plane was located in the east portion of the gauge length.

Beam 6-2: This beam was tested at  $\emptyset$  equal to 1.0 and failed during application of twisting moment. The failure surface was not well defined but was located in the east half of the gauge length. The failure hinge appeared to be located near the bottom face but some aspects of the crack pattern



indicated its location to be in the vicinity of the north side. Readings after failure indicated strains larger than yield strains at gauges 1, 2 and 6.

Beam 6-3: Failure of this beam occurred during application of twisting moment. The test was conducted at  $\phi$  equal to 0.50 and at failure the peak values of moments were maintained through a large rotation. The failure surface was located in the west portion of the gauge length; the failure hinge was near the top surface of the beam. Subsequent to failure, strain gauges 3, 4 and 5 indicated strains larger than yield strains.

Beam 6-4: Failure occurred after application of the twisting portion of the last load increment but peak values of the moments were maintained throughout a very large rotation. The value of  $\phi$  for this test was 0.25 and the failure surface was located in the west portion of the gauge length. Considerable spalling of the concrete at the top surface of the beam occurred. The hinge that formed at failure was located near the top face of the beam. Strain readings taken after failure indicated compressive strain larger than the yield strain at gauge 1 and tensile strains larger than the yield strains at gauges 3, 4 and 5.

(vii) Group 7

In comparison to the beams of Groups 5 and 6, the beams of this group had larger bottom reinforcement and larger spacing of ties. The loading arrangement for this group was the same as for Group 6.

Beam 7-1: This beam was tested at  $\phi$  equal to 2.0 and failed during reading of the deflection gauges. The axis of rotation at failure was in the



vicinity of the bottom face of the beam. The failure surface, which was composed of portions of two sets of spiral cracks, was crowded toward the east end of the gauge length.

Beam 7-2: The value of  $\emptyset$  at which this test was conducted was 1.0 and failure occurred during reading of the strain gauges. The failure hinge formed near the bottom face of the beam and the failure surface was located in the east portion of the gauge length. Strain readings taken after failure indicated strains larger than yield strains at gauges 2 and 8.

Beam 7-3: The failure plane was crowded toward the east end of the gauge length and the hinge was located near the bottom face. However, the width of cracks suggested that another failure plane was about to form in the central portion of the gauge length. This failure plane would have involved a hinge near the north face. This test was carried out at  $\emptyset$  equal to 0.50 and failure occurred during application of twisting moment. Readings obtained after failure indicated strains larger than yield strains at gauges 2, 6, 7 and 8.

Beam 7-4: The test of this beam was carried out at  $\emptyset$  equal to 0.25 and failure occurred during application of twisting moment. The failure surface was a composite of several sets of spiral cracks and was crowded to the west end of the gauge length. The failure hinge was located near the top face of the beam. However, similarly to Beam 7-3, the width of cracks suggested that another failure plane, which would have involved a hinge near the north face, was about to form in the central portion of the



gauge length. Some spalling of the concrete occurred on the top, north corner of the beam. Readings obtained after failure indicated strains larger than yield strains at gauges 3, 4, 7 and 8.



## CHAPTER V

### ANALYSIS OF ULTIMATE STRENGTH IN COMBINED LOADING

#### 5-1 Introduction

The assumptions and the three failure surfaces on which the analysis is based are presented in this chapter. For each failure surface an equation for the torsional strength of a rectangular reinforced concrete beam is derived. In addition, the method used to solve these equations is presented.

#### 5-2 General Remarks

The results presented in CHAPTER IV and in Appendix B indicate that the specimens exhibited two different stages of behavior during a test. Before the concrete cracked, the relationship between torque and twist was nearly linear. In this stage, the reinforcement strains were low and the deflection was zero or slightly positive. After cracking, the slope of the torque-twist relationship decreased considerably and this relationship often became curved. The strains in the reinforcement increased with load at a greater rate and in many cases the deflection became more pronounced. After cracking occurred the specimen appeared to assume a new configuration of equilibrium which was dependent mainly on the value of  $\phi$  and the reinforcement of the beam.

When failure occurred, cracks on three faces of the beam usually widened to define the failure plane. The cracks on two of these



faces decreased in width as they approached the fourth face. Although the fourth face was cracked, these cracks did not widen appreciably at failure. Rotation of the beam at failure occurred about an axis located near this face. Consequently, the failure plane appeared to consist of a tension zone on one side of a neutral axis and a compression zone on the other side, with the neutral axis being the hinge or axis of rotation.

A "Z" shaped crack often developed on the face of the beam adjacent to the hinge, with the central portion of the "Z" being parallel to the longitudinal axis of the beam. However, this is not sufficient justification to assume that the hinge was parallel to the axis of the beam. Furthermore, rotation about a longitudinal hinge could not explain the vertical deflections observed at failure. It is important that any method of analysis used to predict the ultimate strength of beams subjected to combined loading be valid at the limiting cases of pure bending and pure torsion. The neutral axis of a beam subjected to pure bending is perpendicular to the axis of the beam. Adoption of an inclined hinge that is parallel to one face of the beam but which is inclined to the longitudinal axis of the beam was first suggested by Lessig (1959). A smooth transition from pure bending to pure torsion is obtained when this type of hinge is used. The angle this hinge makes with the longitudinal axis is a function of  $\emptyset$  and the properties of the beam and is discussed in SECTION 5.4. Lessig denoted failure on a warped plane with a hinge near the top face of the beam as mode 1; failure on a warped plane with a hinge located near one of the sides was denoted as mode 2. This de-



signation will be followed here, but in addition, failure on a warped plane with a hinge located near the bottom face will be denoted as mode 3.

### 5-3 Assumptions of Analysis

The method of analysis is based on the following assumptions:

- (1) Failure occurs on a warped plane. The boundaries of the warped plane are defined on three sides of the beam by a spiral crack and on the fourth side by a rectangular compression zone which joins the ends of the spiral crack.
- (2) The crack defining the failure plane on three sides of the beam is composed of three straight lines spiralling around the beam at a constant angle. The angle between these cracks and the longitudinal axis of the beam is never less than  $45^\circ$ .
- (3) The concrete outside of the rectangular compression zone is cracked and carries no tension.
- (4) The reinforcement near the face of the beam on which the compression zone is located is neglected.
- (5) No local loads are present within the length of the failure plane.
- (6) The reinforcement has a well defined yield plateau. All reinforcement crossing the failure plane outside of the compression zone yields at failure.
- (7) The cross sectional area of transverse reinforcement intersected by the failure plane is constant per unit length of the beam.



(8) At failure, the characteristics of the concrete in the compression zone are known and the concrete reaches its strength in flexural compression.

Assumptions 1 and 2 define the failure plane. The three lines composing the spiral crack are assumed to be straight for simplicity. The assumption that these cracks do not make an angle of less than  $45^{\circ}$  with the longitudinal axis of the beam is strongly supported by the tests of Chinenkov (1959), Pandit (1965) and this investigation.

The third, fourth and fifth assumptions are introduced to simplify the analysis.

The sixth assumption is strongly supported by the tests of Chinenkov (1959), Lyalin (1959), Yudin (1964), Evans and Sarkar (1965) and this investigation. However, Lessig (1961) has suggested that this assumption is true only within a specific range of the ratio of transverse to longitudinal reinforcement.

The seventh assumption is an idealization which is introduced for simplicity.

The eighth assumption is true only for pure bending i.e.  $\emptyset = 0$ . As torsion is added and the value of  $\emptyset$  is increased, the compression zone is subjected to stress conditions which are complex and which are further complicated by inclined cracks crossing the compression zone. This assumption is discussed further in CHAPTER VIII.



5-4 Failure Surfaces(a) Mode 1

The failure surface for this mode of failure is illustrated in FIGURE 5-1. Three sides of the failure surface are defined by a spiral crack which is located on the bottom and the two vertical faces of the beam. The ends of this spiral crack are joined by a compression zone located at the top face of the beam.

The notation used in the equations which follow is:

$b$  = beam width

$h$  = beam depth

$L_1$  = length of the compression zone

$c_1$  = length of the warped failure plane projected on the longitudinal axis of the beam

$c_{ml}$  = the maximum value that  $c_1$  can have

$A_{sl}$  = area of longitudinal reinforcement at the bottom face of the beam

$f_{y1}$  = yield stress of longitudinal reinforcement

$a_v$  = area of one leg of a tie

$f_{yt}$  = yield stress of tie

$s$  = spacing of ties

$a_1$  = distance from bottom face of beam to centroid of longitudinal reinforcement

$a_{1t}$  = distance from bottom face of beam to centroid of bottom leg of tie

$a_{2t}$  = distance from vertical face of beam to centroid of vertical leg of tie



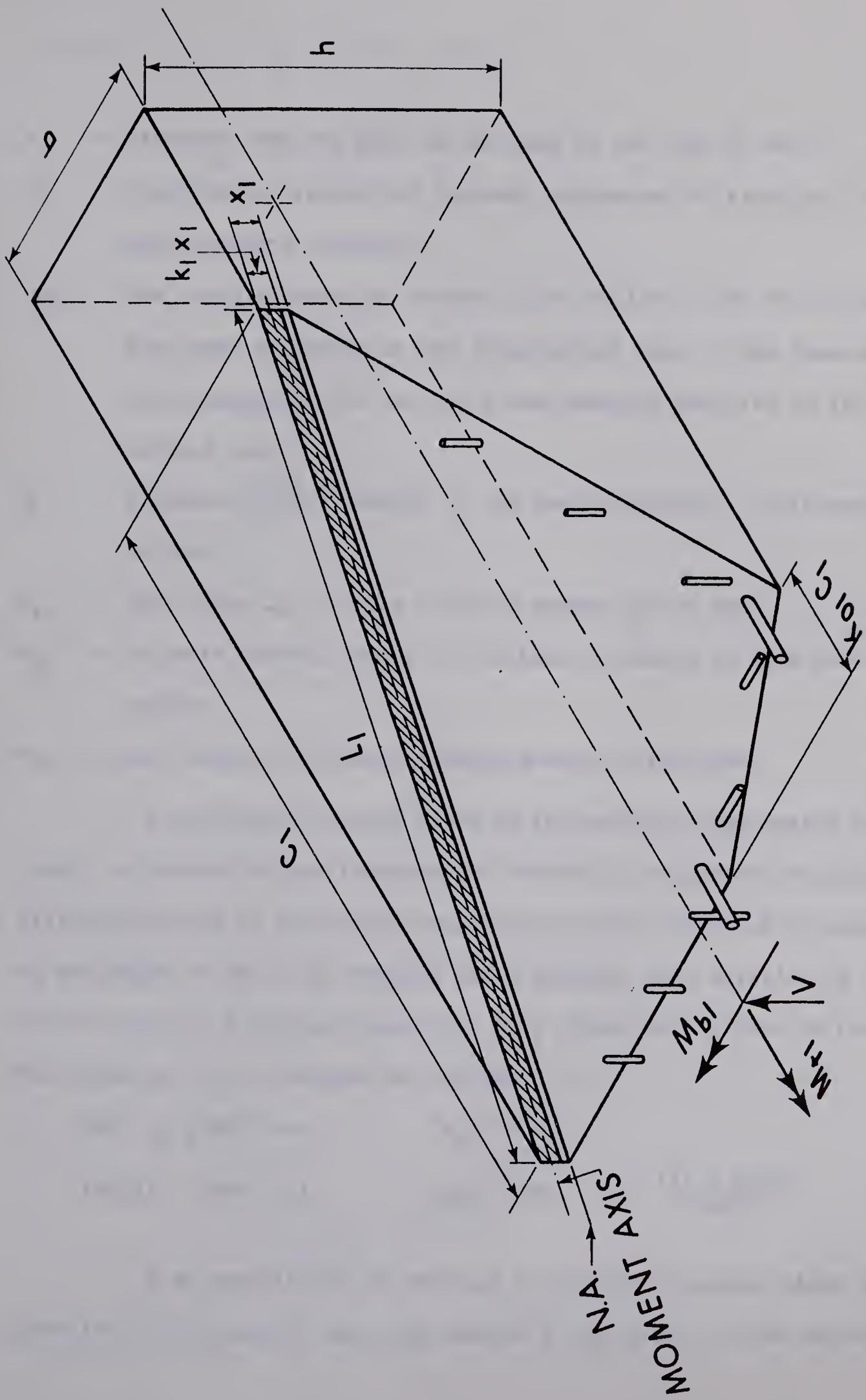


FIGURE 5 - 1 MODE 1 FAILURE SURFACE



$x_1$  = distance from top face of the beam to the neutral axis

$f'_c$  = compressive strength of concrete determined by tests on 6 x 12 inch concrete cylinders

$k_{01}$  = the ratio between the length of the failure crack on the bottom face when projected on the longitudinal axis of the beam and the total length of the failure plane measured parallel to the longitudinal axis

$M_{t1}$  = ultimate twisting moment of the beam according to this mode of failure

$M_{tt}$  = test value of ultimate twisting moment of the beam

$M_{b1}$  = ultimate bending moment of the beam according to this mode of failure

$M_{bt}$  = test value of ultimate bending moment of the beam

A rectangular stress block in the concrete compression zone is used. A concrete stress intensity of  $0.85 f'_c$  is assumed to be uniformly distributed over an equivalent compression zone. This zone is bounded by the edges of the cross section and a straight line parallel to the neutral axis at a distance equal to  $k_1 x_1$  from the top face of the beam.

The value of  $k_1$  is defined as follows:

For  $f'_c \leq 4000$  psi  $k_1 = 0.85$

For  $f'_c > 4000$  psi  $k_1 = 0.85 - 0.05 \frac{(f'_c - 4000)}{1000}$

From equilibrium of internal and external moments about an axis parallel to the neutral axis and located at mid-depth of the equivalent



compression zone, the following equation is obtained.

$$M_{t1} = A_{s1} f_{y1} \left[ \frac{z_1 + p_1 y_1 c_1^2 / bh}{\frac{c_1}{b} + \frac{1}{\phi}} \right] \quad (5-1)$$

where  $z_1 = h_{01} - k_1 x_1 / 2$

$$h_{01} = h - a_1$$

$$\phi = M_{t1} / M_{b1}$$

$$p_1 = \frac{f_{yt} a_v h}{f_{y1} A_{s1} s}$$

$$y_1 = k_{01} (h - a_{1t} - \frac{k_1 x_1}{2}) + \frac{b}{4} (1 - k_{01})(1 - k_{01} - 4a_{2t}/b)$$

$$k_{01} = b/(2h + b)$$

The external moments about this axis are the components of twisting moment and bending moment. The internal moments about this axis are the moments of the forces in the longitudinal reinforcement, the bottom leg of the ties and the vertical legs of the ties. Detailed derivation of this and subsequent equations is presented in Appendix C.

From equilibrium of forces acting perpendicular to the plane which contains the neutral axis and is perpendicular to the top face of the beam, the following equation is obtained.

$$x_1 = \frac{A_{s1} f_{y1} (b + \frac{p_1 k_{01} c_1^2}{h})}{0.85 k_1 f_c' L_1^2} \quad (5-2)$$



To determine the value of  $c_1$  corresponding to the minimum value of  $M_{t1}$ , Equation (5-1) is differentiated with respect to  $c_1$ , equated to zero and solved for  $c_1$ . The result is:

$$c_1 = -\frac{b}{\phi} + b \left[ \frac{1}{\phi^2} + \frac{z_1 h}{p_1 y_1 b} \right]^{1/2} \quad (5-3)$$

Since negative values of  $c_1$  have no physical significance the minimum value of  $c_1$  is zero. In accordance with the second assumption of SECTION 5-3 and neglecting the depth of the compression zone, the maximum value of  $c_1$  is:

$$c_{m1} = 2h + b \quad (5-4)$$

To solve these equations, the value of  $\phi$  and the necessary properties of the section must be computed. The equations may then be solved by an iterative procedure. The initial value of  $x_1$  is assumed to be zero and initial values of  $z_1$  and  $y_1$  are computed. The value of  $c_1$  can then be computed by Equations (5-3) and (5-4). This value of  $c_1$  is used in Equation (5-2) to calculate a value for  $x_1$ . The calculated value of  $x_1$  is then used to compute new values of  $z_1$  and  $y_1$  and the procedure is repeated. With each iteration the difference between the initial and the computed value of  $x_1$  decreases. The process is continued until the desired degree of accuracy is obtained. The ultimate twisting moment is then calculated using Equation (5-1).



(b) Mode 2

The failure surface for this mode of failure is illustrated in FIGURE 5-2. Three sides of the failure surface are defined by a spiral crack which is located on one of the vertical faces of the beam and the two horizontal faces. The ends of this spiral crack are joined by a compression zone located on one side of the beam.

The notation used is the same as for mode 1 with the following exceptions:

$L_2$  = length of the compression zone

$c_2$  = length of the warped failure plane projected on the longitudinal axis of the beam

$c_{m2}$  = the maximum value that  $c_2$  can have

$A_{s2}$  = area of longitudinal reinforcement at the vertical face of the beam opposite to the face on which the compression zone is located

$f_{y2}$  = yield stress of longitudinal reinforcement

$a_2$  = distance from vertical face of beam to centroid of longitudinal reinforcement

$x_2$  = distance from vertical face of beam on which compression zone is located to the neutral axis

$k_{02}$  = the ratio between the length of the failure crack on the vertical face when projected on the longitudinal axis of the beam and the total length of the failure plane measured parallel to the longitudinal axis

$M_{t2}$  = ultimate twisting moment of the beam according to this mode of failure



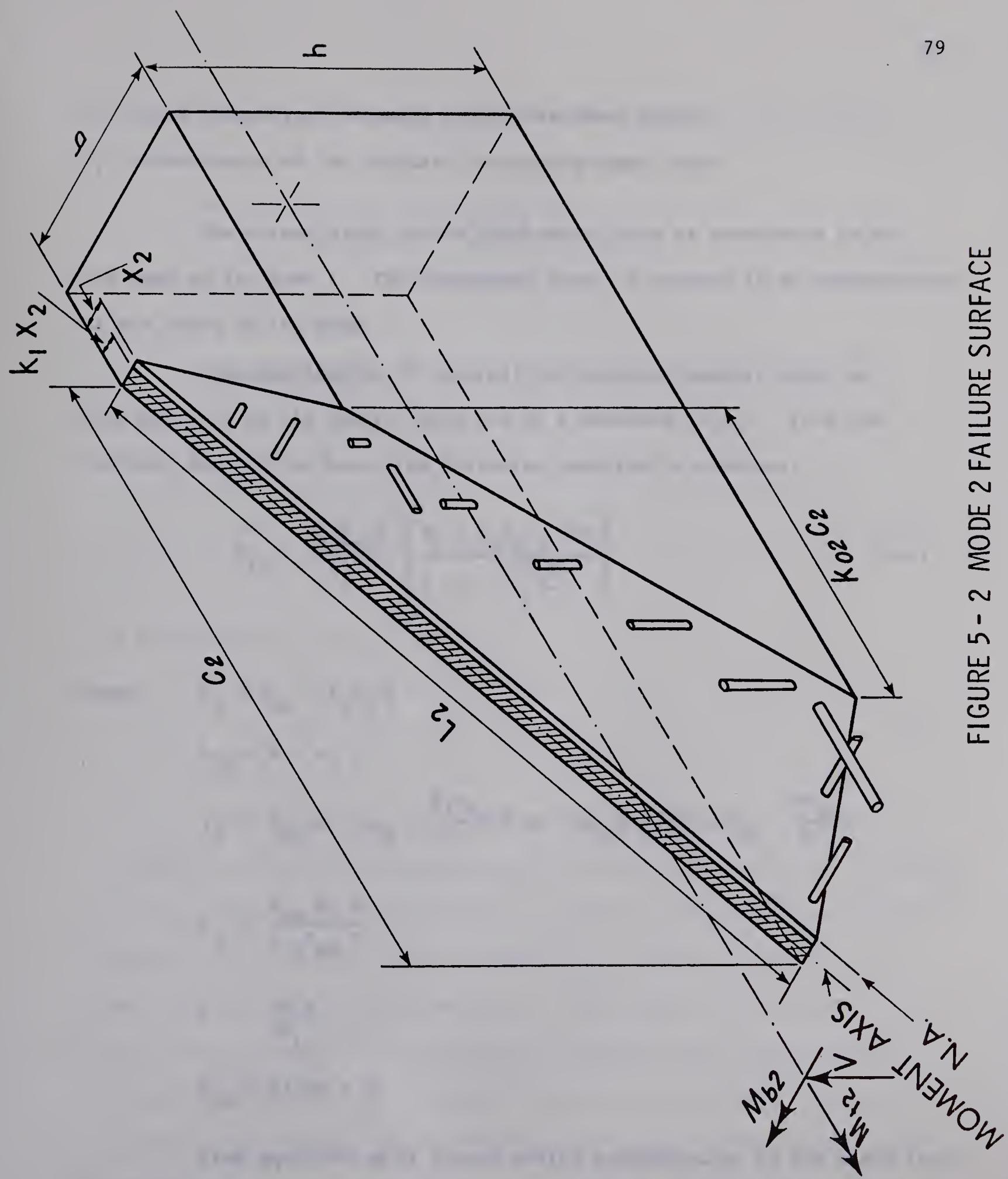


FIGURE 5 - 2 MODE 2 FAILURE SURFACE



$V$  = the theoretical ultimate transverse shear force

$V_t$  = test value of the ultimate transverse shear force

The stress block in the compression zone is considered to be the same as for mode 1. The transverse shear is assumed to be concentrated at mid width of the beam.

From equilibrium of internal and external moments about an axis parallel to the neutral axis and at a distance  $k_1 x_2 / 2$  from the vertical face of the beam, the following equation is obtained.

$$M_{t2} = \frac{A_s 2 f y_2 h}{c_2} \left[ \frac{z_2 + p_2 y_2 c_2^2 / bh}{1 + \delta - \frac{\delta k_1 x_2}{b}} \right] \quad (5-5)$$

where  $z_2 = b_{02} - k_1 x_2 / 2$

$$b_{02} = b - a_2$$

$$y_2 = k_{02} (b - a_{2t} - \frac{k_1 x_2}{2}) + (1 - k_{02}) \frac{h}{4} (1 - k_{02} - \frac{4a_{1t}}{h})$$

$$p_2 = \frac{f_y t a_v b}{f_y^2 A_s^2 S}$$

$$\delta = \frac{V b}{2 M_{t2}}$$

$$k_{02} = h / (2b + h)$$

From equilibrium of forces acting perpendicular to the plane that contains the neutral axis and is perpendicular to the vertical face of the



beam on which the compression zone is located, the following equation is obtained.

$$x_2 = \frac{A_{s2} f_{y2} \left[ h + \frac{p_2 k_{02} c_2^2}{b} \right] - V c_2}{0.85 k_1 f'_c L_2^2} \quad (5-6)$$

An expression for  $c_2$  is determined in the same manner as for mode 1. The resulting equation is:

$$c_2 = \left[ \frac{h z_2 b}{p_2 y_2} \right]^{1/2} \quad (5-7)$$

The maximum value that  $c_2$  can have is:

$$c_{m2} = 2b + h \quad (5-8)$$

After the necessary properties of the section and the value of  $\delta$  have been computed, the equations can be solved by iteration. Assuming that  $x_2$  and  $V$  are equal to zero,  $z_2$  and  $y_2$  are computed. Equations (5-7) and (5-8) are then used to compute  $c_2$ . Using this value of  $c_2$ , Equation (5-6) is used to determine  $x_2$  after which new values of  $z_2$  and  $y_2$  are computed. This procedure is repeated until the difference between the values of  $x_2$  computed from any two consecutive iterations is less than a specified value. The twisting moment is then computed by Equation (5-5) and is used to compute a new value of  $V$  by the following



equation:

$$V = \frac{2M_{t2} \delta}{b} \quad (5-9)$$

Again assuming  $x_2$  to be zero but using the value of  $V$  obtained from Equation (5-9), the above procedure is repeated until a new value of  $V$  is obtained. When the difference between the values of  $V$  computed from any two consecutive iterations is less than a specified value, the procedure is terminated. The values of  $M_{t2}$  and  $V$  determined in the last iteration are the theoretical ultimate values of twisting moment and transverse shear.

(c) Mode 3

The failure surface for this mode of failure is illustrated in FIGURE 5-3. Three sides of the failure surface are defined by a spiral crack which is located on the top and the two vertical faces of the beam. The ends of this spiral crack are joined by a compression zone located at the bottom face of the beam.

The notation used is the same as for mode 1 with the following exceptions:

$L_3$  = length of the compression zone

$c_3$  = length of the warped failure plane projected on the longitudinal axis of the beam

$c_{m3}$  = the maximum value that  $c_3$  can have

$A_{s3}$  = area of longitudinal reinforcement at the top face of the beam



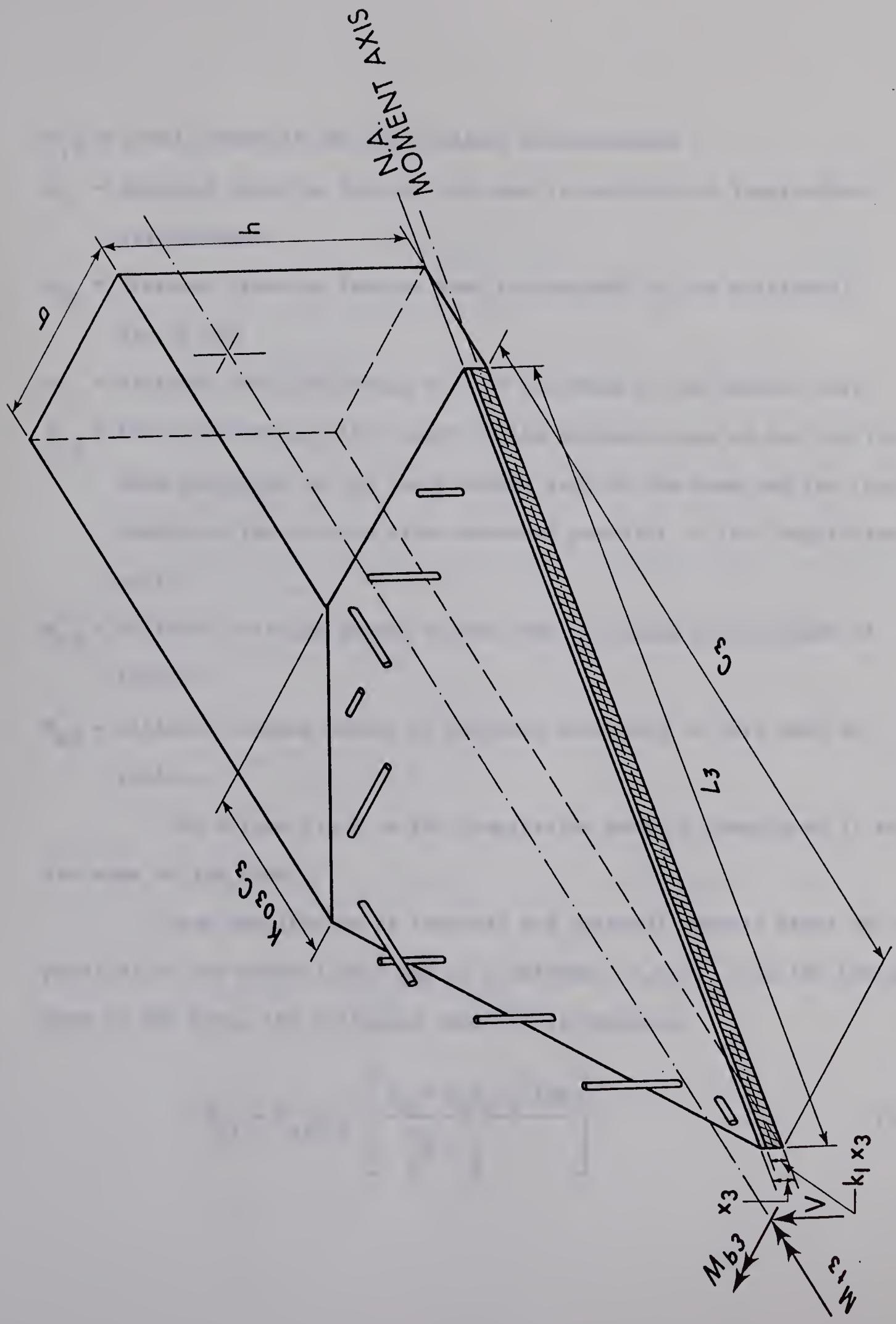


FIGURE 5 - 3 MODE 3 FAILURE SURFACE



$f_{y3}$  = yield stress of the longitudinal reinforcement

$a_3$  = distance from top face of the beam to centroid of longitudinal reinforcement

$a_{3t}$  = distance from top face of beam to centroid of top horizontal leg of tie

$x_3$  = distance from the bottom face of the beam to the neutral axis

$k_{03}$  = the ratio between the length of the failure crack on the top face when projected on the longitudinal axis of the beam and the total length of the failure plane measured parallel to the longitudinal axis

$M_{t3}$  = ultimate twisting moment of the beam according to this mode of failure

$M_{b3}$  = ultimate bending moment of the beam according to this mode of failure

The stress block in the compression zone is considered to be the same as for mode 1.

From equilibrium of internal and external moments about an axis parallel to the neutral axis and at a distance  $k_1 x_3 / 2$  from the bottom face of the beam, the following equation is obtained.

$$M_{t3} = A_{s3} f_{y3} \left[ \frac{\frac{z_3 + p_3 y_3 c_3^2}{c_3^2 / bh}}{\frac{c_3}{b} - \frac{1}{\phi}} \right] \quad (5-10)$$



$$\text{where } z_3 = h_{03} - k_1 x_3 / 2$$

$$h_{03} = h - a_3$$

$$y_3 = k_{03} (h - a_{3t} - \frac{k_1 x_3}{2}) + \frac{b}{4} (1 - k_{03}) (1 - k_{03} - \frac{4a_{2t}}{b})$$

$$p_3 = \frac{f_y t a_v h}{f_y t A_s s_3 S}$$

$$\emptyset = M_{t3} / M_{b3}$$

$$k_{03} = b / (2h + b)$$

From equilibrium of forces acting perpendicular to the plane which contains the neutral axis and is perpendicular to the bottom face of the beam, the following equation is obtained.

$$x_3 = \frac{A_s s_3 f_y t (b + \frac{p_3 k_{03} c_3}{h})^2}{0.85 k_1 f_c' L_3^2} \quad (5-11)$$

The value of  $c_3$  for which  $M_{t3}$  is a minimum is given by:

$$c_3 = \frac{b}{\emptyset} + b \left[ \frac{1}{\emptyset^2} + \frac{z_3 h}{p_3 y_3 b} \right]^{1/2} \quad (5-12)$$

The maximum value that  $c_3$  can have is:

$$c_{m3} = 2h + b \quad (5-13)$$



After the necessary properties of the section and the value of  $\emptyset$  have been computed, the equations can be solved by an iterative procedure identical to that used for mode 1.

### 5-5 Theoretical Ultimate Twisting Moment

The iterative procedure that was used to calculate  $x_1$ ,  $x_2$ , and  $x_3$  was continued until the values calculated in any two consecutive iterations did not differ by an amount greater than 0.001". Similarly, the iterative procedure for mode 2 was continued until the values of  $V$  computed in any two successive iterations did not differ by an amount greater than 0.01 kip.

Three theoretical ultimate twisting moments,  $M_{t1}$ ,  $M_{t2}$  and  $M_{t3}$ , are obtained for each beam by using Equations (5-1), (5-5) and (5-10). The theoretical ultimate twisting moment is designated as  $M_t$  and is equal to the smallest of the three values. The predicted mode of failure is designated as the mode that yields the smallest value of theoretical ultimate twisting moment.



## CHAPTER VI

EVALUATION OF TEST RESULTS6-1 Introduction

This chapter compares the experimental results presented in CHAPTER IV with the theory presented in CHAPTER V. Bending moment versus twisting moment interaction diagrams based on the equations in CHAPTER V are presented. The failure values of twisting moment and bending moment are also shown on the interaction diagrams. Part of an interaction surface for bending moment, twisting moment and shear force is also presented.

The correlation between the experimental results of 109 beams and the theory presented in CHAPTER V is presented in TABLES 6-1 to 6-7. Of these beams, 34 were tested in this investigation and 75 were tested by other investigators.

6-2 Beams Tested in Combined Bending and Torsion(a) Effect of Top Reinforcement

The beams of Groups 1 and 2 were similar in all respects except for the size of the top longitudinal reinforcement. The top reinforcement for beams of Group 1 consisted of 2#3 bars whereas for beams of Group 2 this reinforcement was 2#6. The bottom longitudinal reinforcement for both groups was 2#6.



The twisting moment versus bending moment interaction diagrams for the beams of Groups 1 and 2 are shown in FIGURE 6-1. The lines labeled mode 1, mode 2 and mode 3 were obtained by solving Equations (5-1), (5-5) and (5-10) respectively for values of  $\theta$  ranging from 0 (pure bending) to  $\infty$  (pure torsion). The material and geometrical properties of the beam used in solution of these equations are the average values for the group. The failure values of twisting moment and bending moment are also shown.

(b) Sequence of Loading

The beams of Group 3 were similar to Group C tested by Pandit (1965). Pandit's beams were tested by first applying the transverse load and then twisting the beam to failure. The beams of Group 3 were tested under various sequences of loading. The loading paths for these beams and the failure values of twisting moment and bending moment are shown in FIGURE 6-2.

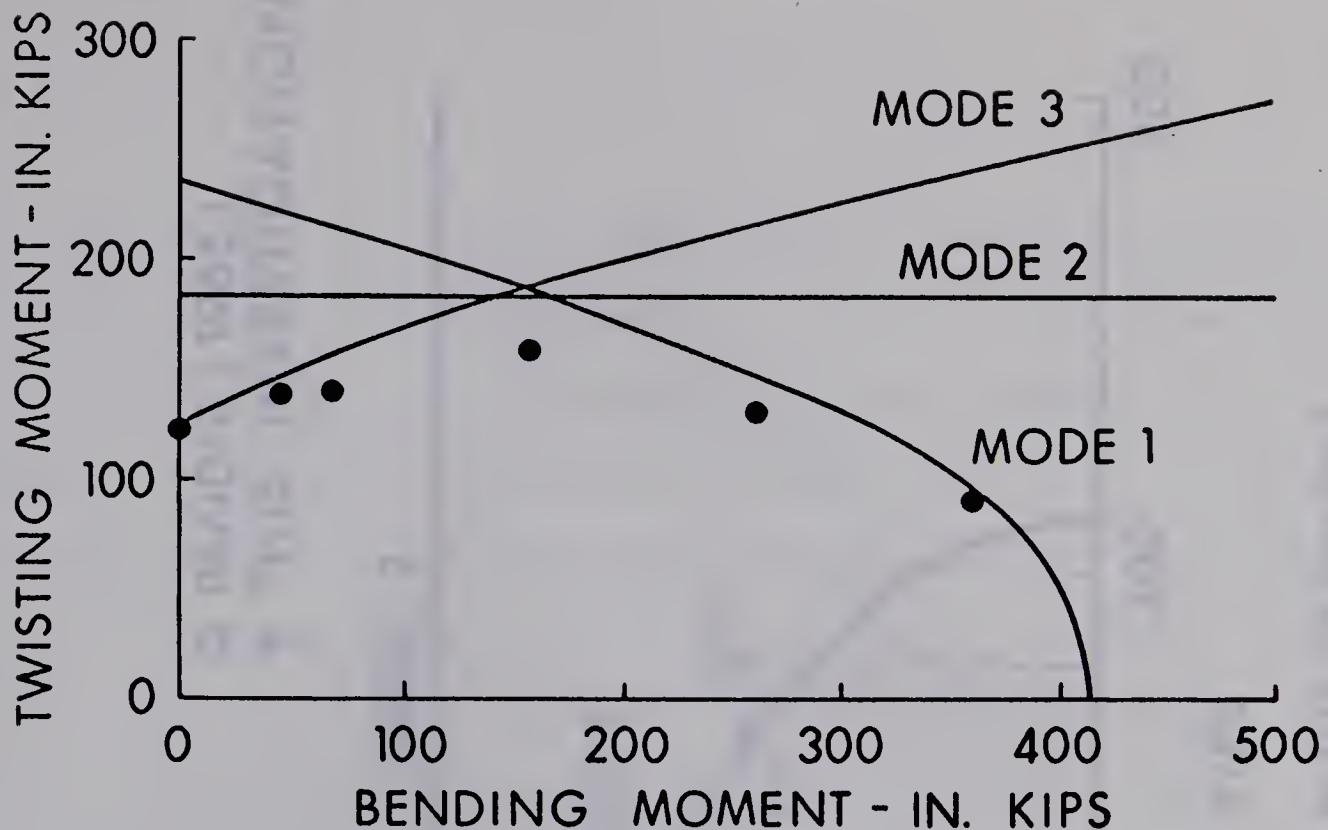
The interaction diagram obtained from Equations (5-1), (5-5) and (5-10) is shown in FIGURE 6-2. The properties of the beam used in solution of these equations are the average values for the nine beams considered.

(c) Effect of Stirrup Spacing

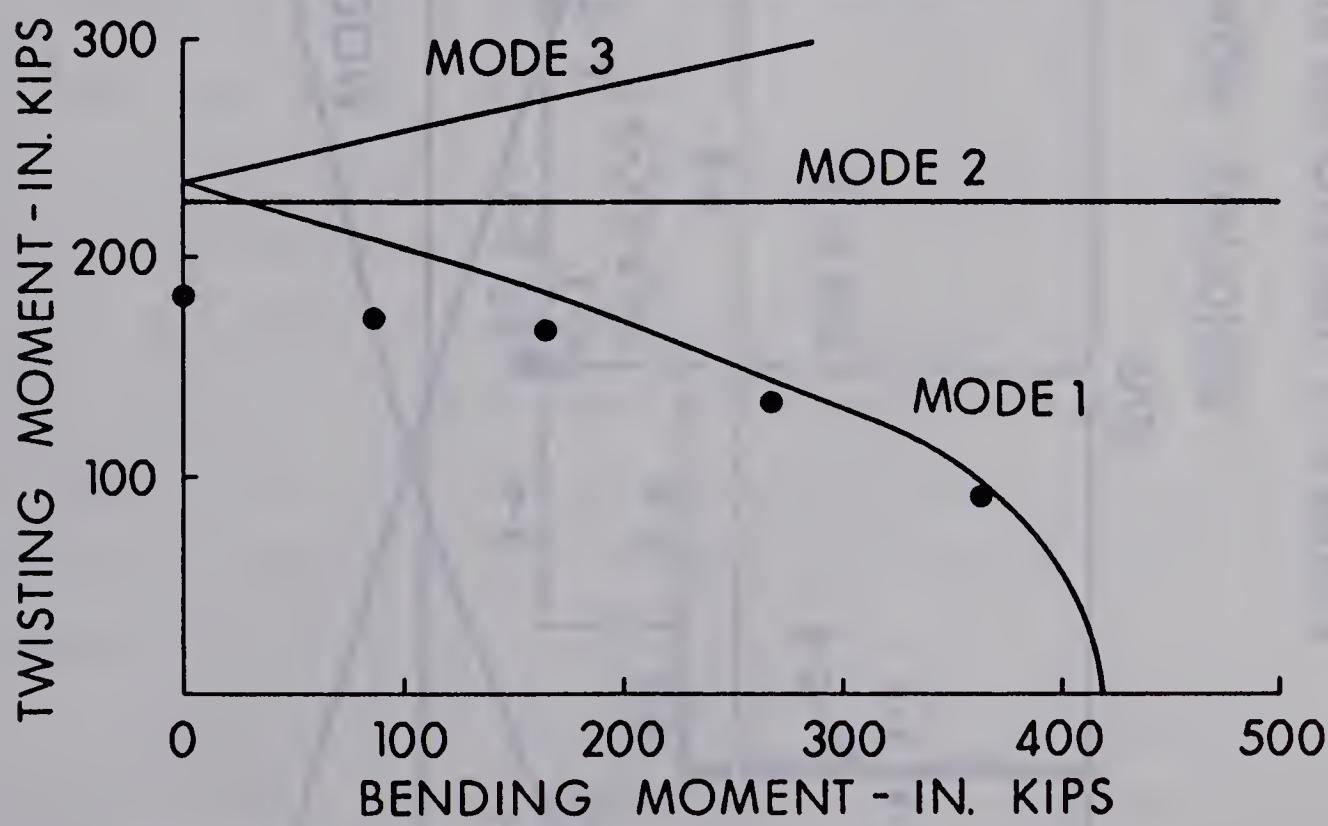
(i) General Remarks

Assumption five of SECTION 5-3 states that the cross section of transverse reinforcement per unit length of the beam is constant over any section of its length. Since this assumption is an idealization in that





(a) GROUP 1



(b) GROUP 2

FIGURE 6 - 1 INTERACTION DIAGRAMS FOR BEAMS OF GROUPS 1 AND 2

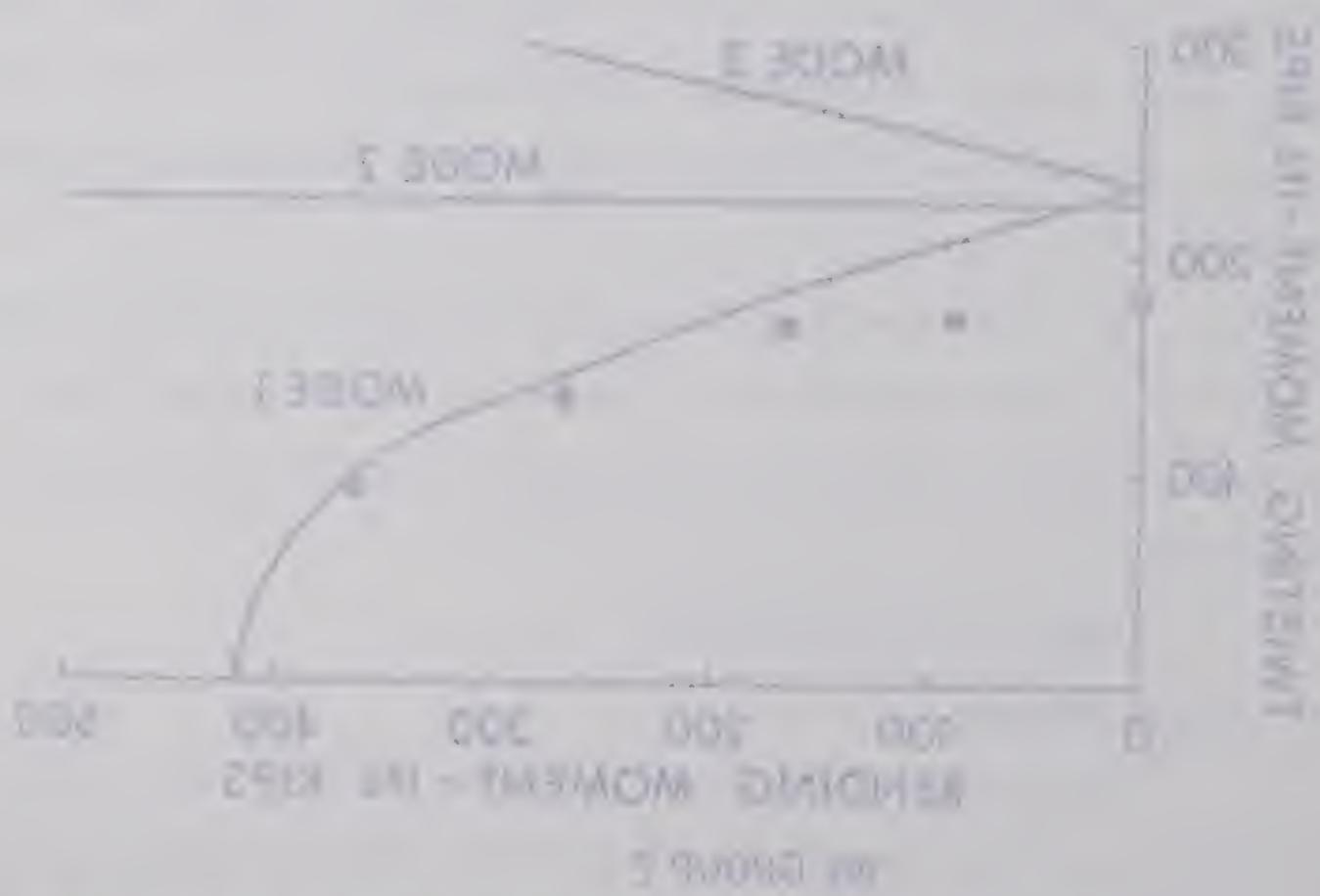
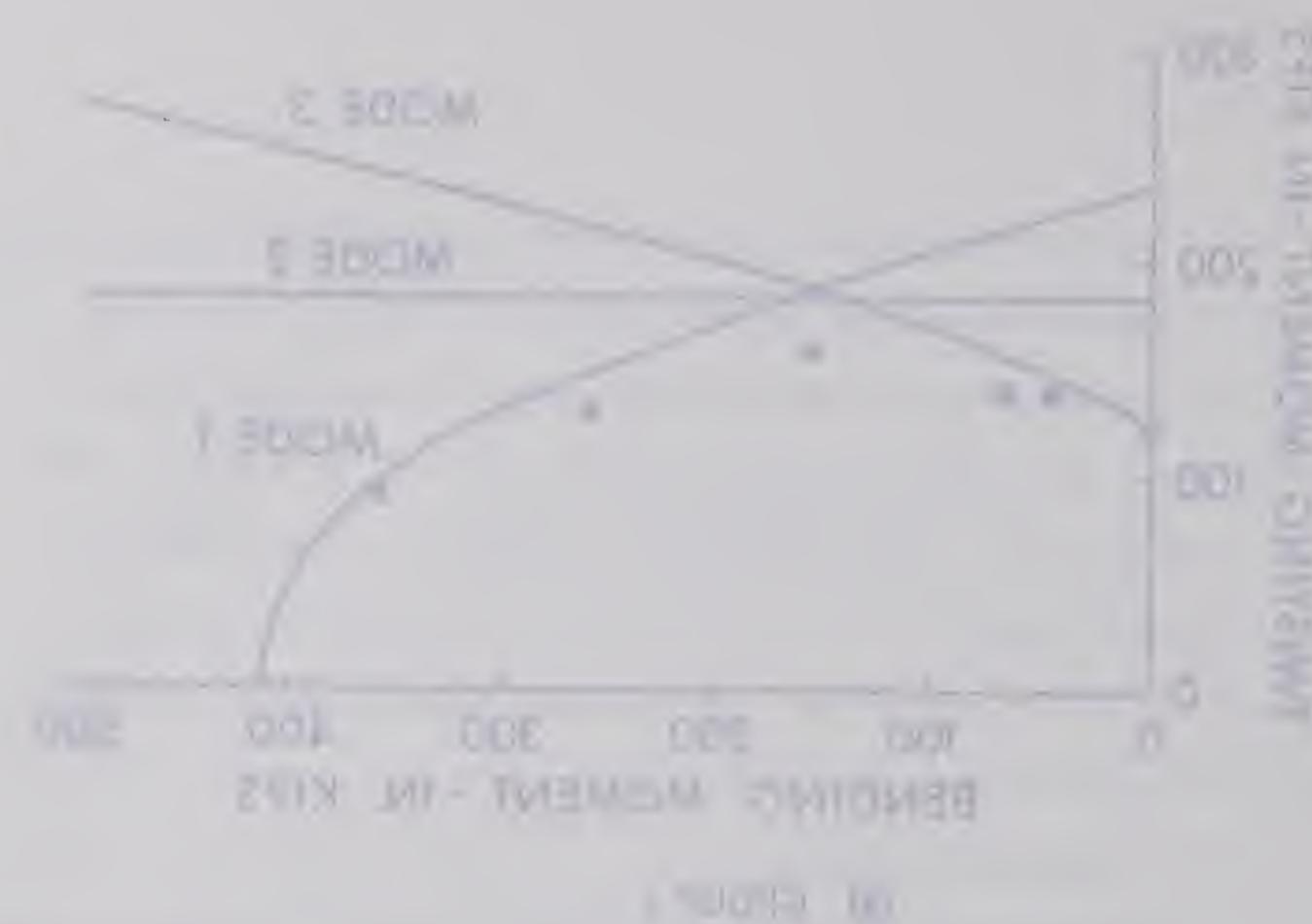


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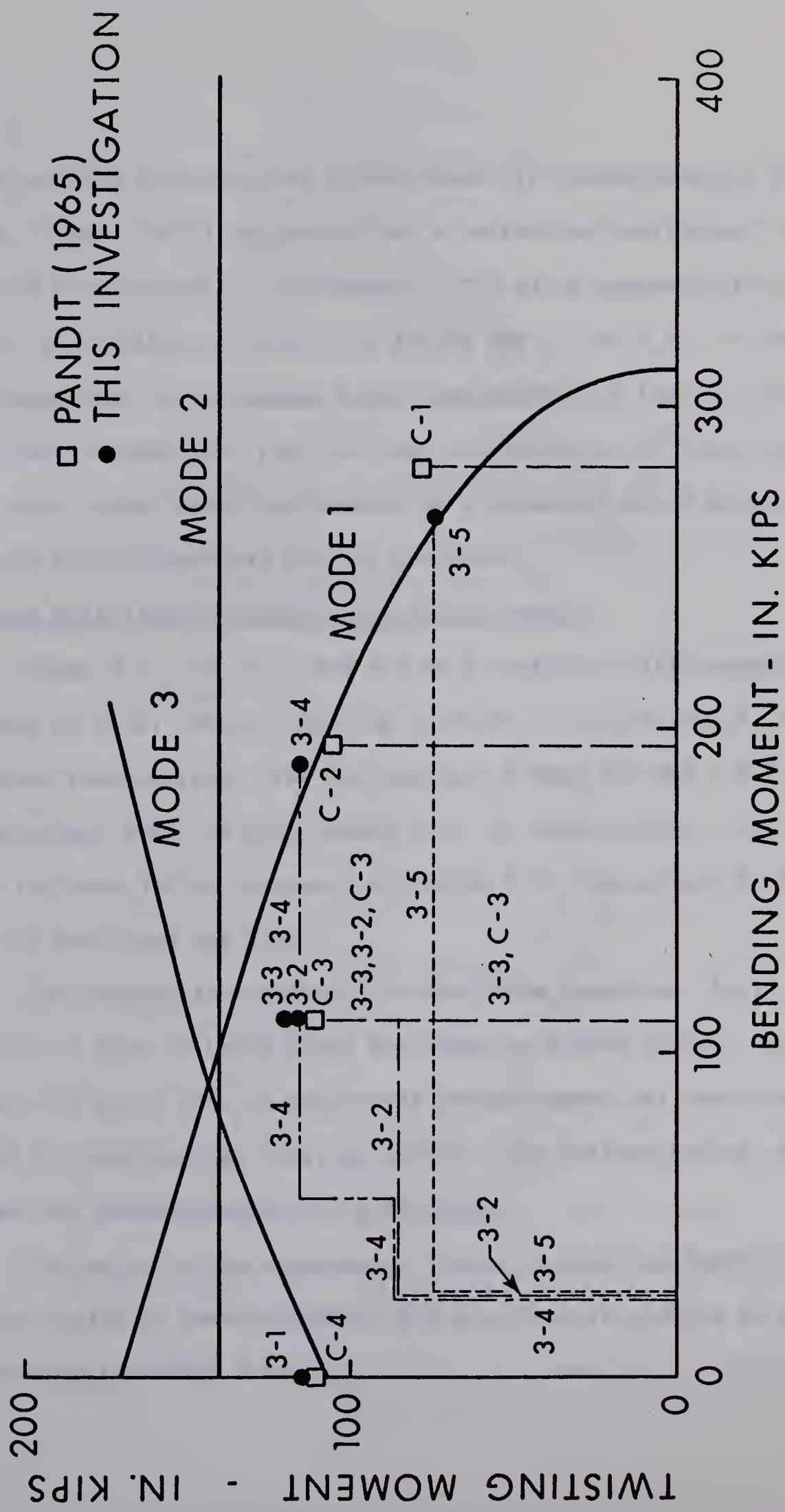


FIGURE 6 - 2 INTERACTION DIAGRAM FOR BEAMS OF GROUP 3



the cross section of transverse reinforcement is concentrated at the tie locations, Lessig (1959) suggested that a correction coefficient be applied to the area of transverse reinforcement. The value suggested for this correction coefficient or efficiency factor was 0.7 or 0.8. To compensate for the transverse reinforcement being concentrated at the tie locations, Pandit (1965) assumed that the ties that are effective in resisting torsion are the least number which are crossed by a potential crack making an angle of  $45^{\circ}$  with the longitudinal axis of the beam.

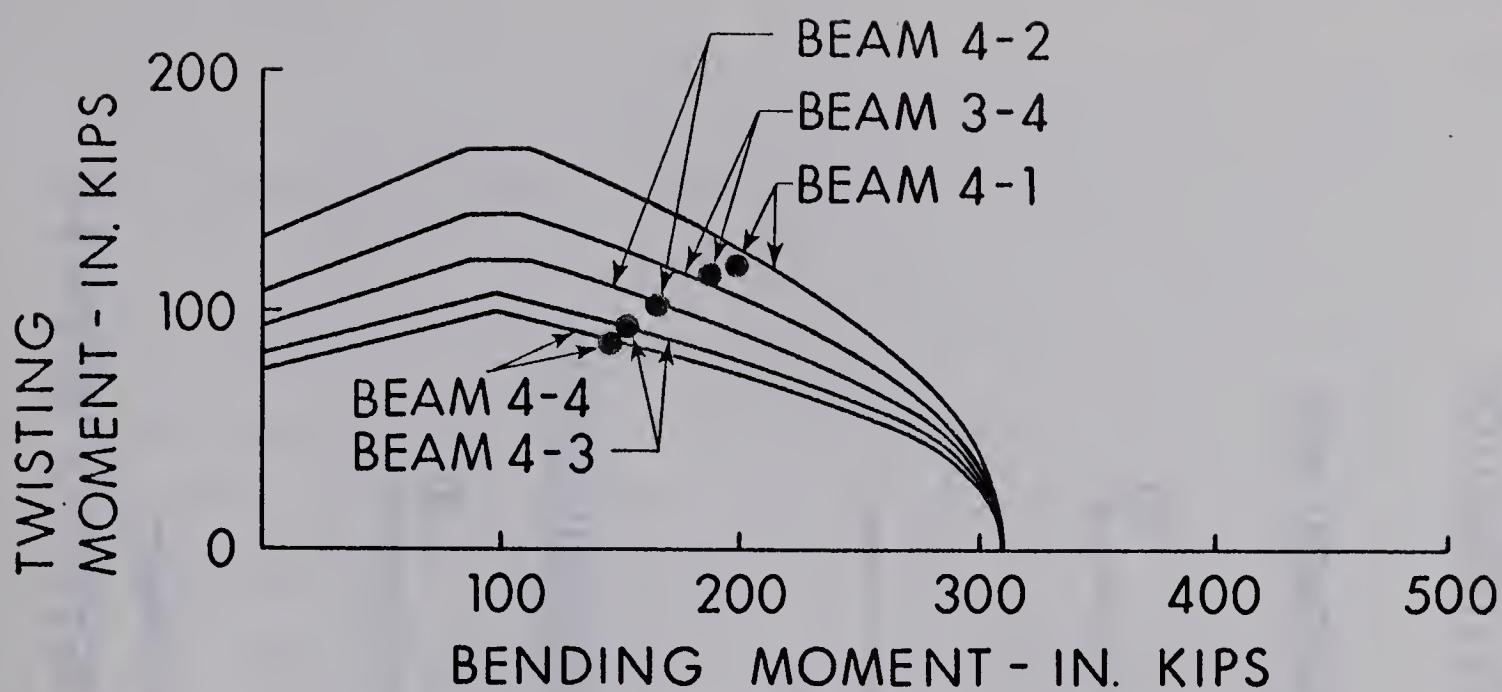
(ii) Beams With Similar Longitudinal Reinforcement

Beams 4-1, 4-2, 4-3, and 4-4 were similar in all respects except the spacing of ties. The tie spacing of Beams 4-1 to 4-4 was 3, 6, 7 1/2 and 9 inches respectively. The tie spacing of Beam 3-4 was 4 1/2 inches. All beams except Beam 3-4 were tested with  $\phi$  equal to 0.61. The loading sequence for Beam 3-4 is indicated in FIGURE 6-2. The value of  $\phi$  at failure for this beam was 0.61.

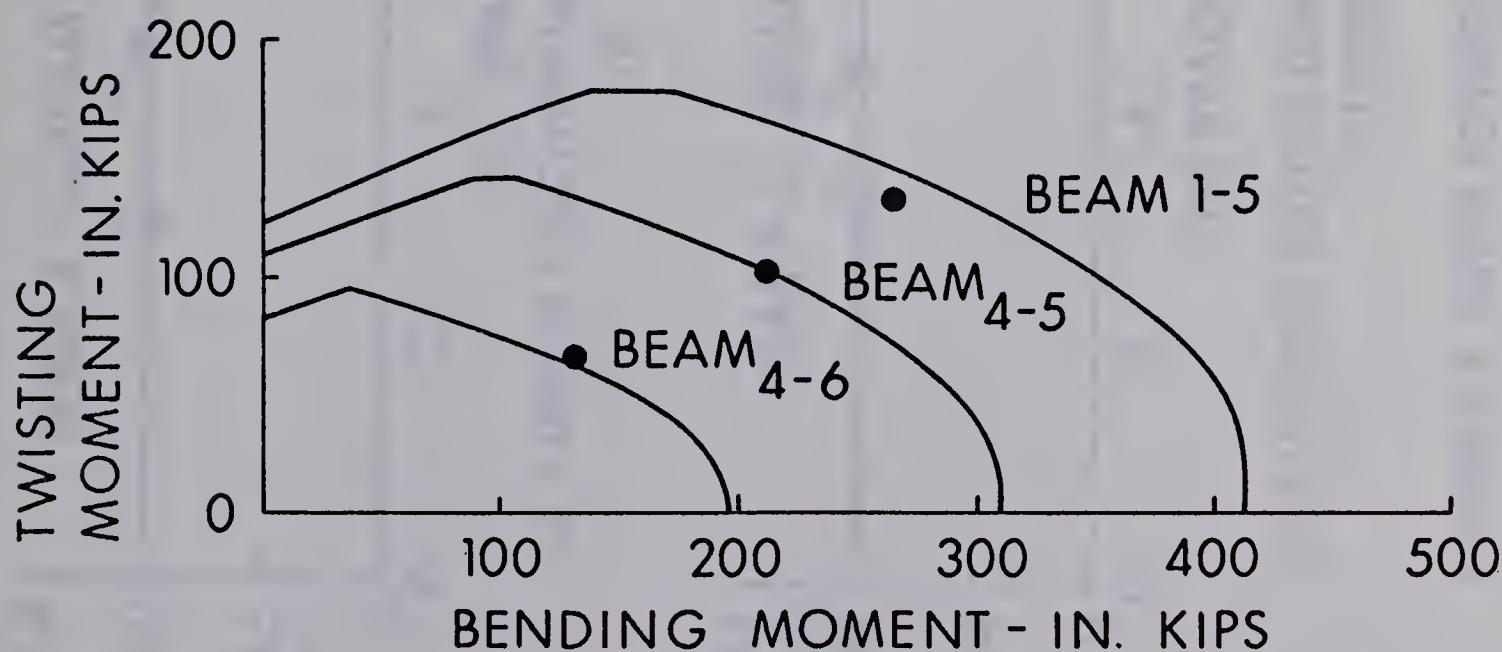
The interaction diagrams obtained from Equations (5-1), (5-5) and (5-10) for each of these beams are shown in FIGURE 6-3(a). In these equations, the total area of transverse reinforcement has been considered effective in resisting the twisting moment. The failure values of twisting moment and bending moment are also shown.

The ratio of the experimental value of twisting moment to the theoretical value of twisting moment for each beam is plotted as a function of tie spacing in FIGURE 6-4(a).





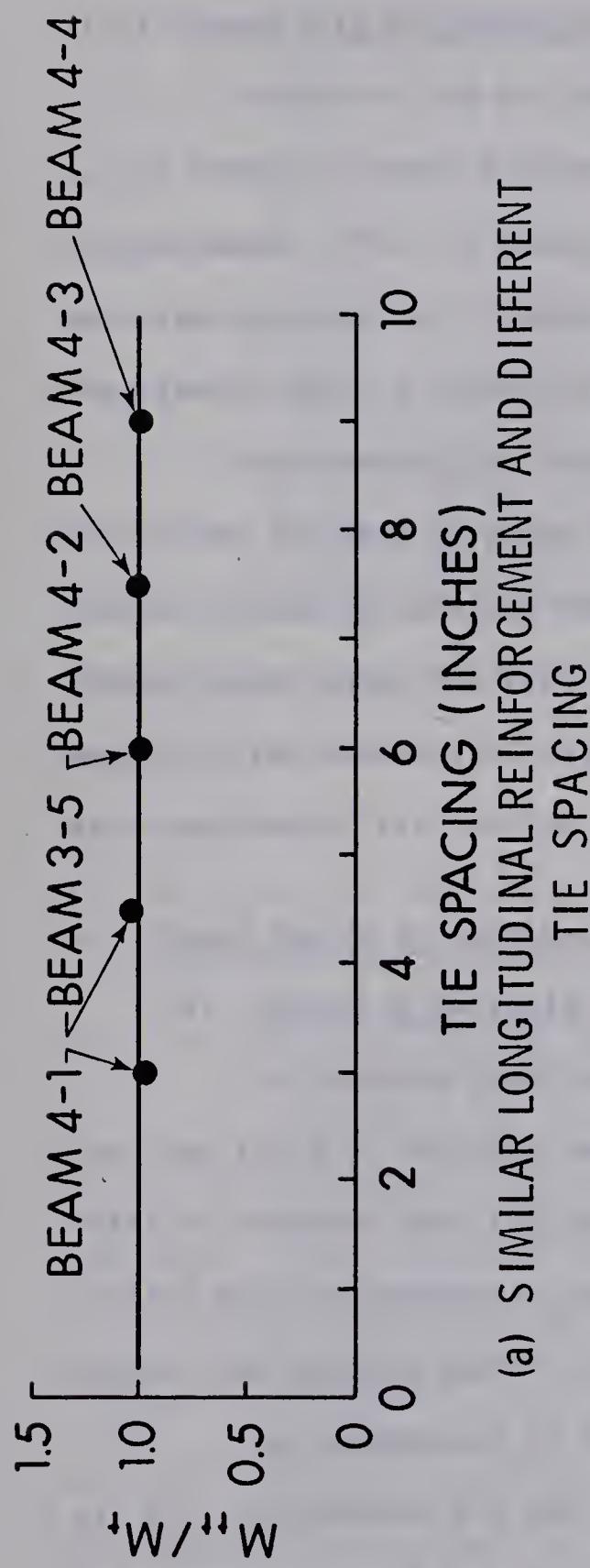
(a) SIMILAR LONGITUDINAL REINFORCEMENT AND DIFFERENT TIE SPACING



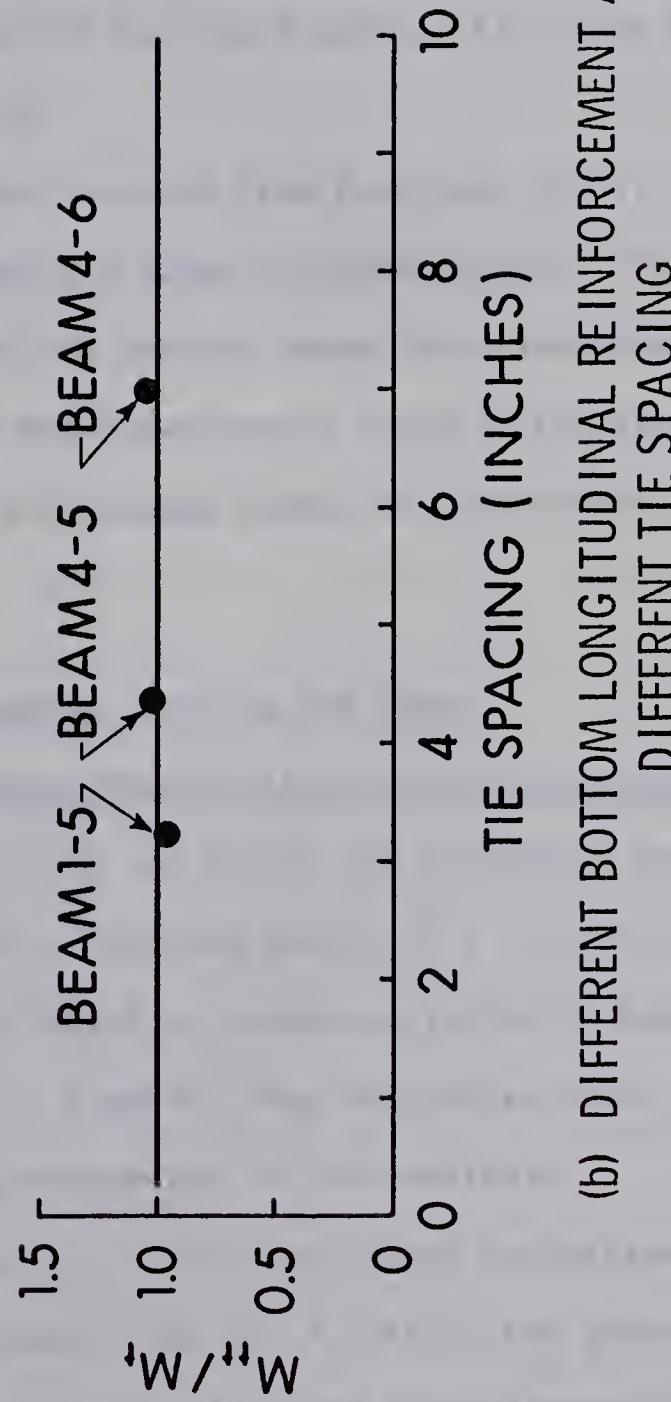
(b) DIFFERENT BOTTOM LONGITUDINAL REINFORCEMENT AND DIFFERENT TIE SPACING

FIGURE 6-3 INTERACTION DIAGRAMS FOR BEAMS WITH DIFFERENT REINFORCEMENT





(a) SIMILAR LONGITUDINAL REINFORCEMENT AND DIFFERENT TIE SPACING



(b) DIFFERENT BOTTOM LONGITUDINAL REINFORCEMENT AND DIFFERENT TIE SPACING

FIGURE 6-4 RATIO BETWEEN EXPERIMENTAL AND THEORETICAL TWISTING MOMENTS AS A FUNCTION OF TIE SPACING



(iii) Beams With Different Longitudinal Reinforcement

Beams 4-5 and 4-6 of Group 4 and Beam 1-5 of Group 1 were similar in all respects except for the spacing of ties and the size of bottom reinforcement. The tie spacing and bottom reinforcement was designed to make the quantity  $p_1$  a constant for the three beams. All three beams were tested with  $\emptyset$  equal to 0.50.

The interaction diagrams obtained from Equations (5-1), (5-5) and (5-10) for each of these beams are shown in FIGURE 6-3(b). The failure values of twisting moment and bending moment are also shown. FIGURE 6-4(b) shows the ratio of the experimental value of twisting moment to the theoretical value of twisting moment for each beam plotted as a function of tie spacing.

6-3 Beams Tested in Combined Bending, Torsion and Shear

(a) Effect of Variable Bending Moment Within the Gauge Length

In deriving Equations (5-1) and (5-10) the assumption was made that the ratio of twisting moment to bending moment,  $\emptyset$ , was known. This ratio is constant over the gauge length of a beam subjected to bending and torsion with no transverse shear. However, when transverse shear is present the bending moment and consequently  $\emptyset$  are variable.

An examination of Equation (5-1) reveals that the minimum value of  $M_{tl}$  is obtained for the minimum value of  $\emptyset$  within the gauge length. In a beam subjected to a constant torsional moment and a linearly varying bending moment, the minimum value of  $\emptyset$  within the gauge length occurs at the cross section subjected to the maximum bending moment. For the beams



of Groups 5, 6 and 7 this cross section was located at the west end of the gauge length.

However, this presupposes that failure occurs at one cross section of the beam. According to FIGURE 5-1, a mode 1 failure occurs on a warped plane extending over a length of beam that is equal to  $c_1$ . The extra reinforcement provided outside of the gauge length forced the failure plane to form within the gauge length. Consequently, it would seem that the failure plane should occur over a length of beam equal to  $c_1$  that is located adjacent to the end of the gauge length subjected to the maximum bending moment. If this is true, the value of  $\emptyset$  which should be used in Equation (5-1) is the value at a cross section located a distance  $c_1/2$  from the west end of the gauge length. This would be the same as using the average bending moment over the length of the warped failure plane to compute the value of  $\emptyset$  to be used in Equation (5-1).

In the beams of Groups 5, 6 and 7 that failed according to mode 1, the failure surface did occur within the region of the gauge length subjected to the highest bending moment. The failure surface appeared to be crowded toward the end of the gauge length and is illustrated by FIGURE 6-5(a). The failure surface in many of these beams appeared to be defined by two sets of spiral cracks. One of these sets contained a crack on the south face which originated at the top of the beam at the west end of the gauge length. This crack progressed down the south face at an angle and continued across the bottom face. The other set





(a) BEAM 6 - 4



(b) BEAM 6 - 1

FIGURE 6 - 5 CRACK PATTERN OF BEAMS 6 - 4 AND 6 - 1



of spiral cracks originated on the bottom face of the beam at the west end of the gauge length and continued up the north face. For a mode 1 failure the reinforcement adjacent to the bottom face of the beam contributes the major portion of the resisting moment. For this reason, there may be justification for considering the critical section to be located a distance equal to  $k_{01}c_1/2$  from the cross section within the gauge length that is subjected to the maximum bending moment rather than  $c_1/2$  as stated previously.

Equation (5-5), which gives the ultimate twisting moment corresponding to a mode 2 failure, does not contain the term  $\emptyset$ . Consequently, a linearly varying bending moment within the gauge length does not affect the location of the failure surface.

An examination of Equation (5-10) reveals that the minimum value of  $M_{t3}$  is obtained for the maximum value of  $\emptyset$  within the gauge length. The maximum value of  $\emptyset$  within the gauge length occurs at the cross section subjected to the minimum bending moment. For the beams of Groups 5, 6 and 7 this cross section was located at the east end of the gauge length. Consequently, the warped failure plane for a mode 3 failure should form over a length of the beam equal to  $c_3$ , measured from the east boundary of the gauge length. In the beams of Groups 5, 6 and 7 that failed according to mode 3, the failure surface was located in the east portion of the gauge length. However, there was a tendency for this failure surface to crowd to the east extremity of the gauge length. This is illustrated by FIGURE 6-5(b). As a result, there may be justification



for considering the critical section to be located a distance equal to  $k_{03}c_3/2$  instead of  $c_3/2$  from the cross section within the gauge length that is subjected to the minimum bending moment.

(b) Solution of Equations When the Bending Moment Varies

For beams having a varying bending moment in their gauge length the value of  $\emptyset$  to be used in Equations (5-1) and (5-10) can not be established until  $c_1$  and  $c_3$  are known but  $c_1$  and  $c_3$  are in turn functions of  $\emptyset$ . Consequently, an iterative procedure was used to solve Equations (5-1) and (5-10). The value of  $\emptyset$  was first computed using the experimental value of twisting moment and the bending moment at the west end of the gauge length. Using this value of  $\emptyset$ , the values of  $c_1$  and  $c_3$  were computed. With these values of  $c_1$  and  $c_3$  a critical section for a mode 1 failure and another critical section for a mode 3 failure were established. Next, the bending moment at each of these critical sections was calculated. Using these bending moments, a new value of  $\emptyset$  was calculated for each critical section and the process was repeated. This procedure was continued for each of the critical sections until the difference between the values of  $\emptyset$  computed in two consecutive iterations was less than one tenth of one percent of  $\emptyset$ .

(c) Effect of Transverse Shear

(i) Comparison of Groups 1, 5 and 6

The beams of Groups 1, 5 and 6 were similar in all respects. The beams of Group 1 were subjected to a uniform twisting moment and a

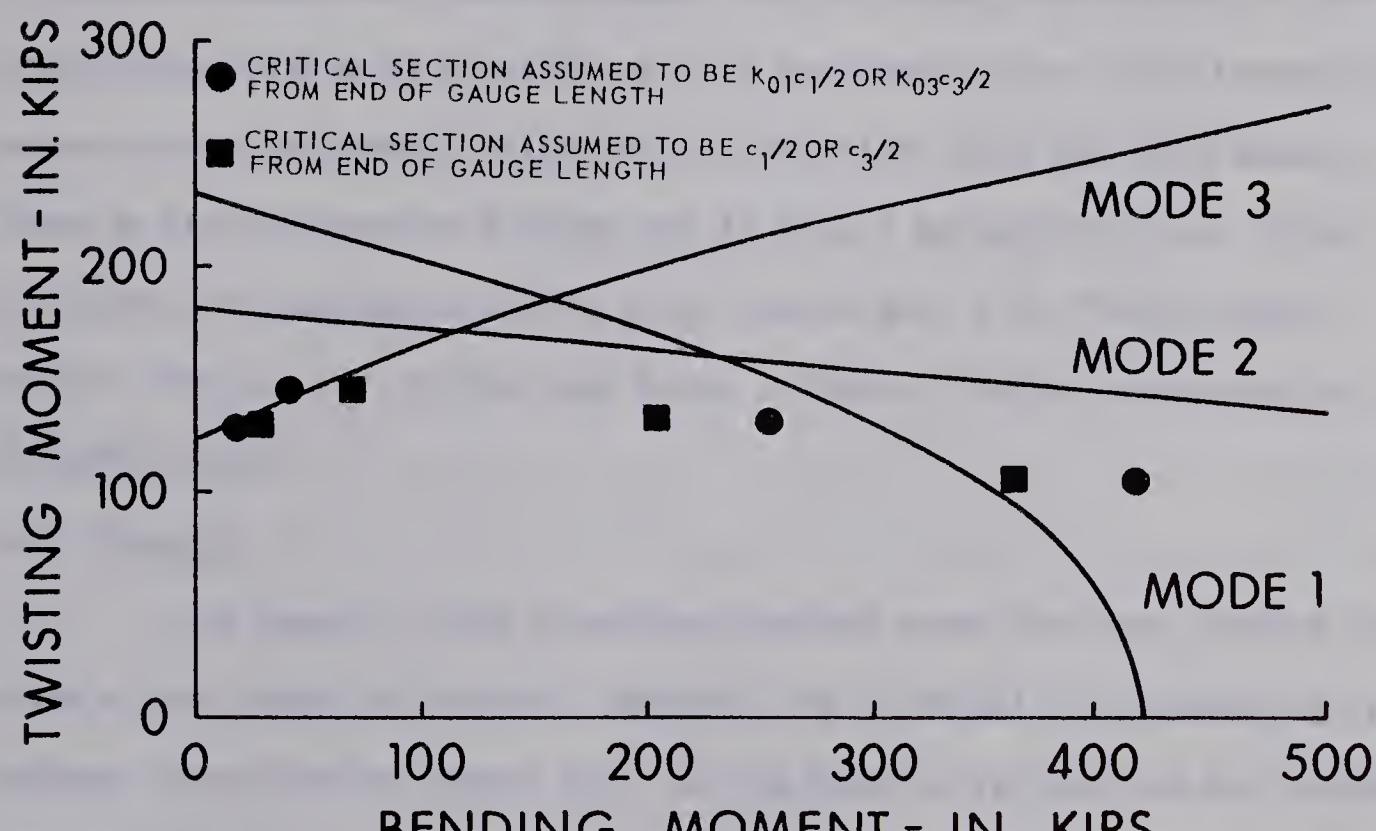
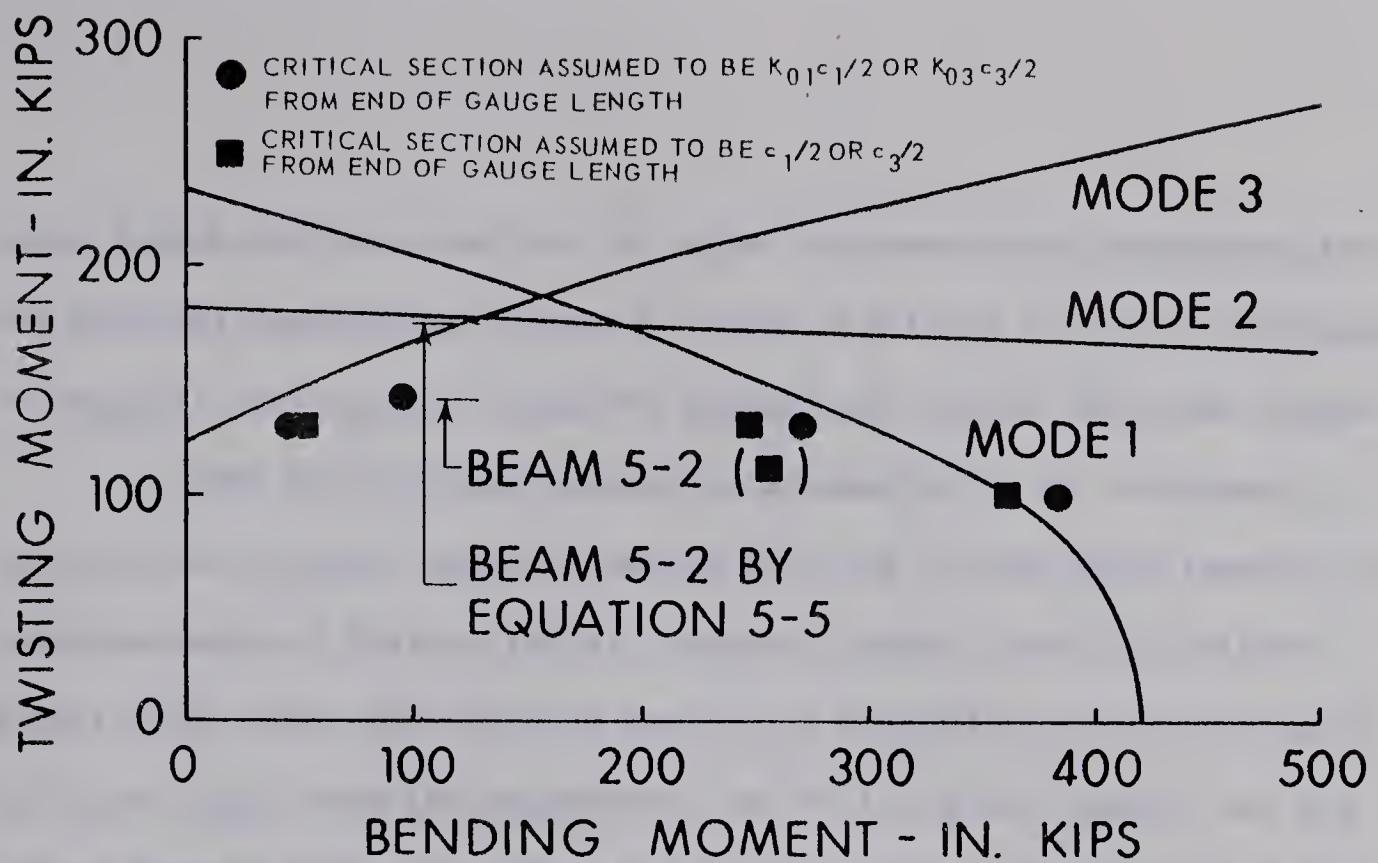


uniform bending moment over their gauge length. However, the beams of Groups 5 and 6 were tested with a single transverse load in addition to the twisting moment. Consequently, the beams of these groups were subjected to a uniform twisting moment, a linearly varying bending moment, and a uniform transverse shear. The magnitude of the transverse shear in any beam of Group 6 is approximately twice that of the corresponding beam of Group 5.

The interaction diagram for the beams of Group 1 is shown in FIGURE 6-1(a). The interaction diagrams for the beams of Groups 5 and 6 are shown in FIGURE 6-6. The lines labeled mode 1, mode 2 and mode 3 were obtained by solving Equations (5-1), (5-5) and (5-10) for various values of  $\emptyset$ . The properties of the beam used in solution of these equations are the average values for the group. Comparison of FIGURES 6-1(a), 6-6(a) and 6-6(b) shows that the curves corresponding to mode 1 and mode 3 are the same for the three interaction diagrams. The reason for this is that the transverse shear does not enter into Equations (5-1) or (5-10). However, the curve corresponding to mode 2, which was a horizontal line in FIGURE 6-1(a), is no longer horizontal in FIGURES 6-6(a) and 6-6(b). This is because the transverse shear does enter into Equation (5-5).

Two sets of failure values of twisting moment and bending moment for each beam have been plotted in FIGURE 6-6. One set of values represents the assumption that the critical section for a mode 1 or mode 3 failure occurs at a distance of  $c_1/2$  or  $c_3/2$  from the appropriate end of the





(b) GROUP 6

FIGURE 6 - 6 INTERACTION DIAGRAMS FOR BEAMS OF GROUPS 5 AND 6



gauge length and the other set of values represents the assumption that the critical section for a mode 1 or mode 3 failure occurs at a distance of  $k_{01}c_1/2$  or  $k_{03}c_3/2$  from the appropriate end of the gauge length.

When the critical section is assumed to be at a distance  $k_{01}c_1/2$  or  $k_{03}c_3/2$  from the appropriate end of the gauge length, the predicted mode of failure for all beams of Groups 5 and 6 is either mode 1 or 3. When the critical section is assumed to be at a distance  $c_1/2$  or  $c_3/2$  from the appropriate end of the gauge length, the predicted mode of failure for Beam 5-2 becomes mode 2. This mode of failure is independent of the bending moment and consequently, the failure values of twisting moment and bending moment are no longer represented on the interaction diagram by a point but by a horizontal line. The theoretical value of twisting moment calculated by Equation (5-5) for this beam is shown on the interaction diagram and is also a horizontal line. This line does not correspond to the line labeled mode 2 in FIGURE 6-6(a) because they are not in the same plane. This is further explained in SECTION 6-3(d).

(ii) Group 7

The beams of this group were tested under the same loading conditions as the beams of Group 6. However, in an effort to increase the ratio between the transverse shear force in the beam at failure and the ultimate transverse shear force which would be expected if the beam were not subjected to torsional moment, the size of the bottom longitudinal bars and the tie spacing were increased. This objective was only partially achieved



since three of the beams of this group failed according to the mode 3 failure surface at relatively low bending moments. Consequently, the transverse shear in the section at failure was also relatively low.

The interaction diagram for the beams of this group is shown in FIGURE 6-7. The lines labeled mode 1, mode 2 and mode 3 were obtained by solving Equations (5-1), (5-5) and (5-10) for various values of  $\emptyset$ . The properties of the beam used in solution of these equations are the average values for the group.

Two sets of failure values of twisting moment and bending moment for each beam are plotted in FIGURE 6-7. One set of values represents the assumption that the critical section for a mode 1 or mode 3 failure occurs at a distance  $c_1/2$  or  $c_3/2$  from the appropriate end of the gauge length and the other set of values represents the assumption that the critical section occurs at a distance of  $k_{01}c_1/2$  or  $k_{03}c_3/2$  from the appropriate end of the gauge length.

Changing the location of the assumed critical section does not change the predicted mode of failure for Beams 7-1 and 7-2 but does change the predicted mode of failure for Beam 7-3 from mode 3 to mode 2 and for Beam 7-4 from mode 1 to mode 2. Consequently, for Beams 7-3 and 7-4 the failure values and the theoretical values of twisting moment and bending moment corresponding to a critical section assumed to be  $c_1/2$  or  $c_3/2$  from the appropriate end of the gauge length are represented on the interaction diagram by horizontal lines. The lines representing the theoretical values of twisting moment and bending moment do not



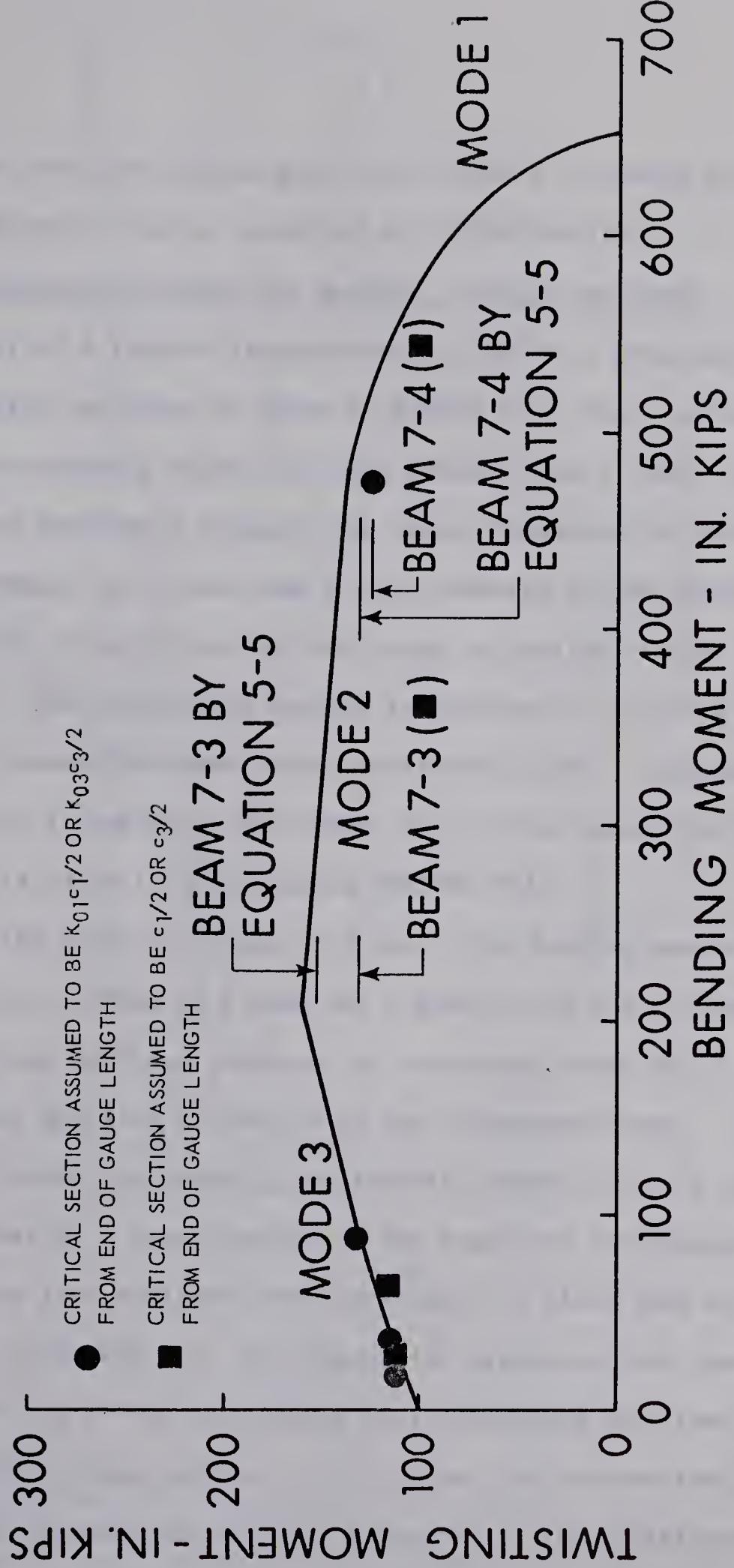


FIGURE 6-7 INTERACTION DIAGRAM FOR BEAMS OF GROUP 7



correspond to the line labeled mode 2 in FIGURE 6-7 because they are not in the same plane. This is explained in SECTION 6-3(d).

(d) Interaction Surface for Bending, Torsion and Shear

Part of a typical interaction surface for a beam subjected to bending, torsion and shear is shown in FIGURE 6-8. This surface is composed of three segments which have been labeled mode 1, mode 2 and mode 3. The mode 1 and the mode 3 segments are single curvature surfaces whereas the mode 2 segment is a plane that slopes downward in the direction of the shear axis. A level line in this plane is parallel to the bending moment axis. This interaction surface is intended to be valid only within the range of transverse shear force considered in this investigation and consequently is incomplete. The upper limit of the transverse shear force for which it is valid is discussed in CHAPTER VIII.

In the tests of Groups 5, 6 and 7, the bending moment at any particular cross section of a beam was a function of the transverse shear and since any one test was conducted at a constant value of  $\emptyset$ , the twisting moment was also a function of the transverse shear. Consequently, the locus of points representing the twisting moment, bending moment and transverse shear at a cross section of the beam from the beginning of the test to failure is a straight line contained in a plane such as plane 0, 1, 2, 3 shown in FIGURE 6-8. An interaction diagram of the type shown in FIGURES 6-6 and 6-7 is obtained by first obtaining the lines of intersection between a plane such as 0, 1, 2, 3 and the interaction surface. These lines of intersection are then projected on the twisting moment-bending



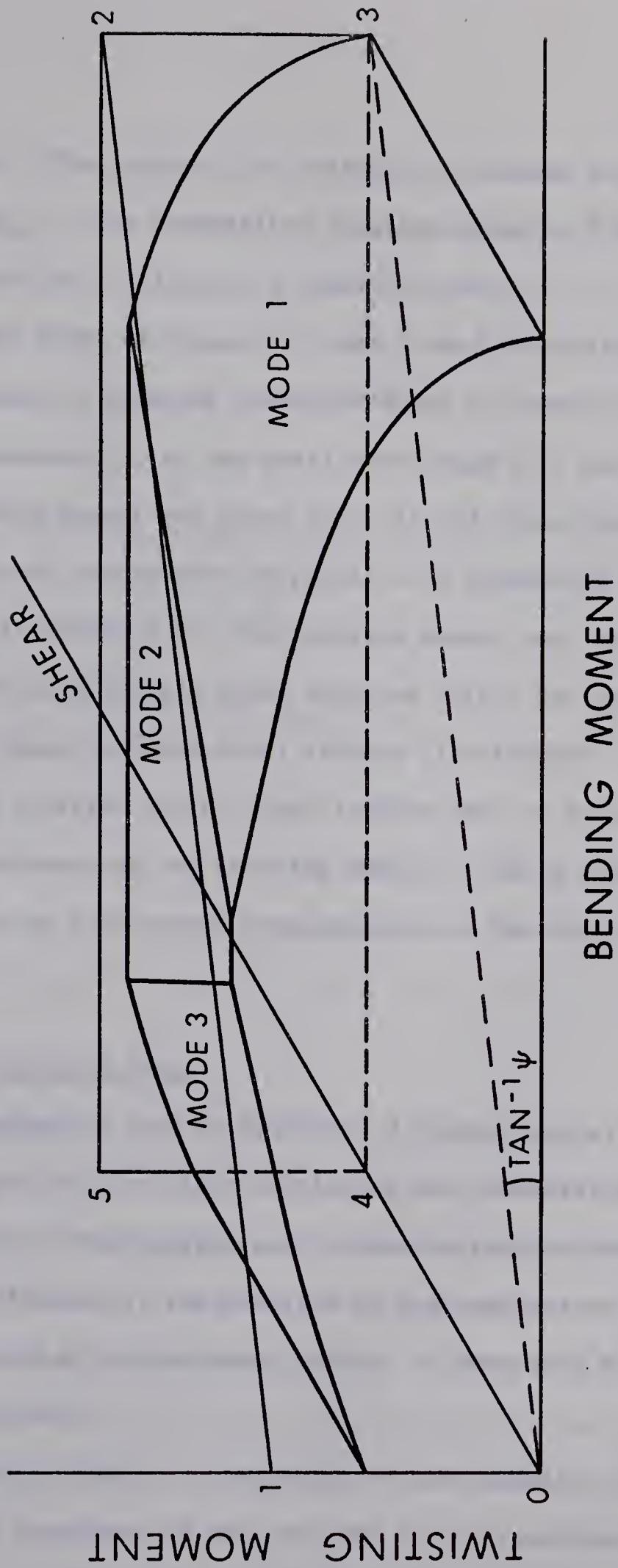


FIGURE 6 - 8 INTERACTION SURFACE FOR BENDING TORSION AND SHEAR



moment plane. The shape of the interaction diagram depends on the value of  $\psi = V_t / M_{bt}$ . The interaction diagrams shown in FIGURES 6-1, 6-2 and 6-3 correspond to a value of  $\psi$  equal to zero.

The beams of Groups 5, 6 and 7 were subjected to a constant twisting moment, a constant shear force and a linearly varying bending moment. Consequently, at any particular stage of a test the twisting moment, bending moment and shear force at all cross sections within the gauge length are represented by points in a plane such as plane 2, 3, 4, 5, shown in FIGURE 6-8. The twisting moment and transverse shear force are the same for all cross sections within the gauge length but the bending moment at each cross section is different. Consequently, changing the location of the cross section that is being considered causes the point representing the twisting moment, bending moment and transverse shear to move in a direction perpendicular to the shear-twisting moment plane.

#### 6-4 Over Reinforced Beams

Assumption four of SECTION 5-3 states that all reinforcing bars which cross the failure plane outside of the compression zone yield at failure. If the amount of longitudinal and transverse reinforcement in a beam is increased sufficiently, the concrete in the compression zone will fail before the tension reinforcement yields. A beam such as this is said to be over reinforced.

Lessig (1961), on the basis of test results, derived two equations to be used to determine if the concrete in the compression zone will fail



before the tension reinforcement yields. The first of these equations establishes a maximum value for  $x_1$  and, according to Lessig, is valid for  $\phi \leq 0.2$ . This equation states that if a beam is not over reinforced,

$$x_1 \leq h_{01} \left[ 0.55 - 0.7(\phi)^{1/2} \right] \quad (6-1)$$

Lessig states that the second of these equations is valid for  $\phi > 0.2$ . This equation is:

$$\begin{aligned} M_{tr} &= 0.07 b^2 h (0.85 k_1 f'_c) \\ M_{tr} &= 0.06 b^2 h k_1 f'_c \end{aligned} \quad (6-2)$$

where  $M_{tr}$  is equal to the maximum torsional moment that can be resisted by a beam that fails due to crushing of the concrete before the reinforcement yields.

#### 6-5 Ratio of Transverse to Longitudinal Reinforcement

Assumption four of SECTION 5-3 states that all reinforcing bars crossing the failure plane outside of the compression zone yield at failure. It is possible that at failure only the longitudinal or only the transverse reinforcement yields, but not both. To ensure that the assumption that both types of reinforcement yield is not violated, it is necessary to restrict the ratio between the transverse and the longitudinal rein-



forcement within certain limits.

Lessig (1961) defined the optimum ratio between transverse and longitudinal reinforcement as the ratio that makes the product of the volume of reinforcement per unit length of beam and the yield stress of the reinforcement a minimum. For a mode 1 failure the following equation was proposed.

$$m_{01} = \frac{1}{1 + \frac{2}{\phi} \left( \frac{b}{2h + b} \right)^{1/2}} \quad (6-3)$$

where  $m_{01}$  = optimum value of  $m_1$

$$m_1 = p_1 b/h$$

For a mode 2 failure the following equation was proposed.

$$m_{02} = 1 \quad (6-4)$$

where  $m_{02}$  = optimum value of  $m_2$

$$m_2 = p_2 h/b$$

The corresponding equation derived on the basis of a mode 3 failure is:

$$m_{03} = \frac{1}{1 - \frac{2}{\phi} \left( \frac{b}{2h + b} \right)^{1/2}} \quad (6-5)$$

where  $m_{03}$  = optimum value of  $m_3$

$$m_3 = p_3 b/h$$



On the basis of experimental evidence, Lessig recommended that the following limits be imposed on the values of  $m_1$  and  $m_2$ .

$$0.5 m_{01} \leq m_1 \leq 1.5 m_{01} \quad (6-6)$$

$$0.5 m_{02} \leq m_2 \leq 1.5 m_{02} \quad (6-7)$$

Since Lessig did not consider a mode 3 failure, no limits on  $m_3$  were suggested.

## 6-6 Correlation of Test Results

### (a) Beams Tested in this Investigation

The theoretical and experimental results for the 34 beams of this investigation are presented in TABLES 6-1 and 6-2. TABLE 6-1 contains the beams tested in combined bending and torsion and TABLE 6-2 contains the beams tested in combined bending, torsion and shear.

The value of  $x$  listed in these tables is the value of  $x_1$ ,  $x_2$ , or  $x_3$  corresponding to the predicted mode of failure. That is, if the predicted failure is mode 2,  $x$  is taken equal to  $x_2$ . Similarly, the value of  $m$  is taken equal to  $m_1$ ,  $m_2$  or  $m_3$  and the value of  $m_o$  is taken equal to  $m_{01}$ ,  $m_{02}$  or  $m_{03}$  for a predicted mode 1, mode 2 or mode 3 failure respectively. When two observed modes of failure are listed, the first one listed is the one that was predominant. These observations are presented in CHAPTER IV.

The value of  $M_{bt}$  listed in TABLE 6-2 is the bending moment



TABLE 6-1  
COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS  
TESTS IN COMBINED BENDING AND TORSION IN THIS INVESTIGATION

| Beam | $M_{bt}$<br>in.kips | $M_{tt}$<br>in.kips | $\phi$   | $M_{tr}$<br>in.kips | $m$  | $m_o$ | $\frac{m}{m_o}$ | $x$<br>inches | $M_{t1}$<br>in.kips | $M_{t2}$<br>in.kips | $M_{t3}$<br>in.kips | $\frac{M_{tt}}{M_t}$ | $\frac{M_{tt}}{M_t}$<br>Failure | Predicted<br>Mode of<br>Failure | Observed<br>Mode of<br>Failure |
|------|---------------------|---------------------|----------|---------------------|------|-------|-----------------|---------------|---------------------|---------------------|---------------------|----------------------|---------------------------------|---------------------------------|--------------------------------|
| 1-1  | 0                   | 122                 | $\infty$ | 114                 | 0.99 | 1.00  | 0.99            | 0.19          | 234                 | 184                 | 125                 | 0.97                 | 3                               | 3                               | 3                              |
| 1-2  | 47                  | 138                 | 2.94     | 109                 | 1.02 | 1.45  | 0.70            | 0.19          | 216                 | 186                 | 151                 | 0.92                 | 3                               | 3                               | 3                              |
| 1-3  | 70                  | 140                 | 2.00     | 106                 | 0.96 | 1.80  | 0.53            | 0.17          | 202                 | 181                 | 157                 | 0.89                 | 3                               | 3                               | 3                              |
| 1-4  | 159                 | 159                 | 1.00     | 118                 | 0.29 | 0.52  | 0.56            | 0.31          | 182                 | 186                 | 206                 | 1.82                 | 1                               | 2 & 1                           | 1                              |
| 1-5  | 267                 | 131                 | 0.49     | 102                 | 0.27 | 0.36  | 0.75            | 0.42          | 138                 | 181                 | 295                 | 1.38                 | 1                               | 1                               | 1                              |
| 1-6  | 362                 | 90                  | 0.25     | 112                 | 0.26 | 0.22  | 1.18            | 0.61          | 91                  | 180                 | 851                 | 0.99                 | 1                               | 1                               | 1                              |
| 2-1  | 0                   | 181                 | $\infty$ | 130                 | 0.53 | 1.00  | 0.53            | 0.41          | 235                 | 230                 | 235                 | 2.30                 | 2                               | 2 & 1                           | 1                              |
| 2-2  | 88                  | 172                 | 1.96     | 119                 | 0.28 | 0.68  | 0.41            | 0.26          | 205                 | 229                 | 268                 | 205                  | 0.84                            | 1                               | 1                              |
| 2-3  | 166                 | 166                 | 1.00     | 112                 | 0.26 | 0.53  | 0.49            | 0.28          | 176                 | 223                 | 293                 | 176                  | 0.94                            | 1                               | 1                              |
| 2-4  | 267                 | 134                 | 0.50     | 117                 | 0.28 | 0.36  | 0.78            | 0.39          | 140                 | 227                 | 404                 | 140                  | 0.96                            | 1                               | 1                              |
| 2-5  | 362                 | 90                  | 0.25     | 123                 | 0.28 | 0.21  | 1.33            | 0.64          | 92                  | 230                 | 1486                | 92                   | 0.98                            | 1                               | 1                              |
| 3-1  | 0                   | 115                 | $\infty$ | 119                 | 0.72 | 1.00  | 0.72            | 0.14          | 171                 | 141                 | 107                 | 1.07                 | 3                               | 3                               | 3                              |
| 3-2  | 111                 | 117                 | 1.05     | 124                 | 0.29 | 0.54  | 0.54            | 0.21          | 134                 | 143                 | 160                 | 134                  | 0.87                            | 1                               | 1                              |
| 3-3  | 111                 | 120                 | 1.08     | 126                 | 0.29 | 0.54  | 0.54            | 0.20          | 135                 | 143                 | 159                 | 135                  | 0.89                            | 1                               | 1                              |
| 3-4  | 188                 | 115                 | 0.61     | 121                 | 0.28 | 0.40  | 0.70            | 0.25          | 112                 | 141                 | 204                 | 112                  | 1.03                            | 1                               | 1                              |
| 3-5  | 265                 | 73                  | 0.28     | 131                 | 0.29 | 0.23  | 1.26            | 0.41          | 73                  | 143                 | 562                 | 73                   | 1.00                            | 1                               | 1                              |
| 4-1  | 200                 | 120                 | 0.60     | 119                 | 0.41 | 0.40  | 1.03            | 0.38          | 123                 | 170                 | 280                 | 123                  | 0.97                            | 1                               | 1                              |
| 4-2  | 166                 | 101                 | 0.61     | 129                 | 0.21 | 0.40  | 0.53            | 0.17          | 102                 | 123                 | 164                 | 102                  | 0.99                            | 1                               | 1                              |
| 4-3  | 153                 | 93                  | 0.61     | 116                 | 0.16 | 0.41  | 0.39            | 0.13          | 93                  | 107                 | 132                 | 93                   | 1.00                            | 1                               | 1 & 2                          |
| 4-4  | 146                 | 85                  | 0.58     | 124                 | 0.14 | 0.39  | 0.36            | 0.11          | 88                  | 100                 | 123                 | 88                   | 0.97                            | 1                               | 1 & 2                          |
| 4-5  | 212                 | 103                 | 0.49     | 127                 | 0.29 | 0.35  | 0.83            | 0.28          | 102                 | 143                 | 241                 | 102                  | 1.01                            | 1                               | 1                              |
| 4-6  | 132                 | 66                  | 0.50     | 121                 | 0.28 | 0.36  | 0.78            | 0.16          | 63                  | 156                 | 156                 | 63                   | 1.04                            | 1                               | 1                              |

Average Deviation 0.055



TABLE 6-2  
COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS  
TESTS IN COMBINED BENDING, TORSION AND SHEAR IN THIS INVESTIGATION

| Beam | $V_t$<br>kips | $M_{bt}$<br>in.kips. | $M_{tt}$<br>in.kips. | $\emptyset$ | $M_{tr}$<br>in.kips | $m$  | $m_o$ | $\frac{m}{m_o}$ | $x$<br>inches | $M_{t1}$<br>in.kips | $M_{t2}$<br>in.kips | $M_{t3}$<br>in.kips | $\frac{M_{tt}}{M_t}$ | Predicted<br>Mode of<br>Failure | Observed<br>Mode of<br>Failure |
|------|---------------|----------------------|----------------------|-------------|---------------------|------|-------|-----------------|---------------|---------------------|---------------------|---------------------|----------------------|---------------------------------|--------------------------------|
| 5-1  | 0.76          | 65                   | 128                  | 1.97        | 124                 | 0.97 | 1.56  | 0.62            | 0.16          | 206                 | 180                 | 151                 | 0.85                 | 3                               | 3                              |
| 5-2  | 1.89          | 143                  | 141                  | 0.99        | 138                 | 0.83 | 1.00  | 0.83            | 0.31          | 186                 | 178                 | 184                 | 0.79                 | 2                               | 2 & 3                          |
| 5-3  | 3.87          | 278                  | 130                  | 0.47        | 119                 | 0.26 | 0.37  | 0.70            | 0.34          | 141                 | 166                 | 262                 | 141                  | 0.92                            | 1                              |
| 5-4  | 5.51          | 389                  | 99                   | 0.25        | 125                 | 0.28 | 0.23  | 1.22            | 0.55          | 98                  | 158                 | 551                 | 98                   | 1.01                            | 1                              |
| 6-1  | 1.81          | 65                   | 129                  | 1.98        | 126                 | 0.97 | 1.27  | 0.76            | 0.16          | 212                 | 176                 | 140                 | 0.92                 | 3                               | 3                              |
| 6-2  | 4.27          | 149                  | 145                  | 0.97        | 115                 | 0.93 | 1.76  | 0.53            | 0.15          | 188                 | 166                 | 153                 | 0.95                 | 3                               | 3 & 2                          |
| 6-3  | 7.67          | 264                  | 132                  | 0.50        | 124                 | 0.28 | 0.42  | 0.67            | 0.32          | 157                 | 157                 | 210                 | 0.84                 | 1                               | 1                              |
| 6-4  | 12.45         | 427                  | 107                  | 0.25        | 129                 | 0.28 | 0.25  | 1.12            | 0.52          | 104                 | 137                 | 411                 | 104                  | 1.03                            | 1                              |
| 7-1  | 1.56          | 56                   | 112                  | 2.00        | 116                 | 0.60 | 1.30  | 0.46            | 0.11          | 226                 | 170                 | 108                 | 1.03                 | 3                               | 3                              |
| 7-2  | 3.32          | 115                  | 115                  | 1.00        | 117                 | 0.63 | 2.02  | 0.31            | 0.11          | 221                 | 166                 | 125                 | 0.92                 | 3                               | 3                              |
| 7-3  | 8.11          | 275                  | 132                  | 0.48        | 129                 | 0.34 | 1.00  | 0.34            | 0.22          | 201                 | 154                 | 172                 | 154                  | 0.86                            | 2                              |
| 7-4  | 14.97         | 505                  | 125                  | 0.25        | 107                 | 0.36 | 1.00  | 0.36            | 0.18          | 157                 | 132                 | 275                 | 132                  | 0.95                            | 2                              |

Average Deviation 0.059  
Average 0.92



at the west end of the gauge length and is the maximum bending moment within the gauge length. The value of  $\phi$  listed is based on this bending moment.

$M_{t1}$  has been calculated on the basis of the assumption that the critical section is located a distance  $c_1/2$  from the west end of the gauge length and  $M_{t3}$  has been calculated on the basis of the assumption that the critical section is located a distance  $c_3/2$  from the east end of the gauge length. Similarly,  $m_{01}$  and  $m_{03}$  were calculated using the value of  $\phi$  existing at the appropriate critical section.

(b) Beams Tested by Other Investigators

The theoretical and experimental results for beams tested by other investigators are presented in TABLES 6-3 to 6-7. The theoretical results are the results obtained using the equations presented in CHAPTER V.

The results for 22 of the beams tested by Pandit (1965) are presented in TABLE 6-3. For those beams tested in combined bending, torsion and shear, the value of  $M_{bt}$  listed in this table is the average value of bending moment over the gauge length. The value of  $\phi$  for each beam has been computed using this value of bending moment. The values of  $M_t$  are based on the assumption that the critical section is located a distance  $c_1/2$  from the west end of the gauge length for a mode 1 failure and a distance  $c_3/2$  from the east end of the gauge length for a mode 3 failure.

Since the analysis presented in CHAPTER V is limited to reinforced concrete beams, the plain concrete beams tested by Pandit are not included in this table. Pandit's Group H beams have also been omitted



TABLE 6-3

## COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

## TESTS BY PANDIT (1965)

| Beam | b<br>inches | h<br>inches | $f'_c$<br>psi | Reinforcement | $V_t$<br>kips | $M_{bt}$<br>in.kips | $M_{tt}$<br>in.kips | $\vartheta$ | $\frac{M_t}{M_t}$ |
|------|-------------|-------------|---------------|---------------|---------------|---------------------|---------------------|-------------|-------------------|
| B-2  | 6.20        | 12.20       | 4640          | 2#3           | #3@6"         | 0                   | 195                 | 72          | 0.37              |
| B-3  | 6.05        | 12.20       | 4690          | 2#3           | 2#4           | #3@6"               | 0                   | 110         | 95                |
| B-4  | 6.10        | 12.20       | 5070          | 2#3           | 2#4           | #3@6"               | 0                   | 0           | 85                |
| C-1  | 6.00        | 12.20       | 4980          | 2#3           | 2#5           | #3@4½               | 0                   | 280         | 78                |
| C-2  | 6.15        | 12.20       | 4820          | 2#3           | 2#5           | #3@4½               | 0                   | 195         | 105               |
| C-3  | 6.10        | 12.20       | 5340          | 2#3           | 2#5           | #3@4½               | 0                   | 110         | 111               |
| C-4  | 6.50        | 12.20       | 5800          | 2#3           | 2#5           | #3@4½               | 0                   | 0           | 111               |
| D-1  | 6.20        | 12.20       | 4880          | 2#5           | 4#6           | #3@4½               | 0                   | 637         | 99                |
| D-2  | 6.20        | 12.20       | 5100          | 2#5           | 4#6           | #3@4½               | 0                   | 365         | 164               |
| D-3  | 6.20        | 12.20       | 4650          | 2#5           | 4#6           | #3@4½               | 0                   | 195         | 156               |
| D-4  | 6.20        | 12.20       | 5080          | 2#5           | 4#6           | #3@4½               | 0                   | 0           | 146               |
| E-1  | 6.20        | 12.20       | 4910          | 2#4           | 2#4           | #3@4½               | 0                   | 195         | 76                |
| E-2  | 6.20        | 12.20       | 5100          | 2#4           | 2#4           | #3@4½               | 0                   | 110         | 101               |
| E-3  | 6.20        | 12.20       | 4700          | 2#4           | 2#4           | #3@4½               | 0                   | 0           | 121               |
| F-1  | 6.10        | 12.20       | 5060          | 3#3           | 3#3           | #3@8"               | 9.2                 | 140         | 58                |
| F-2  | 6.10        | 12.20       | 5060          | 3#3           | 3#3           | #3@8"               | 6.2                 | 95          | 83                |
| F-3  | 6.15        | 12.20       | 4650          | 3#3           | 3#3           | #3@8"               | 3.2                 | 50          | 89                |
| F-4  | 6.10        | 12.20       | 4650          | 3#3           | 3#3           | #3@8"               | 0                   | 0           | 95                |
| G-1  | 6.20        | 12.20       | 4560          | 3#3           | 3#3           | #3@3½               | 10.2                | 155         | 73                |
| G-2  | 6.20        | 12.20       | 4560          | 3#3           | 3#3           | #3@3½               | 6.2                 | 95          | 92                |
| G-3  | 6.15        | 12.20       | 4920          | 3#3           | 3#3           | #3@3½               | 3.2                 | 50          | 103               |
| G-4  | 6.20        | 12.20       | 4920          | 3#3           | 3#3           | #3@3½               | 0                   | 0           | 115               |

Average\* 0.98

Average Deviation\* 0.102

\*Not Including Beam D-3



since they were not provided with any top longitudinal reinforcement.

One of the top longitudinal bars and one of the bottom longitudinal bars in Beam D-3 were provided with strain gauges. Strain readings indicated that neither the top nor the bottom reinforcement had yielded when failure occurred. Consequently, it would appear that this beam was over reinforced.

Beams F-1 to G-4 had three longitudinal bars adjacent to each horizontal face. In the computations for mode 2, the bars at the center of the horizontal faces have been assumed to yield and their reduced moment arm has been taken into consideration.

TABLE 6-4 presents the results for 13 of the beams tested by Chinenkov (1959). Only the beams that were provided with transverse reinforcement adjacent to all faces are included. The cylinder strengths listed in this table were obtained by taking 95 percent of the cube strengths given by Chinenkov.

TABLE 6-5 presents the results for the 12 beams tested by Gesund, Schuette, Buchanan and Gray (1964).

TABLE 6-6 presents the results for 17 of the beams tested by Lyalin (1959). Beams that failed by crushing of the concrete before the reinforcement yielded and beams that failed by breaking of the welds between the transverse and longitudinal reinforcement are not included in this table. The concrete cylinder strengths listed were obtained by taking 95 percent of the cube strengths given by Lyalin. The value of  $M_{bt}$  listed in this table is the maximum bending moment in the



TABLE 6-4  
COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS  
TESTS IN COMBINED BENDING AND TORSION BY CHINENKOV (1959)

| Beam       | b<br>inches | h<br>inches | $f'_c$<br>psi | No. of<br>Bars | Reinforcement             |                              | Ties<br>Diameter<br>inches | Spacing<br>inches | $M_{bt}$<br>in.kips | $M_{tt}$<br>in.kips | $\phi$ | $M_t$<br>in.kips | $\frac{M_{tt}}{M_t}$ |      |
|------------|-------------|-------------|---------------|----------------|---------------------------|------------------------------|----------------------------|-------------------|---------------------|---------------------|--------|------------------|----------------------|------|
|            |             |             |               |                | Top<br>Diameter<br>inches | Bottom<br>Diameter<br>inches |                            |                   |                     |                     |        |                  |                      |      |
| B-2-8-0.1  | 7.90        | 11.80       | 1460          | 2              | 0.79                      | 2                            | 0.79                       | 0.276             | 3.16                | 486                 | 49     | 0.10             | 47                   | 1.04 |
| B-2-9-0.1a | 7.90        | 12.00       | 1460          | 2              | 0.79                      | 2                            | 0.79                       | 0.276             | 3.16                | 469                 | 47     | 0.10             | 46                   | 1.02 |
| B-2-8-0.2  | 7.90        | 12.20       | 1220          | 2              | 0.79                      | 2                            | 0.79                       | 0.276             | 3.16                | 417                 | 83     | 0.20             | 84                   | 0.99 |
| B-2-8-0.2a | 7.90        | 12.00       | 1460          | 2              | 0.79                      | 2                            | 0.79                       | 0.276             | 3.16                | 417                 | 83     | 0.20             | 85                   | 0.97 |
| B-2-8-0.4B | 7.90        | 12.00       | 1460          | 2              | 0.79                      | 2                            | 0.79                       | 0.276             | 3.16                | 347                 | 98     | 0.28             | 100                  | 0.98 |
| B-2-8-0.4  | 7.90        | 12.00       | 3320          | 2              | 0.79                      | 2                            | 0.79                       | 0.276             | 3.16                | 365                 | 146    | 0.40             | 129                  | 1.13 |
| B-2-8-0.4a | 7.90        | 12.00       | 3320          | 2              | 0.79                      | 2                            | 0.79                       | 0.276             | 3.16                | 347                 | 139    | 0.40             | 129                  | 1.08 |
| B-2-8-0.4b | 7.90        | 12.00       | 5140          | 2              | 0.79                      | 2                            | 0.79                       | 0.276             | 3.16                | 365                 | 146    | 0.40             | 129                  | 1.13 |
| B-2-8-0.4c | 7.90        | 11.80       | 5140          | 2              | 0.79                      | 2                            | 0.79                       | 0.276             | 3.16                | 382                 | 153    | 0.40             | 133                  | 1.15 |
| B-2-8-0.4d | 7.90        | 11.80       | 2710          | 2              | 0.79                      | 2                            | 0.79                       | 0.276             | 3.16                | 313                 | 125    | 0.40             | 128                  | 0.98 |
| B-2-8-0.4e | 7.90        | 12.00       | 2710          | 2              | 0.79                      | 2                            | 0.79                       | 0.276             | 3.16                | 330                 | 132    | 0.40             | 129                  | 1.02 |
| B-2-8-0.4f | 7.90        | 11.80       | 2710          | 2              | 0.79                      | 2                            | 0.79                       | 0.394             | 3.16                | 347                 | 139    | 0.40             | 147                  | 0.94 |
| B-2-8-0.4g | 7.90        | 11.80       | 2710          | 2              | 0.79                      | 2                            | 0.79                       | 0.394             | 3.16                | 435                 | 98     | 0.23             | 97                   | 1.02 |

Average Deviation 0.054

Average 1.03



TABLE 6-5  
COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS  
TESTS IN COMBINED BENDING AND TORSION BY GESUND, SCHUETTE, BUCHANAN AND GRAY (1964)

| Beam | b<br>inches | h<br>inches | $f_c'$<br>psi | Reinforcement | $M_{bt}$ | $M_{tt}$ | $M_t$<br>in.kips | $\frac{M_{tt}}{M_t}$ |
|------|-------------|-------------|---------------|---------------|----------|----------|------------------|----------------------|
|      |             |             | Top           | Bottom        | Ties     | Ø        |                  |                      |
| 1    | 8.0         | 8.0         | 5030          | 2#4           | 3#4      | #3@5"    | 79               | 1.00                 |
| 2    | 8.0         | 8.0         | 5300          | 2#4           | 3#4      | #3@2"    | 102              | 1.00                 |
| 3    | 8.0         | 8.0         | 5310          | 2#4           | 3#4      | #3@5"    | 122              | 0.50                 |
| 4    | 8.0         | 8.0         | 4680          | 2#4           | 3#4      | #3@2"    | 134              | 0.50                 |
| 5    | 8.0         | 8.0         | 4240          | 2#4           | 3#4      | #3@5"    | 147              | 0.33                 |
| 6    | 8.0         | 8.0         | 4060          | 2#4           | 3#4      | #3@2"    | 168              | 0.33                 |
| 7    | 8.0         | 8.0         | 5280          | 2#4           | 3#4      | #3@5"    | 173              | 0.25                 |
| 8    | 8.0         | 8.0         | 5740          | 2#4           | 3#4      | #3@2"    | 176              | 0.25                 |
| 9    | 6.0         | 12.0        | 4860          | 2#4           | 3#4      | #3@8"    | 120              | 0.50                 |
| 10   | 6.0         | 12.0        | 3900          | 2#4           | 3#4      | #3@8"    | 176              | 0.25                 |
| 11   | 6.0         | 12.0        | 4860          | 2#4           | 3#4      | #3@4"    | 138              | 0.49                 |
| 12   | 6.0         | 12.0        | 3900          | 2#4           | 3#4      | #3@4"    | 213              | 0.25                 |

Average 0.90

Average Deviation 0.066



TABLE 6-6  
COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS  
TESTS IN COMBINED BENDING, TORSION AND SHEAR BY LYALIN (1959)

| Beam    | b<br>inches | h<br>inches | $f'_c$<br>psi | Reinforcement             |                |                              |                | Ties<br>inches | Spacing<br>inches | $V_t$<br>kips | $M_{bt}$<br>in.kips | $M_{tt}$<br>in.kips | $\vartheta$ | $M_t$<br>in.kips | $\frac{M_{tt}}{M_t}$ |
|---------|-------------|-------------|---------------|---------------------------|----------------|------------------------------|----------------|----------------|-------------------|---------------|---------------------|---------------------|-------------|------------------|----------------------|
|         |             |             |               | Top<br>Diameter<br>inches | No. of<br>Bars | Bottom<br>Diameter<br>inches | No. of<br>Bars |                |                   |               |                     |                     |             |                  |                      |
| 8-0.1   | 7.88        | 12.20       | 2100          | 2                         | 0.788          | 2                            | 0.788          | 0.315          | 3.15              | 12.60         | 521                 | 52                  | 0.10        | 55               | 0.94                 |
| 8-0.1a  | 7.88        | 12.20       | 2100          | 2                         | 0.788          | 2                            | 0.788          | 0.315          | 3.15              | 13.40         | 555                 | 56                  | 0.10        | 57               | 0.98                 |
| 8-0.2   | 7.88        | 11.80       | 1820          | 2                         | 0.788          | 2                            | 0.788          | 0.315          | 3.15              | 11.90         | 452                 | 90                  | 0.20        | 109              | 0.82                 |
| 8-0.2a  | 7.88        | 12.20       | 2100          | 2                         | 0.788          | 2                            | 0.788          | 0.315          | 3.15              | 11.72         | 486                 | 97                  | 0.20        | 104              | 0.93                 |
| 7-0.2   | 7.88        | 12.00       | 1820          | 2                         | 0.788          | 2                            | 0.788          | 0.262          | 3.15              | 11.30         | 468                 | 94                  | 0.20        | 112              | 0.84                 |
| 7-0.2a  | 7.88        | 12.00       | 2300          | 2                         | 0.788          | 2                            | 0.788          | 0.262          | 3.15              | 10.90         | 452                 | 90                  | 0.20        | 111              | 0.81                 |
| 10-0.2  | 7.88        | 12.00       | 2640          | 2                         | 0.788          | 2                            | 0.788          | 0.394          | 3.15              | 12.60         | 521                 | 104                 | 0.20        | 102              | 1.02                 |
| 10-0.2a | 7.88        | 12.20       | 2860          | 2                         | 0.788          | 2                            | 0.788          | 0.394          | 3.15              | 12.60         | 521                 | 104                 | 0.20        | 106              | 0.98                 |
| 1       | 8.66        | 7.88        | 4860          | 2                         | 0.945          | 2                            | 0.945          | 0.307          | 3.15              | 10.90         | 451                 | 90                  | 0.20        | 88               | 1.02                 |
| 1a      | 8.66        | 7.88        | 4670          | 2                         | 0.945          | 2                            | 0.945          | 0.307          | 3.15              | 12.72         | 451                 | 90                  | 0.20        | 93               | 0.97                 |
| 2a      | 8.66        | 11.80       | 4860          | 2                         | 0.945          | 2                            | 0.945          | 0.307          | 3.15              | 16.77         | 694                 | 139                 | 0.20        | 134              | 1.04                 |
| 4       | 8.66        | 11.80       | 4410          | 2                         | 0.945          | 2                            | 0.945          | 0.307          | 3.15              | 20.20         | 555                 | 261                 | 0.47        | 271              | 0.96                 |
| 4a      | 8.66        | 11.80       | 4670          | 2                         | 0.945          | 2                            | 0.945          | 0.307          | 3.15              | 20.10         | 555                 | 278                 | 0.50        | 285              | 0.98                 |
| 5       | 8.66        | 15.80       | 4410          | 2                         | 0.945          | 2                            | 0.945          | 0.307          | 3.15              | 23.50         | 973                 | 195                 | 0.20        | 216              | 0.90                 |
| 5a      | 8.66        | 15.80       | 4930          | 2                         | 0.945          | 2                            | 0.945          | 0.307          | 3.15              | 23.50         | 973                 | 195                 | 0.20        | 193              | 1.01                 |
| 6       | 6.70        | 15.80       | 5030          | 2                         | 0.945          | 2                            | 0.945          | 0.307          | 3.15              | 20.10         | 832                 | 166                 | 0.20        | 176              | 0.95                 |
| 6a      | 6.70        | 15.80       | 4930          | 2                         | 0.945          | 2                            | 0.945          | 0.307          | 3.15              | 21.80         | 903                 | 181                 | 0.20        | 180              | 1.01                 |

Average 0.95  
Average Deviation 0.055



beam and the value of  $\phi$  listed is based on this bending moment. The values of  $M_t$  are based on the assumption that for a mode 1 failure the critical section is located a distance  $c_1/2$  from the cross section subjected to the maximum bending moment and that for a mode 3 failure the critical section is located a distance  $c_3/2$  from the cross section subjected to the minimum bending moment.

The results of 11 of the beams tested by Lessig (1961) are presented in TABLE 6-7. Beams that failed by crushing of the concrete before the reinforcement yielded are not included in this table. The concrete cylinder strengths were obtained by taking 95 percent of the cube strengths given by Lessig. The value of  $M_{bt}$  listed in this table is the maximum bending moment in the beam and the value of  $\phi$  listed is based on this bending moment. The values of  $M_t$  listed are based on the assumption that for a mode 1 failure the critical section is located a distance  $c_1/2$  from the cross section subjected to the maximum bending moment and that for a mode 3 failure the critical section is located a distance  $c_3/2$  from the cross section subjected to the minimum bending moment. In computing  $M_t$  for Beams 19 and 19a, the different diameters of the horizontal and vertical legs of the ties were considered.



TABLE 6-7  
COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS  
TESTS IN COMBINED BENDING, TORSION AND SHEAR BY LESSIG (1961)

| Beam | b<br>inches | h<br>inches | $f'_c$<br>psi | Reinforcement |                |                    | Ties<br>Diameter<br>inches | Spacing<br>inches | $V_t$<br>kips | $M_{bt}$<br>in.kips | $M_{tt}$<br>in.kips | $\phi$   | $M_t$<br>in.kips | $\frac{M_{tt}}{M_t}$ |
|------|-------------|-------------|---------------|---------------|----------------|--------------------|----------------------------|-------------------|---------------|---------------------|---------------------|----------|------------------|----------------------|
|      |             |             |               | Top<br>Bars   | No. of<br>Bars | Diameter<br>inches |                            |                   |               |                     |                     |          |                  |                      |
| 3    | 6.69        | 15.75       | 2140          | 2             | 0.630          | 2                  | 0.394                      | 4.92              | 0             | 0                   | 175                 | $\infty$ | 219              | 0.80                 |
| 13a  | 6.10        | 11.60       | 2900          | 2             | 0.551          | 2                  | 1.100                      | 0.394             | 3.94          | 23.20               | 625                 | 125      | 0.20             | 159                  |
| 14a  | 5.90        | 12.00       | 2960          | 2             | 0.551          | 2                  | 1.100                      | 0.394             | 3.94          | 21.20               | 573                 | 113      | 0.20             | 139                  |
| 15   | 7.88        | 12.20       | 2840          | 2             | 0.551          | 2                  | 0.788                      | 0.394             | 3.94          | 15.50               | 417                 | 156      | 0.37             | 163                  |
| 15a  | 7.88        | 12.00       | 3090          | 2             | 0.551          | 2                  | 0.788                      | 0.394             | 3.94          | 15.50               | 417                 | 151      | 0.36             | 164                  |
| 16   | 6.10        | 12.40       | 3660          | 2             | 0.473          | 2                  | 0.473                      | 0.394             | 3.94          | 4.10                | 156                 | 92       | 0.59             | 92                   |
| 16a  | 6.10        | 12.00       | 3960          | 2             | 0.473          | 2                  | 0.473                      | 0.394             | 3.94          | 4.10                | 156                 | 83       | 0.53             | 86                   |
| 17   | 6.10        | 12.20       | 4260          | 2             | 0.630          | 2                  | 0.630                      | 0.394             | 3.94          | 7.10                | 278                 | 83       | 0.30             | 88                   |
| 17a  | 6.10        | 12.20       | 4440          | 2             | 0.630          | 2                  | 0.630                      | 0.394             | 3.94          | 8.00                | 312                 | 90       | 0.29             | 86                   |
| 19   | 6.10        | 12.00       | 3780          | 2             | 0.630          | 2                  | 0.630                      | 0.236*            | 3.94          | 7.98                | 312                 | 78       | 0.25             | 73                   |
| 19a  | 5.90        | 12.00       | 4020          | 2             | 0.630          | 2                  | 0.630                      | 0.236*            | 3.94          | 7.98                | 312                 | 79       | 0.25             | 71                   |

\*Diameter of horizontal legs of ties = 0.236 inches  
Diameter of vertical legs of ties = 0.394 inches

Average Deviation 0.086  
Average 0.95



## CHAPTER VII

### SIMPLIFIED ANALYSIS

#### 7-1 Introduction

Simplified versions of the equations in CHAPTER V are presented in this chapter. An interaction diagram consisting of three straight lines is also presented along with the applicable equations and the procedure to be used in their solution.

The correlation between the experimental results of 109 beams and the simplified analysis is given in TABLES 7-1 and 7-2. Of these beams, 34 were tested in this investigation and 75 were tested by other investigators.

#### 7-2 Simplification of Equations

In order to reduce the number of calculations and to eliminate the iterative procedures required to solve Equations (5-1), (5-5) and (5-10), the following simplifying assumptions are introduced.

- (1) The critical section is located a distance equal to  $b$  from the cross section of the beam that is subjected to either the maximum or the minimum bending moment within the gauge length.
- (2) The transverse reinforcement that is adjacent to the face opposite to the face on which the hinge forms is located at the level of the longitudinal reinforcement.



- (3) The force in the legs of the ties at the two faces adjacent to the face on which the hinge forms is neglected.
- (4) The depth of the concrete compression zone is zero.

Assumption one defines the cross sections of the beam that should be analysed. If the gauge length is subjected to a uniform bending moment and a uniform twisting moment this assumption does not apply. If a beam has equal top and bottom longitudinal reinforcement, the possibility of a mode 3 failure is precluded and the critical section is assumed to be located a distance equal to  $b$  from the cross section subjected to the maximum bending moment. If a beam has less top reinforcement than bottom, the critical section for a mode 1 failure remains unchanged but the critical section for a mode 3 failure is assumed to be located a distance equal to  $b$  from the cross section subjected to the minimum bending moment. Since a mode 2 failure is independent of the bending moment it is not necessary to define a critical section for a mode 2 failure for beams subjected to a constant transverse shear.

Assumption two reduces the lever arm of the transverse reinforcement by an amount equal to one half the tie diameter plus one half the diameter of the longitudinal bars. Consequently, it reduces the theoretical ultimate torsional moment slightly.

Assumption three neglects the force in the vertical legs of the ties for the mode 1 and mode 3 failure surfaces and neglects the force in the horizontal legs of the ties for the mode 2 failure surface. The contribution of the force in these legs to the torsional moment is small



in comparison to the contribution of the forces in the remainder of the reinforcement. As a result, the theoretical ultimate torsional moment is slightly reduced.

Assumption four increases the lever arm of the force in the longitudinal and transverse reinforcement by a distance equal to one half the depth of the compression zone and therefore increases the ultimate torsional moment. The depth of the compression zone in a beam tested at  $\phi$  equal to zero is significant. However, as the value of  $\phi$  is increased the depth of the compression zone becomes progressively smaller. The depth of compression zone at failure for Beams 1-6 and 2-5 was computed to be 0.47" and 0.50" respectively. These beams were tested at  $\phi$  equal to 0.25. Consequently, increasing the lever arm of the force in the reinforcement by one half the depth of the compression zone increases the theoretical torsional moment by only a small amount.

### 7-3 Simplified Equations

#### (a) Mode 1

The simplifying assumptions outlined in SECTION 7-2 transform Equations (5-1), (5-3) and (5-4) into Equations (7-1) and (7-2).

$$M_{t1} = A_{s1} f_{y1} h_{01} \left[ \frac{1 + p_1 k_{01} c_1^2 / bh}{\frac{c_1}{b} + \frac{1}{\phi}} \right] \quad (7-1)$$

where  $c_1 = -\frac{b}{\phi} + b \left[ \frac{1}{\phi^2} + \frac{h}{p_1 k_{01} b} \right]^{1/2} \leq 2h + b$  (7-2)



Equations (7-1) and (7-2) can then be transformed into Equations (7-3) and (7-4).

$$M_{t1} = A_{s1} f_{y1} h_{01} b \left[ \frac{1 + \frac{c_1^2}{k_{y1} \alpha_1 \beta_1}}{c_1 + \frac{b}{\phi}} \right] \quad (7-3)$$

where  $\beta_1 = 2h + b$

$$\alpha_1 = A_{s1} S/a_v$$

$$k_{y1} = f_{y1}/f_{yt}$$

and  $c_1 = -\frac{b}{\phi} + \left[ \frac{b^2}{\phi^2} + k_{y1} \alpha_1 \beta_1 \right]^{1/2} \leq \beta_1 \quad (7-4)$

(b) Mode 2

The simplifying assumptions of SECTION 7-2 transform Equations (5-5), (5-7) and (5-8) into Equations (7-5) and (7-6).

$$M_{t2} = \frac{A_{s2} f_{y2} h b_{02}}{c_2} \left[ \frac{1 + p_2 k_{02} c_2^2 / bh}{1 + \delta} \right] \quad (7-5)$$

where  $c_2 = \left[ \frac{bh}{p_2 k_{02}} \right]^{1/2} \leq 2b + h \quad (7-6)$

Equations (7-5) and (7-6) can then be transformed into Equations (7-7) and (7-8).



$$M_{t2} = \frac{A_{s2} f_{y2} h b_{02}}{c_2} \left[ \frac{1 + \frac{c_2^2}{k_{y2} \alpha_2 \beta_2}}{1 + \delta} \right] \quad (7-7)$$

where  $\beta_2 = 2b + h$

$$\alpha_2 = A_{s2} S / a_v$$

$$k_{y2} = f_{y2} / f_{yt}$$

and  $c_2 = (k_{y2} \alpha_2 \beta_2)^{1/2} \leq \beta_2$  (7-8)

(c) Mode 3

The simplifying assumptions of SECTION 7-2 transform Equations (5-10), (5-12) and (5-13) into Equations (7-9) and (7-10).

$$M_{t3} = A_{s3} f_{y3} h_{03} \left[ \frac{1 + p_3 k_{03} c_3^2 / bh}{\frac{c_3}{b} - \frac{1}{\emptyset}} \right]^{1/2} \quad (7-9)$$

where  $c_3 = \frac{b}{\emptyset} + b \left[ \frac{1}{\emptyset^2} + \frac{h}{p_3 k_{03} b} \right]^{1/2} \leq 2h + b$  (7-10)

Equations (7-9) and (7-10) can then be transformed into Equations (7-11) and (7-12).

$$M_{t3} = A_{s3} f_{y3} h_{03} b \left[ \frac{1 + \frac{c_3^2}{k_{y3} \alpha_3 \beta_1}}{\frac{c_3}{b} - \frac{1}{\emptyset}} \right]^{1/2} \quad (7-11)$$



$$\text{where } \beta_1 = 2h + b$$

$$\alpha_3 = A_{s3} S/a_v$$

$$k_{y3} = f_{y3}/f_{yt}$$

$$\text{and } c_3 = \frac{b}{\emptyset} + \left[ \frac{b^2}{\emptyset^2} + k_{y3} \alpha_3 \beta_1 \right]^{1/2} \leq \beta_1 \quad (7-12)$$

#### 7-4 Simplified Interaction Diagram

The interaction diagrams shown in CHAPTER VI can be approximated by three straight lines. This simplified interaction diagram is shown in FIGURE 7-1. Points a, b, c, and d are obtained by solving Equations (7-3), (7-7) and (7-11) for values of  $\emptyset$  of  $\infty$ , 1.0, 0.25 and 0. A value of  $\emptyset$  equal to infinity represents a beam subjected to pure torsion;  $\emptyset$  equal to zero represents a beam subjected to pure bending. The values 1.0 and 0.25 have been selected arbitrarily.

The twisting moment at point a is designated  $M_{ta}$  and is equal to the smallest of the twisting moments  $M_{t1}$ ,  $M_{t2}$  and  $M_{t3}$  obtained from Equations (7-3), (7-7) and (7-11) with  $\emptyset$  equal to  $\infty$ . For this value of  $\emptyset$  these equations are transformed into Equations (7-13), (7-15) and (7-17) respectively. The twisting moments obtained from these equations are designated  $M_{t1a}$ ,  $M_{t2a}$  and  $M_{t3a}$  to indicate they are the values of  $M_{t1}$ ,  $M_{t2}$  and  $M_{t3}$  at point a on the simplified interaction diagram.

$$M_{t1a} = A_{s1} f_{y1} h_{01} b \left[ \frac{1 + c_1^2 / k_{y1} \beta_1 \alpha_1}{c_1} \right] \quad (7-13)$$



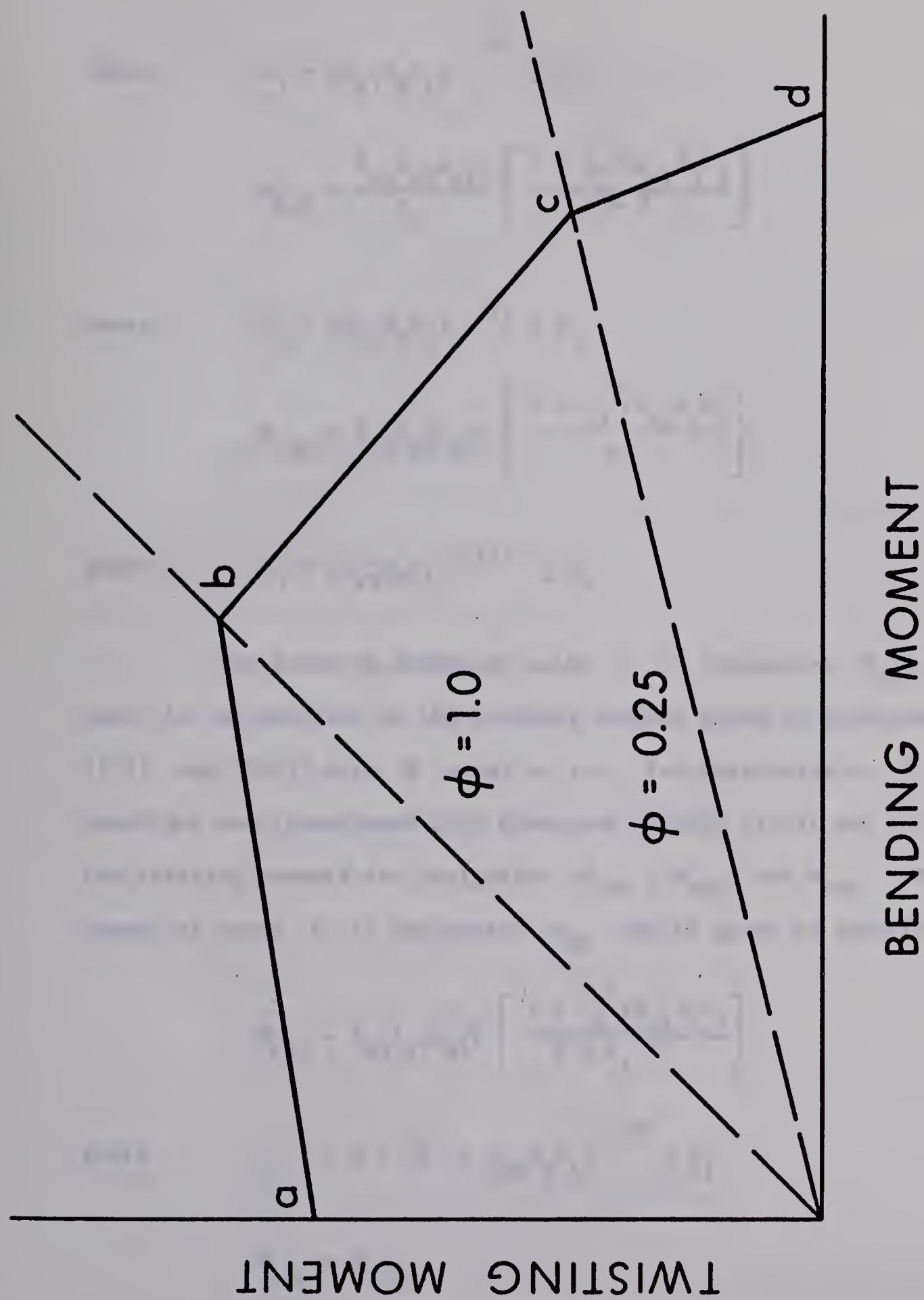


FIGURE 7-1 SIMPLIFIED INTERACTION DIAGRAM



where  $c_1 = (k_{y1} \beta_1 \alpha_1)^{1/2} \leq \beta_1$  (7-14)

$$M_{t2a} = \frac{A_s f_y b_{02} h}{c_2} \left[ \frac{1 + c_2^2 / k_{y2} \beta_2 \alpha_2}{1 + \delta} \right] \quad (7-15)$$

where  $c_2 = (k_{y2} \beta_2 \alpha_2)^{1/2} \leq \beta_2$  (7-16)

$$M_{t3a} = \frac{A_s f_y b_{03} h}{c_3} \left[ \frac{1 + c_3^2 / k_{y3} \beta_3 \alpha_3}{c_3} \right] \quad (7-17)$$

where  $c_3 = (k_{y3} \beta_3 \alpha_3)^{1/2} \leq \beta_1$  (7-18)

The twisting moment at point b is designated  $M_{tb}$  and is equal to the smallest of the twisting moments given by Equations (7-3), (7-7) and (7-11) with  $\emptyset$  equal to 1.0. For this value of  $\emptyset$  these equations are transformed into Equations (7-19), (7-21) and (7-22) and the twisting moments are designated  $M_{t1b}$ ,  $M_{t2b}$  and  $M_{t3b}$ . The bending moment at point b is designated  $M_{bb}$  and is given by Equation (7-24).

$$M_{t1b} = \frac{A_s f_y b_{01} h}{b + c_1} \left[ \frac{1 + c_1^2 / k_{y1} \beta_1 \alpha_1}{b + c_1} \right] \quad (7-19)$$

where  $c_1 = -b + (b^2 + k_{y1} \beta_1 \alpha_1)^{1/2} \leq \beta_1$  (7-20)

$$M_{t2b} = M_{t2a} \quad (7-21)$$



$$M_{t3b} = A_{s3} f_{y3} h_{03} b \left[ \frac{1 + c_3^2 / k_{y3} \beta_1 \alpha_3}{c_3 - b} \right] \quad (7-22)$$

where  $c_3 = b + (b^2 + k_{y3} \beta_1 \alpha_3)^{1/2} \leq \beta_1$  (7-23)

$$M_{bb} = M_{tb} \quad (7-24)$$

The twisting moment at point  $c$  is designated  $M_{tc}$  and is equal to the smallest of the twisting moments given by Equations (7-3), (7-7) and (7-11) with  $\emptyset$  equal to 0.25. For this value of  $\emptyset$  these equations are transformed into Equations (7-25), (7-27) and (7-28) and the twisting moments are designated  $M_{t1c}$ ,  $M_{t2c}$  and  $M_{t3c}$ . The bending moment at point  $c$  is designated  $M_{bc}$  and is given by Equation (7-30).

$$M_{t1c} = A_{s1} f_{y1} h_{01} b \left[ \frac{1 + c_1^2 / k_{y1} \beta_1 \alpha_1}{4b + c_1} \right] \quad (7-25)$$

where  $c_1 = -4b + (16b^2 + k_{y1} \beta_1 \alpha_1)^{1/2} \leq \beta_1$  (7-26)

$$M_{t2c} = M_{t2a} \quad (7-27)$$

$$M_{t3c} = A_{s3} f_{y3} h_{03} b \left[ \frac{1 + c_3^2 / k_{y3} \beta_1 \alpha_3}{c_3 - 4b} \right] \quad (7-28)$$



where  $c_3 = 4b + (16b^2 + k_y^3 \beta_1 \alpha_3)^{1/2} \leq \beta_1$  (7-29)

$$M_{bc} = 4M_{tc} \quad (7-30)$$

Point d on the simplified interaction diagram represents a beam subjected to pure flexure. With  $\emptyset$  equal to zero in Equation (7-1), the twisting moment is found to be zero. The corresponding bending moment is given by Equation (7-31) and is designated  $M_{bu}$ .

$$M_{bu} = A_{s1} f_{y1} (h_{01} - k_1 x_1 / 2) \quad (7-31)$$

where  $x_1 = A_{s1} f_{y1} / 0.85 k_1 f'_c b$  (7-32)

Equations (7-31) and (7-32) are obtained from Equations (5-1) and (5-2) by rearranging terms and using the fact that  $c_1$  is equal to zero for a beam subjected to pure flexure.

### 7-5 Computation Procedure

The following procedure can be used to check the torsional strength of beams that have been tested or to design beams for specified bending moments, twisting moments and transverse shear forces. For design, a trial section is selected and  $\emptyset$  and  $\delta$  are computed using the design values of bending moment, twisting moment and transverse shear.

After the bending moment and twisting moment corresponding to each of points a, b, c, and d have been calculated, the ultimate torsional



moment at any value of  $\phi$  can be obtained by using Equations (7-33), (7-34) and (7-35).

The computation procedure is divided into three parts. The part that is relevant to any particular beam depends on the value of  $\phi$  for that beam. If the top reinforcement of the beam is identical to the bottom, the calculations required by Equations (7-17), (7-18), (7-22), (7-23), (7-28) and (7-29) can be omitted. The three parts of the procedure are:

(a)  $0 \leq \phi \leq 0.25$

(b)  $0.25 < \phi \leq 1.0$

(c)  $\phi > 1.0$

(a)  $0 \leq \phi \leq 0.25$

1. Compute  $M_{tc}$  by Equations (7-25), (7-27) and (7-28)
2. Compute  $M_{bc}$  by Equation (7-30)
3. Compute  $M_{bu}$  by Equation (7-31)
4. Compute  $M_t$  by Equation (7-33)

$$M_t = \frac{M_{bu}}{\frac{M_{bu} - M_{bc}}{M_{tc}} + \frac{1}{\phi}} \quad (7-33)$$

(b)  $0.25 < \phi \leq 1.0$

1. Compute  $M_{tb}$  by Equations (7-19), (7-21) and (7-22)
2. Compute  $M_{tc}$  by Equations (7-25), (7-27) and (7-28)
3. Compute  $M_{bb}$  by Equation (7-24)



4. Compute  $M_{bc}$  by Equation (7-30)

5. Compute  $M_t$  by Equation (7-34)

$$M_t = \frac{M_{tc} + M_{bc} \left[ \frac{M_{tb} - M_{tc}}{M_{bc} - M_{bb}} \right]}{1 + \frac{1}{\phi} \left[ \frac{M_{tb} - M_{tc}}{M_{bc} - M_{bb}} \right]} \quad (7-34)$$

(c)  $\phi > 1.0$

1. Compute  $M_{ta}$  by Equations (7-13), (7-15) and (7-17)

2. Compute  $M_{tb}$  by Equations (7-19), (7-21) and (7-22)

3. Compute  $M_{bb}$  by Equation (7-24)

4. Compute  $M_t$  by Equation (7-35)

$$M_t = \frac{M_{ta}}{1 - \left[ \frac{M_{tb} - M_{ta}}{\phi M_{bb}} \right]} \quad (7-35)$$

## 7-6 Correlation of Test Results

The theoretical and experimental results for the beams previously presented in TABLES 6-1 to 6-7 are presented in TABLES 7-1 and 7-2. The theoretical value of twisting moment,  $M_t$ , listed in these tables was obtained by the procedure presented in SECTION 7-5. For the beams tested in bending, torsion and shear the location of the critical section was established in accordance with assumption one of SECTION 7-2.



TABLE 7-1

## COMPARISON OF SIMPLIFIED ANALYSIS AND EXPERIMENTAL RESULTS

## THIS INVESTIGATION AND PANDIT (1965)

| Beam                    | This Investigation  |                  |                      | Pandit (1965) |                     |                  |                          |  |
|-------------------------|---------------------|------------------|----------------------|---------------|---------------------|------------------|--------------------------|--|
|                         | $M_{tt}$<br>in.kips | $M_t$<br>in.kips | $\frac{M_{tt}}{M_t}$ | Beam          | $M_{tt}$<br>in.kips | $M_t$<br>in.kips | $\frac{M_{tt}}{M_t}$     |  |
| 1-1                     | 122                 | 115              | 1.06                 | B-2           | 72                  | 64               | 1.12                     |  |
| 1-2                     | 138                 | 132              | 1.04                 | B-3           | 95                  | 90               | 1.06                     |  |
| 1-3                     | 140                 | 134              | 1.05                 | B-4           | 85                  | 89               | 0.95                     |  |
| 1-4                     | 159                 | 169              | 0.94                 | C-1           | 78                  | 72               | 1.09                     |  |
| 1-5                     | 131                 | 129              | 1.02                 | C-2           | 105                 | 101              | 1.04                     |  |
| 1-6                     | 90                  | 90               | 1.00                 | C-3           | 111                 | 124              | 0.89                     |  |
| 2-1                     | 181                 | 209              | 0.87                 | C-4           | 111                 | 109              | 1.02                     |  |
| 2-2                     | 172                 | 187              | 0.92                 | D-1           | 99                  | 109              | 0.91                     |  |
| 2-3                     | 166                 | 164              | 1.02                 | D-2           | 164                 | 184              | 0.89                     |  |
| 2-4                     | 134                 | 132              | 1.02                 | D-3           | 156                 | 199              | 0.78                     |  |
| 2-5                     | 90                  | 91               | 0.98                 | D-4           | 146                 | 160              | 0.91                     |  |
| 3-1                     | 115                 | 99               | 1.16                 | E-1           | 76                  | 70               | 1.08                     |  |
| 3-2                     | 117                 | 124              | 0.95                 | E-2           | 101                 | 103              | 0.98                     |  |
| 3-3                     | 120                 | 123              | 0.98                 | E-3           | 121                 | 139              | 0.87                     |  |
| 3-4                     | 115                 | 105              | 1.10                 | F-1           | 58                  | 52               | 1.12                     |  |
| 3-5                     | 73                  | 71               | 1.03                 | F-2           | 83                  | 70               | 1.18                     |  |
| 4-1                     | 120                 | 116              | 1.03                 | F-3           | 89                  | 82               | 1.08                     |  |
| 4-2                     | 101                 | 96               | 1.06                 | F-4           | 95                  | 94               | 1.01                     |  |
| 4-3                     | 93                  | 85               | 1.09                 | G-1           | 73                  | 53               | 1.37                     |  |
| 4-4                     | 85                  | 80               | 1.07                 | G-2           | 92                  | 68               | 1.35                     |  |
| 4-5                     | 103                 | 96               | 1.07                 | G-3           | 103                 | 77               | 1.34                     |  |
| 4-6                     | 66                  | 59               | 1.13                 | G-4           | 115                 | 86               | 1.33                     |  |
| 5-1                     | 128                 | 130              | 0.99                 |               |                     |                  | Average * 1.08           |  |
| 5-2                     | 141                 | 147              | 0.96                 |               |                     |                  | Average Deviation* 0.124 |  |
| 5-3                     | 130                 | 124              | 1.05                 |               |                     |                  | *Not Including Beam D-3  |  |
| 5-4                     | 99                  | 96               | 1.03                 |               |                     |                  |                          |  |
| 6-1                     | 129                 | 123              | 1.05                 |               |                     |                  |                          |  |
| 6-2                     | 145                 | 124              | 1.17                 |               |                     |                  |                          |  |
| 6-3                     | 132                 | 126              | 1.05                 |               |                     |                  |                          |  |
| 6-4                     | 107                 | 98               | 1.09                 |               |                     |                  |                          |  |
| 7-1                     | 112                 | 95               | 1.18                 |               |                     |                  |                          |  |
| 7-2                     | 115                 | 107              | 1.07                 |               |                     |                  |                          |  |
| 7-3                     | 132                 | 132              | 1.00                 |               |                     |                  |                          |  |
| 7-4                     | 125                 | 118              | 1.06                 |               |                     |                  |                          |  |
| Average 1.04            |                     |                  |                      |               |                     |                  |                          |  |
| Average Deviation 0.051 |                     |                  |                      |               |                     |                  |                          |  |



TABLE 7-2

## COMPARISON OF SIMPLIFIED ANALYSIS AND EXPERIMENTAL RESULTS

CHINENKOV (1959), GESUND, SCHUETTE, BUCHANAN AND

GRAY (1964), LYALIN (1959) AND LESSIG (1961)

| Beam  | $M_{tt}$<br>in.kips | $M_t$<br>in.kips | $\frac{M_{tt}}{M_t}$ | Beam                    | $M_{tt}$<br>in.kips | $M_t$<br>in.kips | $\frac{M_{tt}}{M_t}$ |
|---|---------------------|------------------|----------------------|-------------------------|---------------------|------------------|----------------------|
| Chinenkov (1959)                              |                     |                  |                      | Lyalin (1959)           |                     |                  |                      |
| 0.1   | 49                  | 41               | 1.20                 | 8-0.1                   | 52                  | 57               | 0.92                 |
| 0.1a  | 47                  | 40               | 1.17                 | 8-0.1a                  | 56                  | 58               | 0.96                 |
| 0.2   | 83                  | 78               | 1.06                 | 8-0.2                   | 90                  | 110              | 0.82                 |
| 0.2a  | 83                  | 80               | 1.04                 | 8-0.2a                  | 97                  | 105              | 0.92                 |
| 0.4B  | 98                  | 98               | 1.00                 | 7-0.2                   | 94                  | 101              | 0.93                 |
| 0.4   | 146                 | 121              | 1.21                 | 7-0.2a                  | 90                  | 99               | 0.90                 |
| 0.4a  | 139                 | 121              | 1.15                 | 10-0.2                  | 104                 | 111              | 0.94                 |
| 0.4b  | 146                 | 121              | 1.21                 | 10-0.2a                 | 104                 | 114              | 0.91                 |
| 0.4c  | 153                 | 125              | 1.23                 | 1                       | 90                  | 97               | 0.92                 |
| 0.4d  | 125                 | 121              | 1.04                 | 1a                      | 90                  | 102              | 0.88                 |
| 0.4e  | 132                 | 122              | 1.09                 | 2a                      | 139                 | 138              | 1.01                 |
| 0.4f  | 139                 | 141              | 0.99                 | 4                       | 261                 | 225              | 1.16                 |
| 0.4g  | 98                  | 96               | 1.02                 | 4a                      | 278                 | 241              | 1.15                 |
| Average 1.11                                  |                     |                  |                      | 5                       | 195                 | 206              | 0.95                 |
| Average Deviation 0.080                       |                     |                  |                      | 5a                      | 195                 | 188              | 1.04                 |
| Gesund, Schuette, Buchanan<br>and Gray (1964) |                     |                  |                      | 6                       | 166                 | 160              | 1.04                 |
| Average 0.97                                  |                     |                  |                      | 6a                      | 181                 | 163              | 1.11                 |
| Average Deviation 0.077                       |                     |                  |                      | Lessig (1961)           |                     |                  |                      |
| 1   | 79                  | 90               | 0.88                 | 3                       | 175                 | 202              | 0.87                 |
| 2   | 102                 | 120              | 0.85                 | 13a                     | 125                 | 141              | 0.89                 |
| 3   | 61                  | 66               | 0.93                 | 14a                     | 113                 | 122              | 0.92                 |
| 4   | 67                  | 78               | 0.86                 | 15                      | 156                 | 130              | 1.20                 |
| 5   | 49                  | 52               | 0.94                 | 15a                     | 151                 | 129              | 1.17                 |
| 6   | 56                  | 58               | 0.96                 | 16                      | 92                  | 87               | 1.06                 |
| 7   | 43                  | 43               | 1.01                 | 16a                     | 83                  | 81               | 1.02                 |
| 8   | 44                  | 46               | 0.95                 | 17                      | 83                  | 82               | 1.01                 |
| 9   | 60                  | 71               | 0.85                 | 17a                     | 90                  | 81               | 1.11                 |
| 10  | 44                  | 54               | 0.82                 | 19                      | 78                  | 59               | 1.32                 |
| 11  | 68                  | 87               | 0.78                 | 19a                     | 79                  | 57               | 1.38                 |
| 12  | 53                  | 61               | 0.87                 | Average 1.09            |                     |                  |                      |
| Average Deviation 0.055                       |                     |                  |                      | Average Deviation 0.136 |                     |                  |                      |



## CHAPTER VIII

### DISCUSSION

#### 8-1 General Behavior

The behavior of the test specimens prior to cracking of the concrete was affected very little by the reinforcement provided. However, after cracking occurred the specimens appeared to assume a new configuration of equilibrium that was dependent on the value of  $\theta$  at which the test was being conducted and the reinforcement that was provided. The behavior of the beams has been presented in detail in CHAPTER IV and Appendix B.

Three modes of failure were observed in this investigation and they have been designated mode 1, mode 2, and mode 3. A mode 1 failure is characterized by the formation of a hinge adjacent to the top face of the beam, yielding of the bottom reinforcement and positive vertical deflection of the beam. A mode 2 failure is characterized by the formation of a hinge adjacent to one of the vertical faces of the beam, yielding of the reinforcement adjacent to one or both of the vertical faces and little or no vertical deflection of the beam. A mode 3 failure is characterized by the formation of a hinge adjacent to the bottom face of the beam, yielding of the top reinforcement and negative vertical deflection of the beam. The failure surfaces corresponding to these modes of failure have been idealized and are presented in FIGURES 5-1, 5-2 and 5-3.

The theoretical ultimate torsional moment of a beam has been



defined as the smallest of the three moments computed by Equations (5-1), (5-5) and (5-10). Each of these equations has been derived for a different mode of failure. The predicted mode of failure has been designated as the mode that yields the smallest value of theoretical ultimate torsional moment.

The observed mode of failure was determined principally by noting the development and width of cracks and by observing the behavior of the beam at failure. Additional information regarding the mode of failure was obtained from the deflection readings and the strains in the longitudinal reinforcement.

TABLES 6-1 and 6-2 indicate that for most of the beams the observed mode of failure agreed with the predicted mode of failure. When the observed mode of failure did not agree with the predicted mode of failure the beam usually exhibited two failure surfaces corresponding to different modes of failure. The one that appeared to be the best developed was selected as the observed failure surface and the corresponding mode of failure has been recorded in TABLES 6-1 and 6-2. For example, the failure of Beam 1-4 was recorded as a mode 2 failure but it was noted that this beam also had a fairly well developed mode 1 failure surface. The predicted mode of failure for this beam is mode 1. The theoretical torsional moment corresponding to a mode 2 failure surface is 2.6 percent higher than the moment corresponding to a mode 1 failure surface. This is also evident from FIGURE 6-1(a). This interaction diagram indicates that a Group 1 beam tested at  $\phi$  equal to 1.0 would have virtually the



same strength according to either a mode 1 or a mode 2 failure surface.

### 8-2 Effect of Reinforcement

Provision of top longitudinal reinforcement that was less in area than the bottom longitudinal reinforcement did not affect the behavior or the ultimate strength of the beams that were tested at low values of  $\phi$ . Low values of  $\phi$  have been defined as 0.5 or less. Comparison of the test results for Beams 1-5 and 2-4 and Beams 1-6 and 2-5 illustrates this. The observed mode of failure of all four of these beams was mode 1.

Provision of top longitudinal reinforcement that was less in area than the bottom longitudinal reinforcement did affect the behavior and ultimate strength of the beams that were tested at  $\phi$  equal to 1.0. Comparison of the test results of Beams 1-4 and 2-3 illustrates this. The observed mode of failure of Beam 2-3 was mode 1. The observed mode of failure of Beam 1-4 was mode 2 but a fairly well developed mode 1 failure surface was also evident. Although the deflection of both beams was positive, the deflection of Beam 1-4 remained almost constant in the latter stages of the test whereas the deflection of Beam 2-3 continued to increase.

Provision of top longitudinal reinforcement that was less in area than the bottom longitudinal reinforcement significantly affected the behavior and ultimate strength of the beams that were tested at a high value of  $\phi$ . A high value of  $\phi$  has been defined as 2.0 or greater. Comparison of the test results of Beams 1-1 and 2-1 and Beams 1-3 and 2-2 illustrates this. Failure of Beams 1-1 and 1-3 was by mode 3 whereas



failure of Beams 2-1 and 2-2 was by mode 2 and mode 1 respectively.

After cracking of the concrete occurred, Beams 1-1 and 1-3 exhibited significant negative deflections whereas Beams 2-1 and 2-2 exhibited small negative and small positive deflections respectively.

Increasing the tie spacing while keeping the longitudinal reinforcement and  $\emptyset$  constant did not affect the behavior of the beams prior to cracking. However, after cracking occurred, the slope of the twisting moment-angle of twist relationship decreased with an increase in tie spacing. In addition, the ultimate torsional moment of the beams decreased. The deflection of the beams at a given value of torsional moment was not affected by a variation in the tie spacing.

Comparison of the plots presented in Appendix B for Beams 1-5, 4-5 and 4-6 indicates that the behavior of beams tested at the same value of  $\emptyset$  but in which the amount of longitudinal and transverse reinforcement was progressively reduced was the same until cracking of the concrete occurred. After cracking occurred, the slope of the twisting moment-angle of twist relationship decreased with a decrease in the amount of reinforcement provided. In addition, the ultimate torsional moment of the beams decreased. The deflection of the beams at a given value of torsional moment increased with a decrease in the amount of reinforcement provided.

TABLES 6-1 and 6-2 indicate that many of the beams tested in this investigation were over reinforced if Equation (6-2) is the criterion used to establish the limit of over reinforcement. However, the results



presented in CHAPTER IV and Appendix B indicate that in most cases the reinforcement had yielded at failure of a beam. Consequently, it appears that Equation (6-2) is unnecessarily restrictive and that the coefficient 0.06 should be increased. Strain readings taken after failure of Beams 2-1 and 2-2 indicated that in each of these beams one of the bottom reinforcing bars did not yield. Consequently, it appears that these two beams were slightly over reinforced. If  $M_{tr}$  is taken equal to the test value of torsional moment for each of the two beams and the value of the coefficient in Equation (6-2) is calculated, the result is 0.084 and 0.087 for Beams 2-1 and 2-2 respectively. On the basis of the results of these two beams it appears that the coefficient 0.06 in Equation (6-2) could be increased to 0.08.

TABLES 6-1 and 6-2 indicate that all beams tested in this investigation except Beams 2-2, 2-3, 4-3, 4-4, 7-1, 7-2, 7-3 and 7-4 had a value of  $m$  between 0.5 and 1.5 times  $m_o$ . The value of  $m$  for the beams noted above was less than 0.5  $m_o$ . The minimum value of the ratio  $m/m_o$  occurred in Beam 7-2 and was equal to 0.31. The maximum value of this ratio occurred in Beam 2-5 and was equal to 1.33. Lessig (1961) suggested that this ratio should be between 0.5 and 1.5. A ratio of less than 0.5 implies that there is insufficient transverse reinforcement present to provide a proper balance between the two types of reinforcement and that the longitudinal reinforcement will not yield. However, in the beams noted above for which this ratio was less than 0.5 the longitudinal reinforcement did yield except in Beam 2-2. Consequently, the suggested



lower limit of 0.5 appears to be conservative.

### 8-3 Effect of Twisting Moment to Bending Moment Ratio

The effect of different ratios of  $\emptyset$  on the deflection, mode of failure and torsional strength of beams with equal and unequal top and bottom reinforcement has been discussed in SECTION 8-2.

Plots of the twisting moment-angle of twist relationship are presented in Appendix B. The initial straight portion of this relationship represents uncracked behavior of the beam. The decreasing length of this straight portion with decreasing values of  $\emptyset$  indicates that the presence of bending moment reduced the torsional moment at which cracking of the concrete occurred.

The angle of twist at a particular value of twisting moment for the beams of Group 1 did not show any consistent variation with  $\emptyset$  and appeared to be influenced by the mode of failure of the beams. For a given value of twisting moment greater than the cracking moment, the angle of twist was greater for the beams that failed by mode 3 than the ones that failed by mode 1. For the beams of Group 2 that failed by mode 1, the angle of twist at a particular value of twisting moment decreased with decreasing values of  $\emptyset$ .

### 8-4 Sequence of Loading

Pandit (1965) applied the transverse load first and then twisted the specimens to failure. Cowan and Armstrong (1955) and Gesund, Schuette, Buchanan and Gray (1964) used equipment that increased the bending moment



and twisting moment at a constant ratio. Chinenkov (1959), Lyalin (1959), Lessig (1961) and Evans and Sarkar (1965) used equipment that permitted a change in the ratio of twisting moment to bending moment at any stage of a test. However, their tests were carried out by incrementing first one type of load and then the other so that for any one test  $\emptyset$  was a constant at the end of each complete load increment. Goode and Helmy (1965) tested beams by first applying a twisting moment and then a bending moment and stated that these beams sustained an ultimate bending moment at least equal to the ultimate bending moment that would be expected in pure flexure.

The interaction diagram for the beams of Group 3 of this investigation and the beams of Group C of Pandit (1965) is shown in FIGURE 6-2. In addition, the failure values of twisting moment and bending moment and the loading paths are shown. All of the Group 3 beams except Beam 3-1 were first subjected to a small bending moment due to the weight of the loading equipment before further load was applied. FIGURE 6-2 indicates that the sequence of loading did not significantly affect the ultimate torsional strength of these beams.

Beam 3-3, which was subjected to a substantial bending moment before it was twisted to failure, exhibited a crack pattern that consisted essentially of two sets of cracks, one set vertical and the other set inclined at approximately  $45^{\circ}$  to the longitudinal axis of the beam. Beams 3-2, 3-4 and 3-5, which were subjected to a substantial twisting moment before the major portion of the bending moment was applied, exhibited crack patterns that consisted of cracks on all faces at approximately  $45^{\circ}$



to the axis of the beam. The major effect of the last bending sequence applied to these three beams was to widen the cracks that had first appeared during the twisting sequence. However, Beams 3-2, 3-3, 3-4 and 3-5 all developed numerous cracks while the loads were at their peak values.. This indicates that the formation of the failure surface was not entirely dependent on the cracks that formed at lower loads and that a redistribution of stress occurred at failure.

Prior to the final bending sequence, Beam 3-4 was subjected to a twisting moment of 115 inch kips and a bending moment of 55 inch kips. At this stage strain readings indicated yield in one of the top bars, both of the bottom bars and the north leg of the tie that was instrumented. However, failure did not occur until the bending moment had been increased to 188 inch kips. This shows that yielding of the reinforcement is not sufficient to indicate that failure is about to occur. Failure does not occur until the concrete in the compression zone has reached its ultimate strength.

It is stated in SECTION 8-3 that the presence of bending moment reduced the torsional moment at which cracking of the concrete occurred. First cracking in beams tested at a constant value of  $\emptyset$  occurred under the action of combined bending and torsion. At low values of  $\emptyset$  the tensile stresses in the concrete due to bending were appreciable and consequently the first cracks appeared on the bottom face and lower part of the vertical faces and were inclined to the longitudinal axis of the beam. At high values of  $\emptyset$  the first cracks developed at approximately mid depth



of the vertical faces and were inclined to the longitudinal axis of the beam. An interesting exception to this is illustrated by Beams 3-2 and 3-3. The cracking torque for both of these beams was approximately 45 inch kips. At this stage the bending moment in Beam 3-2 was 24 inch kips whereas in Beam 3-3 it was 111 inch kips. When twisting of Beam 3-3 was commenced, vertical bending cracks were already present. These bending cracks would relieve some of the tensile stress in the uncracked sections of concrete between the cracks. As a result, the first torsional cracks did not appear on the bottom and lower part of the vertical faces of Beam 3-3 but appeared at approximately mid depth of the vertical faces and were inclined to the longitudinal axis of the beam. Consequently, the sequence of loading used by Pandit (1965) appears to be relevant with regard to his finding that the magnitude of flexure had no significant effect on the value of the cracking torque.

#### 8-5 Effect of Transverse Shear

The beams of Groups 1 to 4 were subjected to a uniform twisting moment and a uniform bending moment whereas the beams of Groups 5 to 7 were subjected to a uniform twisting moment, a linearly varying bending moment and a uniform transverse shear. The reinforcement and geometrical details of the beams of Groups 1, 5 and 6 were similar in all respects. The failure surface in the beams that were not subjected to transverse shear was located in different parts of the gauge length in different beams. However, the failure surface in the beams that were subjected to transverse shear generally formed at either the east end or at the west end of



the gauge length. For all beams of Groups 5, 6 and 7 the bending moment at the east end of the gauge length was less than at the west end. Failure surfaces occurring at the east end of the gauge length were always of the mode 3 type; those occurring at the west end were always mode 1. Since these beams were subjected to a linearly varying bending moment,  $\phi$  also varied linearly. Equations (5-1) and (5-10) indicate that the ultimate torsional moment of a beam is a function of  $\phi$ . Equation (5-1) indicates that a mode 1 failure will occur in the region subjected to the lowest values of  $\phi$  and Equation (5-10) indicates that a mode 3 failure will occur in the region subjected to the highest values of  $\phi$ . This is in agreement with the observed behavior.

Since  $\phi$  varies linearly in beams subjected to a uniform transverse shear, the question arises as to what value should be used in Equations (5-1) and (5-10). The minimum values of torsional moment are obtained by using the minimum value of  $\phi$  in Equation (5-1) and the maximum value of  $\phi$  in Equation (5-10). However, since the failure surface develops over a length of the beam and not at one cross section, the use of these values is unnecessarily conservative. Another possibility is to use the values of  $\phi$  at distances of  $c_1/2$  and  $c_3/2$  from the cross sections at which the minimum and maximum values of  $\phi$  occur respectively. Observation of the failure surfaces that formed in these tests indicated that this would sometimes be unconservative since the failure surface often appeared to crowd to one end of the gauge length.



This was most noticeable in those beams subjected to a high bending moment gradient within the gauge length. Some value of  $\phi$  between these two limiting cases would probably conform more closely with the actual behavior of the beams. Using the values of  $\phi$  at distances of  $c_1/4$  and  $c_3/4$  from the cross sections at which the minimum and maximum values of  $\phi$  occur respectively changes the average of the ratio  $M_{tt}/M_t$  for the beams listed in TABLE 6-2 to 0.96 and the average deviation of this ratio to 0.067. The assumption that is made in the simplified analysis is that the values of  $\phi$  which should be used in the computations are the values occurring at a distance  $b$  from the cross sections at which the minimum and maximum values of  $\phi$  occur. This is a compromise which simplifies the procedure but decreases its accuracy.

Equations (5-1), (5-5) and (5-10) indicate that the transverse shear enters into only a mode 2 failure. Because of the effect of the varying bending moment it is difficult to ascertain if this is entirely true. Although the presence of transverse shear did affect the initial development of the cracks, the observations and test results indicate that transverse shear within the range covered by these tests does not significantly affect the behavior at failure or the ultimate torsional strength of a beam except as provided for by a mode 2 failure surface.

The upper limit of the range of transverse shear covered by these tests can be indicated by using any one of three parameters. The first of these parameters is the ratio  $V_t/V_u$ , where  $V_u$  is the total ultimate transverse shear capacity of the beam when subjected to only bending and



shear. The value of  $V_u$  has been computed by the method outlined in Chapter 17 of the American Concrete Institute Building Code (1963). The maximum value of this ratio occurred in Beam 7-4 and was equal to 0.50. The second of these parameters is  $\delta$  and is defined in CHAPTER V. The maximum value of  $\delta$  occurred in Beam 6-4 and was equal to 0.37. The third parameter,  $\psi$ , relates the transverse shear to the bending moment and is defined in CHAPTER VI. The maximum value of  $\psi$  in the beams tested in this investigation occurred in Beam 6-4 at the east end of the gauge length and was equal to 0.123. The value of  $\psi$  for this beam at the west end of the gauge length was 0.029.

#### 8-6 Validity of Assumptions

The first and second assumptions of SECTION 5-3 define the failure surface. Observations made during these tests indicate that failure occurs on a warped plane and that at failure the spiral cracks widen on three faces of the beam but not on the fourth face. The cracks on the two faces adjacent to the fourth face decrease in width as they approach the fourth face. These cracks do not always spiral around the beam at a constant angle. The beams tested with  $\phi$  equal to 0.25 exhibited cracks on the vertical faces that were nearly vertical near the bottom and became more horizontal as they progressed upward. The angle that the cracks on the bottom face made with the axis of the beam was generally close to the same as the angle made by the bottom portion of the cracks on the vertical faces. However, for values of  $\phi$  equal to or greater than 0.5 the cracks on all faces were at nearly a constant angle to the longitudinal



axis of the beam.

The third assumption of SECTION 5-3 states that the concrete cracks to the level of the neutral axis. This is not entirely true but the moment of the tensile force in the portion of uncracked concrete is very small in comparison to the ultimate torsional moment of the beam.

The fourth assumption neglects the forces in the reinforcement near the face of the beam on which the hinge is located. Comparison of the test results for Beams 1-5 and 2-4 and Beams 1-6 and 2-5 indicates that neither axial nor dowel forces in the top longitudinal reinforcement contributed significantly to the ultimate torsional strength of these beams. Since the lever arm of these forces is small the moment due to these forces is small in comparison to the ultimate torsional moment of the beam.

The sixth assumption states that all reinforcement crossing the failure plane outside of the compression zone yields at failure. The strain readings obtained from gauges mounted on the longitudinal reinforcement indicate that the longitudinal bars adjacent to the face opposite the hinge did yield at failure. The difficulties encountered in measuring the strain in the ties intersecting the failure plane are indicated in SECTION 4-3(c). However, the strain readings obtained from the ties that did intersect the failure plane and the crack widths observed at failure indicate that this assumption is justified with regard to the ties as well as the longitudinal reinforcement.

The seventh assumption of SECTION 5-3 was introduced to simplify the equations. It states that the cross sectional area of transverse rein-



forcement intersected by the failure plane is constant per unit length of the beam. The area of transverse reinforcement is not constant per unit length of the beam, it is concentrated at the locations of the ties. This assumption is accurate for a tie spacing of zero and becomes progressively more inaccurate as the tie spacing is increased. To allow for the fact that a failure crack can cross only a whole number of ties, Pandit (1965) suggested that the number of ties resisting torsion be taken as the smallest number intersected by a potential crack at  $45^{\circ}$  to the axis of the beam. Similarly, Lessig (1959) suggested that an efficiency factor of less than 1.0 be applied to the transverse reinforcement.

The beams of Group 4 and Beams 1-5 and 3-4 were used to determine the effect of tie spacing on the torsional strength of beams. The interaction diagrams and the failure values of twisting moment and bending moment for these beams are presented in FIGURE 6-3. The correlation between the experimental and theoretical torsional moments is plotted as a function of tie spacing in FIGURE 6-4. This correlation is good within the range of tie spacing covered by these tests. The ratio of tie spacing to beam depth varied from 0.25 to 0.74; the ratio of tie spacing to beam width varied from 0.48 to 1.44. The value of  $\theta$  at which the beams were tested was 0.61 for Beams 3-4, 4-1, 4-2, 4-3 and 4-4 and 0.50 for Beams 1-5, 4-5 and 4-6.

If the tie spacing of a beam is sufficiently large it should behave in the same manner as a beam with longitudinal reinforcement only. Gesund and Boston (1964) reported tests on beams with longitudinal rein-



forcement only and stated that most of the beams failed suddenly. In contrast, all beams tested in this investigation exhibited appreciable ductility. This is not surprising for the majority of the beams since the tie spacing was generally less than the beam width. However, even Beam 4-4 which had a tie spacing of 9" exhibited great ductility. The first cracks on the bottom and two vertical faces of this beam were observed after application of load increment 4. At this load stage the bending moment was 44 percent of the ultimate bending moment and the torsional moment was 47 percent of the ultimate torsional moment. The first cracks on the top face were observed after application of load increment 5 when the bending and torsional moments were 54 percent and 56 percent of their ultimate values respectively. The unit angle of twist after application of load increment 9 was  $1019 \times 10^{-6}$  radians per inch. Failure occurred during application of load increment 10. Readings taken subsequent to failure indicated that the beam was supporting 100 percent of the ultimate bending moment, 90 percent of the ultimate torsional moment and that the unit angle of twist was  $1777 \times 10^{-6}$  radians per inch.

The eighth assumption of SECTION 5-3 states that the characteristics of the concrete in the compression zone are known and that the concrete reaches its strength in flexural compression. This assumption is true only for pure bending and becomes progressively more inaccurate as the amount of torsion a beam is subjected to is increased. When a beam is subjected to combined bending and torsion, the compression zone shown in FIGURES 5-1, 5-2 and 5-3 is subjected to a complex state of stress that



is further complicated by cracks crossing the compression zone. Subjecting a beam to transverse shear as well as bending and torsion increases the complexity of this problem.

Observation of the beams at failure suggested that the ends of the compression zone were less effective in resisting compression than the central portion. This can be illustrated by referring to FIGURE 5-1. The compression zone shown in this failure surface is intersected by cracks that cross the top face of the beam. These cracks, in conjunction with other cracks that formed shortly before failure, permitted displacement of part of the left half of the compression zone shown in FIGURE 5-1. The other segment of the beam showed similar displacements in the other half of the compression zone. If the ends of the compression zone are less effective in resisting compression than the central portion, the depth of the compression zone should be computed using an effective length of compression zone that is less than the full length shown in FIGURES 5-1, 5-2 and 5-3. In addition, when failure occurred it often appeared that the capacity of the compression zone was exhausted by pieces of concrete in the compression zone breaking away from the remainder of the beam. This is illustrated by FIGURES 4-5 and 4-10. Consequently, it appears that for beams subjected to combined loading the parameters  $0.85f'_c$  and  $k_1$  in Equations (5-2), (5-6) and (5-11) should be modified or replaced by other parameters.

If a factor  $k_e$  is introduced into Equations (5-2), (5-6) and (5-11) to reduce the effective length of the compression zone and to



modify the parameters  $0.85f'_c$  and  $k_1$ , Equations (8-1), (8-2) and (8-3) are obtained.

$$x_1 = \frac{A_{s1}f_{y1}(b + p_1k_{01}c_1^2/h)}{0.85 k_e k_1 f'_c L_1^2} \quad (8-1)$$

$$x_2 = \frac{A_{s2}f_{y2}(h + p_2k_{02}c_2^2/b) - v c_2}{0.85 k_e k_1 f'_c L_2^2} \quad (8-2)$$

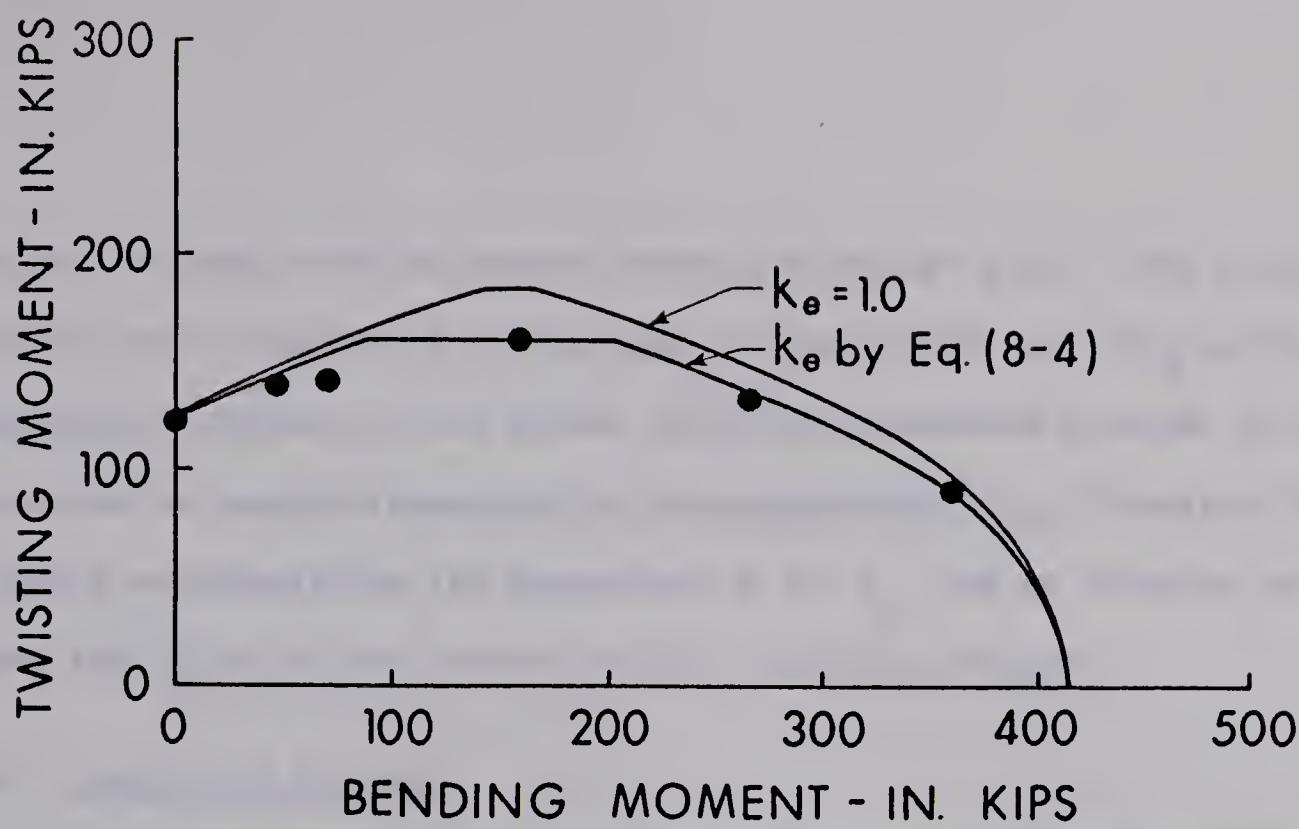
$$x_3 = \frac{A_{s3}f_{y3}(b + p_3k_{03}c_3^2/h)}{0.85 k_e k_1 f'_c L_3^2} \quad (8-3)$$

To show the effect of this factor on the theoretical ultimate torsional moment of a beam, the interaction diagrams for the beams of Groups 1 and 2 are shown in FIGURE 8-1. In addition, the failure values of twisting moment and bending moment are shown. The interaction diagrams for  $k_e = 1.0$  are the same as the ones shown in FIGURE 6-1. The other interaction diagrams in FIGURE 8-1 are plotted with the value of  $k_e$  defined by Equation (8-4).

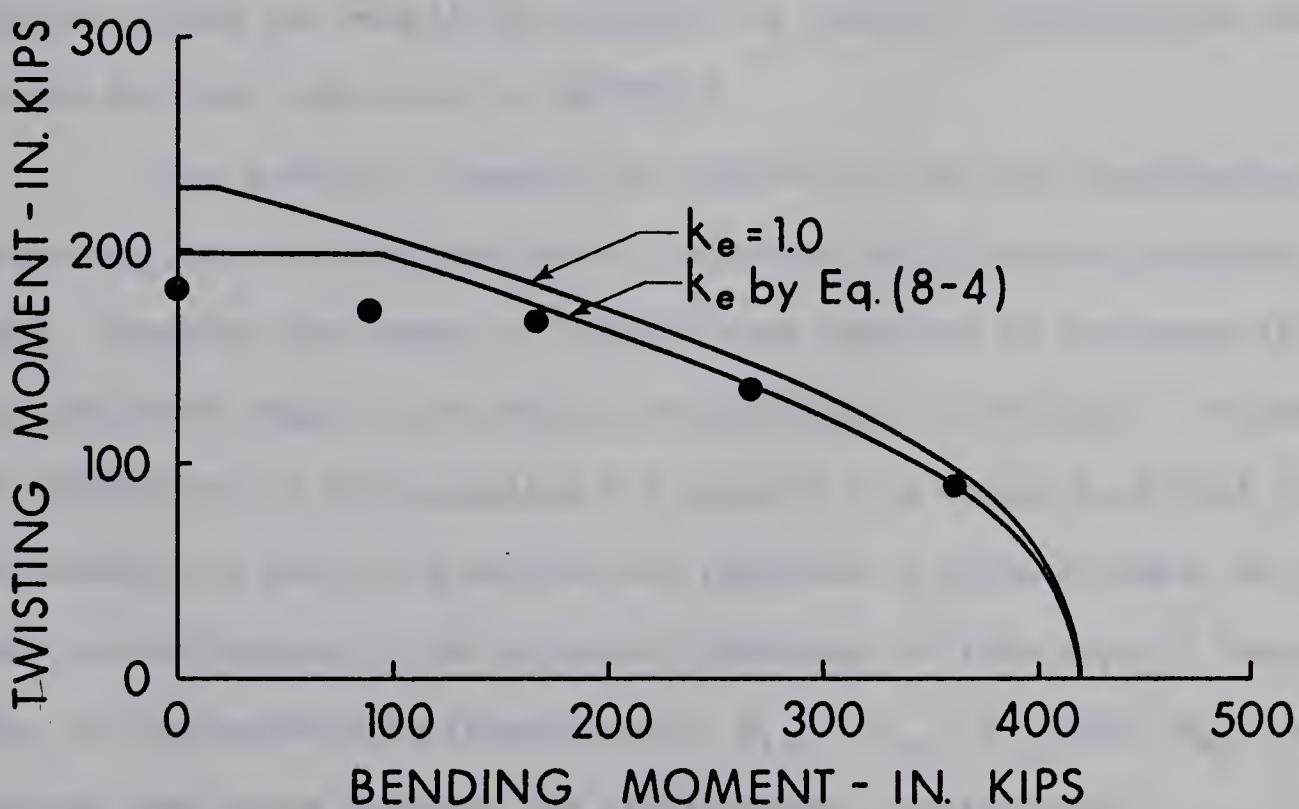
$$k_e = 1.0 - 1.6 c/c_m \geq 0.2 \quad (8-4)$$

For values of  $c/c_m$  greater than 0.5 this equation assigns a value to  $k_e$  of 0.2. For values of  $c/c_m$  between zero and 0.5 the value assigned





(a) GROUP 1



(b) GROUP 2

FIGURE 8 - 1 MODIFIED INTERACTION DIAGRAMS FOR BEAMS OF GROUPS 1 AND 2



to  $k_e$  by this equation varies linearly from 1.0 to 0.2. The selection of the lower limit of  $k_e$  as 0.2 and the upper limit of  $c/c_m$  as 0.5 is completely arbitrary. The linear variation is assumed in order to provide a simple but smooth transition in the range  $0 < c < 0.5 c_m$ . Equation (8-4) greatly oversimplifies the determination of  $k_e$  and is intended only to show the effect of this factor on the interaction diagram.

### 8-7 Simplified Analysis

A simplified analysis and a simplified interaction diagram are presented in CHAPTER VII. Comparison of the values of the ratio  $M_{tt}/M_t$  listed in TABLES 6-1 to 6-7 with the values listed in TABLES 7-1 and 7-2 indicates that the simplified analysis is generally conservative compared to the analysis presented in CHAPTER V.

The analysis presented in CHAPTER VII has the disadvantage of requiring numerous computations to determine the torsional strength of a beam. However, the number of calculations required is decreased if top reinforcement equal to the bottom reinforcement is provided. In addition, the parameters in the equations are grouped in a manner such that if adjustments to the trial section are required, a minimum number of quantities must be recalculated. The principal advantage of this type of analysis is that it is adaptable to tabulation of  $M_{ta}$ ,  $M_{tb}$ ,  $M_{tc}$  and  $M_{bu}$  for various beam cross sections and reinforcement combinations.

This analysis is applicable only to rectangular, concrete beams that are provided with both longitudinal and transverse reinforcement adjacent to all faces. The amount of reinforcement provided must be suf-



ficient to ensure that failure does not occur when first cracking of the concrete occurs. If this requirement is not satisfied the beam should be analysed as a plain concrete beam. This analysis should not be used if the magnitude of transverse shear exceeds the limits discussed in SECTION 8-5.



## CHAPTER IX

### SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

#### 9-1 Summary

The experimental phase of this investigation consisted of testing 34 reinforced concrete beams subjected to various combinations of load. All beams had a nominal 6" x 12" cross section and a nominal concrete strength of 5000 psi. Various combinations of reinforcement were provided but except for the beams of Group 4 all beams within one group had similar reinforcement. All beams were provided with both longitudinal and transverse reinforcement.

The 34 beams were divided into 7 groups. The beams of Groups 1, 2, 3 and 4 were subjected to various combinations of bending and torsion and the beams of Groups 5, 6 and 7 were subjected to various combinations of bending, torsion and shear.

All beams were tested to failure by applying load in a series of increments. Each increment consisted of increasing to a predetermined level the transverse load or the twisting load or both, depending on the type of test. In the cases where both types of load were applied in the same increment, the transverse load was applied first. All beams except those of Group 3 and Beam 1-2 were subjected to loads such that for any one test the ratio of twisting moment to bending moment at the end of each increment was a constant. The loading sequences for the beams of Group 3



and Beam 1-2 have been outlined in SECTION 3-4.

Based on the observations made in the experimental phase of this investigation three idealized failure surfaces have been presented, two of which were first suggested by Lessig (1959). Equations for the ultimate torsional strength of a beam based on each of the three failure surfaces have been developed and their method of solution has been presented. These equations have then been simplified and an interaction diagram consisting of three straight lines has been presented along with the applicable equations.

The correlation between the experimental results of 109 beams and the results of both the analysis presented in CHAPTER V and the simplified analysis presented in CHAPTER VII has been given. Of these beams, 34 are the beams tested in this investigation and 75 are beams that have been tested by other investigators.

## 9-2 Conclusions

The following conclusions are based on the results of this investigation and are limited to rectangular, concrete beams with both longitudinal and transverse reinforcement.

- (1) Reinforced concrete beams that are subjected to bending, torsion and moderate amounts of transverse shear can fail by three different modes. These modes of failure are characterized by the formation of a hinge adjacent to one face of the beam and yielding of the reinforcement adjacent to the face opposite to the hinge. The modes of failure predicted by the analysis agree with the observed modes of failure.



- (2) In beams subjected to a constant twisting moment and a linearly varying bending moment a mode 3 failure occurs in the region subjected to the lowest bending moment and a mode 1 failure occurs in the region subjected to the highest bending moment.
- (3) A beam that is provided with less top longitudinal reinforcement than bottom longitudinal reinforcement and is subjected to a small bending moment in addition to a torsional moment exhibits a greater torsional strength than if the same beam is subjected to pure torsion.
- (4) A beam that is provided with less top longitudinal reinforcement than bottom longitudinal reinforcement and is subjected to either pure torsion or torsion in combination with a small bending moment deflects upwards after cracking of the concrete has occurred and exhibits an upward deflection at failure.
- (5) The presence of flexure does not increase the torsional strength of a beam provided with equal top and bottom reinforcement.
- (6) A small or moderate amount of transverse shear does not affect the torsional strength of a beam except as provided for by a mode 2 failure surface.
- (7) The top longitudinal reinforcement does not contribute significantly to the torsional strength of a beam that fails by mode 1.
- (8) The assumption that at failure the full length of the concrete compression zone reaches its flexural strength gives an upper bound of the torsional strength of a beam.



- (9) Increasing the tie spacing decreases the torsional strength of a beam.
- (10) The sequence of loading does not significantly affect the torsional strength of a beam.
- (11) The behavior of a beam prior to cracking of the concrete is not significantly affected by the reinforcement provided. After cracking occurs, the behavior depends on the reinforcement and on the ratio of twisting moment to bending moment.
- (12) In beams tested at a constant ratio of twisting moment to bending moment the presence of bending moment reduces the torsional moment at which cracking of the concrete occurs.
- (13) The limit of over reinforcement suggested by Lessig for ratios of twisting moment to bending moment greater than 0.2 is conservative.
- (14) The lower limit of the ratio of transverse to longitudinal reinforcement suggested by Lessig is conservative.

### 9-3 Recommendations

The scope of this investigation is necessarily limited and before a comprehensive design procedure can be established further research is necessary. In particular, the following aspects of the problem should be investigated.

- (1) Determination of the effective length of the concrete compression zone and the criterion of failure for the concrete in the compression zone is necessary.



- (2) The amount of reinforcement necessary to cause failure of a beam before the reinforcement yields must be determined.
- (3) The minimum amount of reinforcement necessary to prevent failure from occurring when first cracking of the concrete occurs must be determined.
- (4) The maximum tie spacing for which this analysis applies must be determined.
- (5) The effect of placing the longitudinal bars at various locations within the cross section should be investigated.
- (6) Beams with height to width ratios of less than 1.0 should be tested.
- (7) The location of the critical plane in beams subjected to a torsional moment and a variable bending moment must be more accurately established.
- (8) Beams subjected to torsion, bending and high transverse shear should be tested. Also, it is necessary to develop a method to predict the torsional strength of a beam subjected to only torsion and transverse shear. This is necessary not only as a limiting case but to permit determination of the torsional strength of a beam at an inflection point.
- (9) Beams with cross sections other than rectangular or square should be tested. Equations for the torsional strength of these beams could be developed on the basis of the failure surfaces that are observed.



## **LIST OF REFERENCES**



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## APPENDIX A

### PROPERTIES OF MATERIALS AND FABRICATION OF SPECIMENS



## APPENDIX A

### PROPERTIES OF MATERIALS AND FABRICATION OF SPECIMENS

#### A-1 Materials

##### (a) Sand

A sieve analysis of the sand used is given in TABLE A-1. The average moisture content of the sand was found to be 4.5 percent.

TABLE A-1  
Sieve Analysis of Sand

| Sieve Size       | Weight Retained<br>(gms.) | % Retained | Cumulative<br>% Retained | A.S.T.M.<br>Standard |
|------------------|---------------------------|------------|--------------------------|----------------------|
| #4               | 17.5                      | 3.0        | 3.0                      | 0 - 5                |
| #8               | 85.2                      | 14.7       | 17.7                     |                      |
| #16              | 54.6                      | 9.5        | 27.2                     | 20 - 55              |
| #30              | 60.0                      | 10.3       | 37.5                     |                      |
| #50              | 208.4                     | 35.8       | 73.3                     | 70 - 90              |
| #100             | 122.9                     | 21.1       | 94.4                     | 90 - 98              |
| Pan              | 17.8                      | 3.1        | --                       |                      |
| Silt             | 14.4                      | 2.5        | --                       |                      |
| Total            | 580.8                     | 100.0      | 253.1                    |                      |
| Fineness Modulus | 2.53                      |            |                          |                      |

##### (b) Coarse Aggregate

The coarse aggregate was 3/4" maximum size crushed rock. The average moisture content was 1.7 percent. The results of a sieve analysis are presented in TABLE A-2.



TABLE A-2  
Sieve Analysis of Coarse Aggregate

| Sieve Size | Weight Retained<br>(lbs.) | % Retained | Cumulative<br>% Retained |
|------------|---------------------------|------------|--------------------------|
| 3/4"       | 0.30                      | 1.1        | 1.1                      |
| 3/8"       | 15.63                     | 58.4       | 59.5                     |
| #4         | 10.03                     | 37.5       | 97.0                     |
| Pan        | 0.80                      | 3.0        | 100.0                    |
| Total      | 26.76                     | 100.0      | 257.6                    |

(c) Concrete

The mix design used for all specimens was:

- (1) Cement 114 lbs.
- (2) Sand 294 lbs.
- (3) Coarse Aggregate 428 lbs.

The quantity of water used was approximately 57 lbs. but minor adjustments were made as required to produce a mix of approximately 3" slump. The yield of this mix was about six cubic feet.

High early strength cement supplied in paper bags was used.

(d) Reinforcement

All reinforcement used in the specimens was of the deformed type. The reinforcement of one size was all from the same heat except for the ties. These were received in two batches and were designated as lot number 1 and lot number 2. All ties within one lot were from the same heat. Coupon tests were performed on three specimens from each group in order to determine the yield and ultimate strength of the reinforcement. The results of these tests are shown in TABLE A-3.



TABLE A-3  
Strength Properties of Reinforcement

| Group          | Average Yield Stress (k.s.i.) | Average Ultimate Stress (k.s.i.) |
|----------------|-------------------------------|----------------------------------|
| #3 Bars        | 53.0                          | 79.5                             |
| #4 Bars        | 47.0                          | 73.9                             |
| #5 Bars        | 48.9                          | 76.1                             |
| #6 Bars        | 46.9                          | 76.7                             |
| #8 Bars        | 43.8                          | 81.8                             |
| #3 Ties Lot #1 | 55.0                          | 80.4                             |
| #3 Ties Lot #2 | 53.7                          | 78.4                             |

#### A-2 Fabrication of Specimens

The longitudinal reinforcement was first cut to the required length and then the reinforcement cages were fabricated by wiring the ties to the longitudinal bars at each intersection. At each point where an electrical strain gauge was required, the reinforcement was ground to a smooth finish and covered with a piece of rigid insulation which was securely taped to the bar. Before concrete was cast, the forms were oiled and the reinforcing cages were placed and properly positioned in them.

The concrete was mixed in a nine cubic foot capacity laboratory concrete mixer. One batch was sufficient for each beam along with its control cylinders. Before mixing began, a one cubic foot butter mix was used to condition the mixer. All concrete was mixed for approximately five minutes. The water content was adjusted until a slump of 3" was obtained. When the proper slump was obtained the concrete was deposited in the forms and vibrated with a high frequency internal vibrator.



Four six by twelve inch concrete cylinders were cast with each beam. These cylinders were compacted by rodding and were cured and stored under the same conditions as the beams. Two cylinders were tested in compression and the other two were subjected to the splitting tensile test. A beam and the cylinders corresponding to it were tested on the same day.

The beams and cylinders were cured for three days by covering them with wet burlap and a plastic sheet. Following this, the forms were stripped and the specimens were stored in air until the day of test.

Before the day of test, the pieces of insulation covering the smooth areas of the reinforcement were removed in order to expose the prepared surfaces of the bar. These surfaces were then cleaned and SR-4 electrical strain gauges were glued to them.



## APPENDIX B

### APPENDIX B

#### APPENDIX B OBSERVED DATA



APPENDIX B  
OBSERVED DATA

B-1 Test Results

The test results for each beam are given in TABLES B-1 to B-34. Reinforcement stresses that are enclosed in parentheses indicate that the reinforcement had yielded before the strain began to decrease. For computation of these stresses it has been assumed that the maximum recorded strain was equal to the maximum strain that occurred.

B-2 Deformation Characteristics and Reinforcement Stresses

The angle of twist, deflection and reinforcement stress for each beam are plotted against twisting moment in FIGURES B-1 to B-18. No deflections were measured for Beams 3-1 or 3-4. Upward (negative) deflections are plotted on the left side of zero and downward (positive) deflections on the right side. Compressive (negative) stresses are plotted on the left side of zero and tensile (positive) stresses on the right side. The unusual shape of the plots for the beams of Group 3 is due to the sequence of loading that was used. The sequence of loading for the beams is presented in CHAPTER III.

The plots of twisting moment versus deflection and reinforcement stress have been selected as representative of the results obtained. The word east, west or center beside a plot of deflection indicates the deflection gauge to which the plot corresponds. The number beside a plot



of reinforcement stress indicates the strain gauge to which the plot corresponds. The location of the deflection gauges and the numbering system of the strain gauges are presented in CHAPTER III.



TABLE B-1  
TEST RESULTS FOR BEAM 1-1

| Load Stage | Torque in. kips | Bending Moment in. kips | ∅  | Twist                 |                 | Deflection |        |      |
|------------|-----------------|-------------------------|----|-----------------------|-----------------|------------|--------|------|
|            |                 |                         |    | in. x 10 <sup>6</sup> | Radians per in. | West       | Center | East |
| 0          | 0               | --                      | -- | 0                     | 0               | 0          | 0      | 0    |
| 1          | 23              | --                      | -- | 30                    | 0               | - 0.5      | - 0.5  |      |
| 2          | 46              | --                      | -- | 65                    | 0               | 0          | - 0.5  |      |
| 3          | 55              | --                      | -- | 90                    | 0               | 0          | - 0.5  |      |
| 4          | 65              | --                      | -- | 243                   | - 1.0           | - 1.5      | - 1.5  |      |
| 5          | 79              | --                      | -- | 503                   | - 4.0           | - 5.0      | - 4.5  |      |
| 6          | 98              | --                      | -- | 813                   | - 9.0           | - 10.5     | - 9.0  |      |
| 7          | 107             | --                      | -- | 960                   | - 11.0          | - 12.5     | - 11.0 |      |
| 8          | 117             | --                      | -- | 1233                  | - 16.0          | - 19.0     | - 16.5 |      |
| 9*         | 122             | --                      | -- | --                    | --              | --         | --     |      |

| Load Stage | Reinforcement Stresses (ksi) |       |      |      |      |       |       |      |
|------------|------------------------------|-------|------|------|------|-------|-------|------|
|            | Gauge                        |       |      |      |      |       |       |      |
| 1          | 2                            | 3     | 4    | 5    | 6    | 7     | 8     |      |
| 0          | 0                            | 0     | 0    | 0    | 0    | 0     | 0     | 0    |
| 1          | +0.1                         | +0.2  | +0.3 | +0.3 | +0.6 | +0.3  | -0.1  | +0.2 |
| 2          | 0.3                          | 0.7   | 1.0  | 0.9  | 1.7  | 2.7   | -0.3  | 0.5  |
| 3          | 0.9                          | 1.5   | 1.9  | 1.7  | 3.0  | 7.8   | -0.1  | 1.0  |
| 4          | 18.8                         | 15.3  | 8.1  | 7.0  | 10.8 | 26.4  | +10.5 | 6.0  |
| 5          | 35.8                         | 38.5  | 15.0 | 11.6 | 18.7 | 38.2  | 15.3  | 12.9 |
| 6          | 52.7                         | Yield | 21.5 | 17.3 | 28.5 | 48.6  | 18.6  | 24.8 |
| 7          | Yield                        | Yield | 22.8 | 19.7 | 32.2 | 52.5  | 21.0  | 32.4 |
| 8          | Yield                        | Yield | 23.5 | 23.8 | 37.2 | Yield | 23.7  | 39.0 |
| 9*         | --                           | --    | --   | --   | --   | --    | --    | --   |

\*Failure



TABLE B-2  
TEST RESULTS FOR BEAM 1-2

| Load Stage | Torque in.kips | Bending Moment in.kips | ∅    | Twist                             |                          | Deflection |        |      |
|------------|----------------|------------------------|------|-----------------------------------|--------------------------|------------|--------|------|
|            |                |                        |      | Radians per in. x 10 <sup>6</sup> | inches x 10 <sup>2</sup> | West       | Center | East |
| 0          | 0              | 0                      | 0    | 0                                 | 0                        | 0          | 0      | 0    |
| 1          | 49             | 24                     | 2.04 | 85                                | + 0.5                    | + 1.0      | + 1.0  |      |
| 2          | 60             | 27                     | 2.22 | 183                               | 1.0                      | 0.5        | 1.5    |      |
| 3          | 71             | 30                     | 2.37 | 363                               | 0.5                      | 0          | 0      |      |
| 4          | 81             | 32                     | 2.53 | 535                               | - 1.0                    | - 1.5      | - 1.0  |      |
| 5          | 91             | 35                     | 2.60 | 683                               | - 2.0                    | - 3.5      | - 3.0  |      |
| 6          | 102            | 38                     | 2.69 | 838                               | - 4.0                    | - 5.5      | - 4.0  |      |
| 7          | 107            | 39                     | 2.74 | 905                               | - 5.0                    | - 6.0      | - 4.5  |      |
| 8          | 112            | 40                     | 2.80 | 990                               | - 5.5                    | - 7.0      | - 6.5  |      |
| 9          | 117            | 42                     | 2.79 | 1073                              | - 6.0                    | - 8.0      | - 7.0  |      |
| 10         | 123            | 43                     | 2.86 | 1173                              | - 7.0                    | - 9.0      | - 8.0  |      |
| 11         | 127            | 44                     | 2.89 | 1328                              | - 8.5                    | - 11.0     | - 9.0  |      |
| 12         | 133            | 46                     | 2.90 | 1435                              | -11.0                    | -13.0      | -10.5  |      |
| 13*        | 138            | 47                     | 2.94 | --                                | --                       | --         | --     |      |

| Load Stage | Reinforcement Stresses (ksi) |       |      |      |      |       |      |       |
|------------|------------------------------|-------|------|------|------|-------|------|-------|
|            | Gauge                        |       |      |      |      |       |      |       |
| 1          | 2                            | 3     | 4    | 5    | 6    | 7     | 8    |       |
| 0          | 0                            | 0     | 0    | 0    | 0    | 0     | 0    | 0     |
| 1          | -0.1                         | +0.2  | +2.5 | +2.7 | +2.7 | +4.6  | -0.7 | +0.2  |
| 2          | +3.3                         | 2.9   | 9.8  | 7.1  | 6.6  | 14.1  | +2.0 | 2.9   |
| 3          | 17.1                         | 13.3  | 15.6 | 13.1 | 11.1 | 25.1  | 5.3  | 13.3  |
| 4          | 29.5                         | 26.6  | 20.7 | 16.2 | 14.4 | 30.9  | 9.0  | 26.6  |
| 5          | 38.7                         | 34.5  | 23.7 | 18.6 | 17.4 | 36.5  | 12.6 | 34.5  |
| 6          | 48.7                         | 42.2  | 26.3 | 20.8 | 21.2 | 41.1  | 16.0 | 42.2  |
| 7          | Yield                        | 45.2  | 27.3 | 21.8 | 23.1 | 42.9  | 17.7 | 45.2  |
| 8          | Yield                        | 48.5  | 28.6 | 23.0 | 25.1 | 44.9  | 19.5 | 48.5  |
| 9          | Yield                        | 51.9  | 30.0 | 24.3 | 27.5 | 47.6  | 21.8 | 51.9  |
| 10         | Yield                        | Yield | 31.1 | 25.6 | 30.0 | 49.6  | 24.3 | Yield |
| 11         | Yield                        | Yield | 32.2 | 27.3 | 32.5 | 51.2  | 26.7 | Yield |
| 12         | Yield                        | Yield | 34.0 | 29.1 | 35.6 | 52.6  | 30.0 | Yield |
| 13*        | Yield                        | Yield | 36.2 | 31.0 | 38.4 | Yield | 34.6 | Yield |

\*Failure



TABLE B-3  
TEST RESULTS FOR BEAM 1-3

| Load Stage | Torque in.kips | Bending Moment in.kips | ∅    | Twist Radians per in. $\times 10^6$ | Deflection inches $\times 10^2$ |        |      |
|------------|----------------|------------------------|------|-------------------------------------|---------------------------------|--------|------|
|            |                |                        |      |                                     | West                            | Center | East |
| 0          | 0              | 0                      | --   | 0                                   | 0                               | 0      | 0    |
| 1          | 48             | 24                     | 2.00 | 110                                 | 1.5                             | 0.5    | 0.5  |
| 2          | 60             | 30                     | 2.00 | 288                                 | 1.0                             | 0      | 0.5  |
| 3          | 71             | 35                     | 2.03 | 495                                 | 1.0                             | - 0.5  | -0.5 |
| 4          | 80             | 40                     | 2.00 | 633                                 | 0                               | - 1.5  | -1.0 |
| 5          | 91             | 46                     | 1.98 | 813                                 | -0.5                            | - 3.0  | -2.5 |
| 6          | 102            | 51                     | 2.00 | 995                                 | -1.5                            | - 4.0  | -4.0 |
| 7          | 113            | 57                     | 1.98 | 1200                                | -2.5                            | - 6.0  | -5.0 |
| 8          | 124            | 62                     | 2.00 | 1410                                | -3.5                            | - 7.5  | -6.0 |
| 9          | 129            | 65                     | 1.99 | 1570                                | -4.5                            | -10.0  | -7.5 |
| 10         | 134            | 67                     | 2.00 | 1730                                | -5.5                            | -11.5  | -8.5 |
| 11*        | 140            | 70                     | 2.00 | --                                  | --                              | --     | --   |

| Load Stage | Reinforcement Stresses (ksi) |       |      |      |      |      |      |      |
|------------|------------------------------|-------|------|------|------|------|------|------|
|            | Gauge                        |       |      |      |      |      |      |      |
| 1          | 2                            | 3     | 4    | 5    | 6    | 7    | 8    |      |
| 0          | 0                            | 0     | 0    | 0    | 0    | 0    | 0    | 0    |
| 1          | 0                            | 1.3   | 2.6  | 3.3  | 0.5  | 1.6  | -0.4 | 5.7  |
| 2          | 14.5                         | 9.9   | 10.4 | 11.2 | 4.8  | 21.2 | +9.1 | 21.5 |
| 3          | 23.7                         | 21.0  | 14.7 | 15.6 | 8.9  | 26.7 | 15.8 | 27.9 |
| 4          | 29.7                         | 27.3  | 17.5 | 19.2 | 12.1 | 30.0 | 20.7 | 33.0 |
| 5          | 37.5                         | 36.5  | 20.2 | 22.6 | 18.0 | 33.1 | 27.3 | 38.4 |
| 6          | 43.6                         | 45.2  | 22.8 | 25.8 | 23.2 | 36.9 | 31.0 | 42.6 |
| 7          | 50.5                         | Yield | 25.5 | 29.4 | 29.8 | 39.8 | 34.1 | 47.0 |
| 8          | Yield                        | Yield | 28.2 | 32.6 | 36.0 | 43.0 | 37.4 | 49.9 |
| 9          | Yield                        | Yield | 29.3 | 34.5 | 39.4 | 45.5 | 38.9 | 51.1 |
| 10         | Yield                        | Yield | 31.2 | 36.4 | 44.5 | 46.3 | 40.9 | 51.0 |
| 11*        | Yield                        | Yield | 33.3 | 38.9 | 49.3 | 45.3 | --   | --   |

\*Failure



TABLE B-4

## TEST RESULTS FOR BEAM 1-4

| Load Stage | Torque in. kips | Bending Moment in. kips | $\emptyset$ | Twist                         |      | Deflection inches $\times 10^2$ |      |   |
|------------|-----------------|-------------------------|-------------|-------------------------------|------|---------------------------------|------|---|
|            |                 |                         |             | Radians per in. $\times 10^6$ | West | Center                          | East |   |
| 0          | 0               | 0                       | --          | 0                             | 0    | 0                               | 0    | 0 |
| 1          | 23              | 24                      | 0.96        | 28                            | 1.0  | 0.5                             | 0    | 0 |
| 2          | 38              | 38                      | 1.00        | 50                            | 1.5  | 1.5                             | 1.0  |   |
| 3          | 52              | 51                      | 1.02        | 100                           | 3.5  | 3.5                             | 3.0  |   |
| 4          | 65              | 65                      | 1.00        | 195                           | 5.5  | 6.0                             | 5.0  |   |
| 5          | 78              | 78                      | 1.00        | 315                           | 7.5  | 7.0                             | 6.0  |   |
| 6          | 98              | 98                      | 1.00        | 553                           | 8.0  | 7.5                             | 6.5  |   |
| 7          | 118             | 119                     | 0.99        | 813                           | 8.5  | 8.0                             | 7.0  |   |
| 8          | 132             | 132                     | 1.00        | 1013                          | 8.0  | 7.5                             | 6.5  |   |
| 9          | 145             | 146                     | 0.99        | 1258                          | 8.5  | 7.0                             | 6.0  |   |
| 10*        | 159             | 159                     | 1.00        | --                            | -    | -                               | -    |   |

| Load Stage | Reinforcement Stresses (ksi) |      |       |       |      |      |      |      |
|------------|------------------------------|------|-------|-------|------|------|------|------|
|            | Gauge                        |      |       |       |      |      |      |      |
| 1          | 2                            | 3    | 4     | 5     | 6    | 7    | 8    |      |
| 0          | 0                            | 0    | 0     | 0     | 0    | 0    | 0    | 0    |
| 1          | -0.6                         | -0.5 | +1.0  | +0.8  | +0.1 | +0.3 | -0.2 | +0.4 |
| 2          | -2.0                         | -1.6 | 3.2   | 3.7   | 0.6  | 1.1  | -0.5 | 1.1  |
| 3          | -3.3                         | -2.5 | 6.6   | 8.7   | 1.8  | 3.1  | -1.0 | 5.8  |
| 4          | -3.4                         | -1.9 | 13.6  | 16.6  | 5.3  | 20.1 | +1.5 | 20.7 |
| 5          | +2.7                         | +3.6 | 21.6  | 23.8  | 7.9  | 26.7 | 10.4 | 28.2 |
| 6          | 11.1                         | 14.1 | 30.0  | 32.7  | 13.5 | 34.8 | 15.8 | 37.8 |
| 7          | 18.4                         | 27.0 | 38.1  | 40.8  | 21.0 | 42.9 | 17.9 | 45.3 |
| 8          | 24.8                         | 31.5 | 44.0  | 45.3  | 25.8 | 45.3 | 18.9 | 49.4 |
| 9          | 33.4                         | 37.2 | Yield | Yield | 31.8 | 47.7 | 20.1 | 53.4 |
| 10*        | --                           | --   | --    | --    | --   | --   | --   | --   |

\*Failure



TABLE B-5  
TEST RESULTS FOR BEAM 1-5

| Load Stage | Torque in.kips | Bending Moment in.kips | $\emptyset$ | Twist Radians per in. $\times 10^6$ | Deflection inches $\times 10^2$ |
|------------|----------------|------------------------|-------------|-------------------------------------|---------------------------------|
|            |                |                        |             |                                     | West Center East                |
| 0          | 0              | 0                      | --          | 0                                   | 0 0 0                           |
| 1          | 13             | 24                     | 0.54        | 18                                  | 0.5 0.5 0                       |
| 2          | 27             | 51                     | 0.53        | 43                                  | 2.5 2.0 1.5                     |
| 3          | 39             | 78                     | 0.50        | 80                                  | 5.0 5.0 4.5                     |
| 4          | 53             | 105                    | 0.50        | 153                                 | 8.0 9.5 7.0                     |
| 5          | 67             | 132                    | 0.51        | 250                                 | 11.0 12.5 10.5                  |
| 6          | 86             | 173                    | 0.50        | 443                                 | 15.5 17.0 15.0                  |
| 7          | 107            | 213                    | 0.50        | 695                                 | 19.0 21.0 17.5                  |
| 8          | 113            | 227                    | 0.50        | 805                                 | 20.5 22.5 19.5                  |
| 9          | 120            | 240                    | 0.50        | 898                                 | 22.0 24.0 19.5                  |
| 10         | 128            | 254                    | 0.50        | 1210                                | 26.0 29.0 24.5                  |
| 11*        | 131            | 267                    | 0.49        | --                                  | -- -- --                        |
| 12**       | 111            | 267                    | 0.42        | 2325                                | 57.0 62.5 50.5                  |

| Load Stage | Reinforcement Stresses (ksi) |      |       |       |       |       |      |       |
|------------|------------------------------|------|-------|-------|-------|-------|------|-------|
|            | Gauge                        |      |       |       |       |       |      |       |
| 1          | 2                            | 3    | 4     | 5     | 6     | 7     | 8    |       |
| 0          | 0                            | 0    | 0     | 0     | 0     | 0     | 0    | 0     |
| 1          | -0.6                         | -0.4 | +0.6  | +0.9  | +0.5  | +0.3  | +0.3 | +0.2  |
| 2          | -3.1                         | -2.9 | 3.2   | 4.5   | 1.6   | 1.0   | 0.5  | 0.8   |
| 3          | -6.6                         | -5.1 | 9.1   | 10.7  | 3.9   | 2.8   | 0.4  | 9.9   |
| 4          | -8.3                         | -5.7 | 16.8  | 18.7  | 6.5   | 9.5   | 0.6  | 17.7  |
| 5          | -9.2                         | -4.8 | 23.0  | 26.6  | 8.4   | 15.7  | 1.7  | 25.8  |
| 6          | -5.4                         | -0.9 | 34.7  | 41.4  | 11.1  | 25.7  | 10.8 | 34.3  |
| 7          | -3.8                         | +3.2 | 44.8  | Yield | 19.9  | 32.3  | 15.9 | 45.9  |
| 8          | -3.9                         | 3.0  | Yield | Yield | 23.7  | 34.7  | 18.3 | 49.5  |
| 9          | -4.2                         | 3.6  | Yield | Yield | 27.7  | 36.3  | 20.2 | 53.6  |
| 10         | -1.2                         | 5.7  | Yield | --    | 35.9  | 39.8  | 26.7 | Yield |
| 11*        | --                           | --   | --    | --    | --    | --    | --   | --    |
| 12**       | -8.7                         | -5.9 | --    | --    | Yield | Yield | 37.5 | Yield |

\*Failure

\*\*Past peak loads



TABLE B-6  
TEST RESULTS FOR BEAM 1-6

| Load Stage | Torque in. kips | Bending Moment in. kips | ∅    | Twist Radians per in. x 10 <sup>6</sup> | Deflection inches x 10 <sup>2</sup> |        |       |
|------------|-----------------|-------------------------|------|---|-------------------------------------|--------|-------|
|            |                 |                         |      |   | West                                | Center | East  |
| 0          | 0               | 0                       | --   | 0                                       | 0                                   | 0      | 0     |
| 1          | 10              | 38                      | 0.26 | 13                                      | 1.0                                 | 1.5    | 1.0   |
| 2          | 19              | 78                      | 0.24 | 30                                      | 3.5                                 | 5.0    | 4.0   |
| 3          | 29              | 119                     | 0.24 | 55                                      | 7.5                                 | 9.0    | 8.0   |
| 4          | 39              | 159                     | 0.25 | 90                                      | 11.5                                | 13.0   | 14.0  |
| 5          | 49              | 200                     | 0.25 | 135                                     | 15.5                                | 17.5   | 15.5  |
| 6          | 60              | 240                     | 0.25 | 188                                     | 19.5                                | 22.0   | 19.0  |
| 7          | 70              | 281                     | 0.25 | 243                                     | 23.0                                | 26.5   | 23.0  |
| 8          | 80              | 321                     | 0.25 | 315                                     | 27.0                                | 31.0   | 27.0  |
| 9*         | 90              | 362                     | 0.25 | -                                       | --                                  | --     | --    |
| 10**       | 76              | 362                     | 0.21 | -                                       | 173.0                               | 207.5  | 177.0 |

| Load Stage | Reinforcement Stresses (ksi) |       |       |       |       |       |      |      |
|------------|------------------------------|-------|-------|-------|-------|-------|------|------|
|            | Gauge                        |       |       |       |       |       |      |      |
| 1          | 2                            | 3     | 4     | 5     | 6     | 7     | 8    |      |
| 0          | 0                            | 0     | 0     | 0     | 0     | 0     | 0    | 0    |
| 1          | - 1.6                        | - 0.9 | +1.5  | +1.7  | +0.4  | +0.1  | +0.4 | +0.1 |
| 2          | - 6.6                        | - 4.3 | 7.8   | 7.1   | 0.9   | 0     | 0.9  | 0    |
| 3          | -11.3                        | - 7.5 | 15.9  | 16.0  | 3.2   | -0.1  | 1.2  | -0.3 |
| 4          | -15.5                        | -10.1 | 23.6  | 24.3  | 4.2   | +0.3  | 1.7  | +0.1 |
| 5          | -19.6                        | -12.3 | 30.3  | 32.4  | 5.2   | 0.9   | 2.0  | 0.8  |
| 6          | -24.3                        | -14.4 | 36.6  | 40.5  | 6.2   | 2.5   | 2.4  | 1.9  |
| 7          | -28.4                        | -15.9 | 42.9  | Yield | 7.4   | 3.9   | 2.7  | 8.4  |
| 8          | -32.9                        | -17.6 | Yield | Yield | 9.1   | 8.7   | 3.6  | 15.6 |
| 9*         | -37.2                        | -17.8 | Yield | Yield | 21.0  | --    | --   | --   |
| 10**       | Yield                        | -44.3 | --    | --    | Yield | Yield | 42.7 | 46.5 |

\*Failure

\*\*Past peak loads



TABLE B-7  
TEST RESULTS FOR BEAM 2-1

| Load Stage | Torque in. kips | Bending Moment in. kips | ∅  | Twist Radians per in. x 10 <sup>6</sup> | Deflection inches x 10 <sup>2</sup> |        |      |
|------------|-----------------|-------------------------|----|---|-------------------------------------|--------|------|
|            |                 |                         |    |   | West                                | Center | East |
| 0          | 0               | --                      | -- | 0                                       | 0                                   | 0      | 0    |
| 1          | 20              | --                      | -- | 30                                      | -0.5                                | 0      | 0    |
| 2          | 40              | --                      | -- | 58                                      | 0                                   | 0      | 0    |
| 3          | 60              | --                      | -- | 258                                     | +0.5                                | 0      | -1.5 |
| 4          | 80              | --                      | -- | 463                                     | -0.5                                | -1.5   | -1.0 |
| 5          | 100             | --                      | -- | 668                                     | -1.5                                | -2.0   | -2.0 |
| 6          | 120             | --                      | -- | 895                                     | -2.0                                | -3.0   | -3.0 |
| 7          | 140             | --                      | -- | 1145                                    | -3.0                                | -4.0   | -3.5 |
| 8          | 161             | --                      | -- | 1483                                    | -4.0                                | -4.5   | -4.0 |
| 9*         | 181             | --                      | -- | --                                      | --                                  | --     | --   |
| 10**       | 162             | --                      | -- | 2748                                    | -5.0                                | -16.5  | -5.5 |

| Load Stage | Reinforcement Stresses (ksi) |       |       |      |       |       |       |       |
|------------|------------------------------|-------|-------|------|-------|-------|-------|-------|
|            | Gauge                        |       |       |      |       |       |       |       |
| 1          | 2                            | 3     | 4     | 5    | 6     | 7     | 8     |       |
| 0          | 0                            | 0     | 0     | 0    | 0     | 0     | 0     | 0     |
| 1          | +0.1                         | 0     | 0     | +0.2 | +0.1  | +0.3  | 0     | -0.2  |
| 2          | 0.6                          | +0.5  | +0.4  | 0.7  | 1.0   | 0.9   | -0.2  | +1.5  |
| 3          | 8.9                          | 10.4  | 8.7   | 6.0  | 10.5  | 18.5  | +10.6 | 27.2  |
| 4          | 15.7                         | 16.8  | 14.7  | 12.3 | 16.6  | 28.4  | 16.1  | 32.4  |
| 5          | 20.1                         | 21.3  | 17.4  | 18.6 | 22.6  | 37.6  | 21.0  | 40.4  |
| 6          | 23.6                         | 24.5  | 20.8  | 22.8 | 29.7  | 45.9  | 25.6  | 46.1  |
| 7          | 27.3                         | 27.7  | 24.2  | 26.6 | 37.9  | Yield | 30.0  | 51.9  |
| 8          | 32.2                         | 32.5  | 30.0  | 30.2 | 46.9  | Yield | 34.9  | Yield |
| 9*         | 40.6                         | 42.0  | 36.7  | 37.8 | Yield | Yield | --    | --    |
| 10**       | Yield                        | Yield | Yield | 34.4 | Yield | --    | Yield | Yield |

\*Failure

\*\*Past peak loads



TABLE B-8

## TEST RESULTS FOR BEAM 2-2

| Load Stage | Torque in.kips | Bending Moment in.kips | ∅    | Twist                             |                          | Deflection <sup>2</sup> |        |      |
|------------|----------------|------------------------|------|-----------------------------------|--------------------------|-------------------------|--------|------|
|            |                |                        |      | Radians per in. x 10 <sup>6</sup> | inches x 10 <sup>2</sup> | West                    | Center | East |
| 0          | 0              | 0                      | --   | 0                                 | 0                        | 0                       | 0      | 0    |
| 1          | 48             | 24                     | 2.00 | 208                               | 1.0                      | 1.5                     | 3.5    |      |
| 2          | 59             | 30                     | 1.97 | 320                               | 2.0                      | 1.0                     | 1.5    |      |
| 3          | 70             | 35                     | 2.00 | 408                               | 2.5                      | 2.5                     | 2.0    |      |
| 4          | 81             | 40                     | 2.02 | 520                               | 2.5                      | 2.0                     | 0.5    |      |
| 5          | 99             | 50                     | 1.98 | 733                               | 3.0                      | 3.5                     | 3.0    |      |
| 6          | 119            | 59                     | 2.02 | 988                               | 3.5                      | 4.0                     | 4.0    |      |
| 7          | 138            | 69                     | 2.00 | 1233                              | 4.0                      | 4.0                     | 3.5    |      |
| 8          | 157            | 78                     | 2.01 | 1593                              | 4.0                      | 4.5                     | 4.0    |      |
| 9*         | 172            | 88                     | 1.96 | --                                | -                        | -                       | -      |      |
| 10**       | 154            | 88                     | 1.58 | 2190                              | 7.0                      | 4.5                     | 7.0    |      |

| Load Stage | Reinforcement Stresses (ksi) |      |       |      |       |       |      |        |
|------------|------------------------------|------|-------|------|-------|-------|------|--------|
|            | 1                            | 2    | 3     | 4    | 5     | 6     | 7    | 8      |
| 0          | 0                            | 0    | 0     | 0    | 0     | 0     | 0    | 0      |
| 1          | 4.9                          | 5.6  | 6.8   | 10.8 | 11.0  | 22.3  | 11.1 | 17.2   |
| 2          | 9.9                          | 11.7 | 17.7  | 13.9 | 14.1  | 26.9  | 14.6 | 23.0   |
| 3          | 12.3                         | 13.5 | 21.6  | 16.0 | 16.4  | 31.2  | 17.3 | 25.5   |
| 4          | 14.7                         | 15.0 | 25.8  | 18.0 | 19.6  | 35.7  | 20.0 | 30.7   |
| 5          | 17.1                         | 16.4 | 31.4  | 23.6 | 28.8  | 41.3  | 28.0 | 43.8   |
| 6          | 21.3                         | 18.9 | 37.5  | 30.0 | 35.4  | 45.2  | 34.4 | Yield  |
| 7          | 24.8                         | 21.9 | 43.8  | 34.6 | 42.8  | 50.1  | 39.9 | Yield  |
| 8          | 27.8                         | 26.0 | Yield | 39.9 | 53.0  | Yield | 43.2 | (51.7) |
| 9*         | --                           | --   | --    | --   | --    | --    | --   | --     |
| 10**       | 33.6                         | 29.4 | Yield | 42.9 | Yield | Yield | 48.3 | Yield  |

\*Failure

\*\*Past peak loads



TABLE B-9  
TEST RESULTS FOR BEAM 2-3

| Load Stage | Torque in.kips | Bending Moment in.kips | ∅    | Twist                 |             | Deflection inches x 10 <sup>2</sup> |        |      |
|------------|----------------|------------------------|------|-----------------------|-------------|-------------------------------------|--------|------|
|            |                |                        |      | in. x 10 <sup>6</sup> | Radians per | West                                | Center | East |
| 0          | 0              | 0                      | --   | 0                     | 0           | 0                                   | 0      | 0    |
| 1          | 25             | 24                     | 1.04 | 33                    | 0.5         | 0                                   | 0.5    |      |
| 2          | 38             | 38                     | 1.00 | 90                    | 1.5         | 2.0                                 | 2.0    |      |
| 3          | 51             | 51                     | 1.00 | 200                   | 4.0         | 4.5                                 | 4.0    |      |
| 4          | 64             | 65                     | 0.98 | 333                   | 5.0         | 6.0                                 | 5.5    |      |
| 5          | 78             | 78                     | 1.00 | 478                   | 6.5         | 7.0                                 | 6.5    |      |
| 6          | 97             | 98                     | 0.99 | 753                   | 8.0         | 8.5                                 | 7.5    |      |
| 7          | 118            | 119                    | 0.99 | 1055                  | 10.5        | 11.5                                | 9.5    |      |
| 8          | 132            | 132                    | 1.00 | 1240                  | 11.0        | 12.0                                | 10.5   |      |
| 9          | 145            | 146                    | 0.99 | 1485                  | 13.0        | 14.0                                | 11.5   |      |
| 10         | 159            | 159                    | 1.00 | 1850                  | 16.5        | 17.5                                | 14.5   |      |
| 11*        | 166            | 166                    | 1.00 | --                    | --          | --                                  | --     |      |
| 12**       | 149            | 166                    | 0.90 | 2703                  | 29.5        | 27.0                                | 27.5   |      |

| Load Stage | Reinforcement Stresses (ksi) |      |       |       |       |        |      |       |
|------------|------------------------------|------|-------|-------|-------|--------|------|-------|
|            | Gauge                        |      |       |       |       |        |      |       |
| 1          | 2                            | 3    | 4     | 5     | 6     | 7      | 8    |       |
| 0          | 0                            | 0    | 0     | 0     | 0     | 0      | 0    | 0     |
| 1          | -0.5                         | -0.1 | +0.6  | +0.7  | +0.5  | +0.2   | -0.3 | 0     |
| 2          | -0.8                         | +1.6 | 2.9   | 4.6   | 6.3   | -0.9   | +1.2 | 17.2  |
| 3          | +3.9                         | 3.3  | 12.7  | 10.4  | 7.8   | +1.1   | 3.0  | 24.9  |
| 4          | 6.0                          | 11.5 | 18.0  | 18.7  | 9.9   | 4.6    | 5.5  | 30.3  |
| 5          | 7.6                          | 15.0 | 22.0  | 23.5  | 13.3  | 24.0   | 8.5  | 32.7  |
| 6          | 9.3                          | 18.4 | 27.9  | 30.3  | 20.4  | 37.2   | 13.5 | 40.8  |
| 7          | 14.7                         | 21.2 | 35.1  | 36.0  | 28.7  | 46.1   | 19.9 | 49.5  |
| 8          | 16.2                         | 22.6 | 39.9  | 40.1  | 33.1  | 51.7   | 23.5 | Yield |
| 9          | 17.4                         | 24.1 | 45.0  | 44.3  | 37.4  | Yield  | 27.3 | Yield |
| 10         | 20.0                         | 27.0 | Yield | Yield | 46.2  | Yield  | 32.4 | Yield |
| 11*        | --                           | --   | --    | --    | --    | --     | --   | --    |
| 12**       | 13.8                         | 27.3 | Yield | Yield | Yield | (40.0) | 47.2 | Yield |

\*Failure

\*\*Past peak loads



TABLE B-10

## TEST RESULTS FOR BEAM 2-4

| Load Stage | Torque in.kips | Bending Moment in.kips | ∅    | Twist<br>in. x 10 <sup>6</sup> | Deflection<br>inches x 10 <sup>2</sup> |        |      |
|------------|----------------|------------------------|------|--------------------------------|--|--------|------|
|            |                |                        |      |                                | West                                   | Center | East |
| 0          | 0              | 0                      | --   | 0                              | 0                                      | 0      | 0    |
| 1          | 12             | 24                     | 0.50 | 15                             | 0                                      | 0.5    | 0.5  |
| 2          | 25             | 51                     | 0.49 | 30                             | 1.0                                    | 2.5    | 1.5  |
| 3          | 38             | 78                     | 0.49 | 60                             | 4.0                                    | 5.0    | 4.0  |
| 4          | 52             | 105                    | 0.50 | 148                            | 8.0                                    | 9.0    | 7.5  |
| 5          | 66             | 132                    | 0.50 | 250                            | 11.0                                   | 12.5   | 10.5 |
| 6          | 86             | 173                    | 0.50 | 428                            | 15.0                                   | 17.5   | 14.5 |
| 7          | 106            | 213                    | 0.50 | 623                            | 18.5                                   | 22.0   | 19.0 |
| 8          | 120            | 240                    | 0.50 | 780                            | 21.0                                   | 24.5   | 21.0 |
| 9          | 126            | 254                    | 0.50 | 875                            | 23.0                                   | 26.0   | 22.0 |
| 10*        | 1.34           | 267                    | 0.50 | --                             | --                                     | --     | --   |
| 11**       | 112            | 267                    | 0.42 | 2030                           | 55.0                                   | 59.5   | 48.5 |

| Load Stage | Reinforcement Stresses (ksi) |       |       |       |       |      |      |      |
|------------|------------------------------|-------|-------|-------|-------|------|------|------|
|            | Gauge                        |       |       |       |       |      |      |      |
| 1          | 2                            | 3     | 4     | 5     | 6     | 7    | 8    |      |
| 0          | 0                            | 0     | 0     | 0     | 0     | 0    | 0    | 0    |
| 1          | -0.4                         | -0.2  | +0.6  | +0.6  | +0.2  | 0    | 0    | +0.3 |
| 2          | -2.1                         | -1.5  | 2.6   | 2.7   | 0.6   | 0    | +0.1 | 0.6  |
| 3          | -4.5                         | -3.1  | 7.1   | 7.9   | 2.0   | 0    | -0.3 | 6.9  |
| 4          | -4.4                         | -1.6  | 14.8  | 22.9  | 4.3   | +8.2 | -1.0 | 12.3 |
| 5          | -3.9                         | -1.2  | 22.2  | 32.1  | 6.8   | 22.5 | -0.1 | 16.5 |
| 6          | -3.4                         | -1.2  | 35.5  | 44.2  | 11.3  | 29.6 | +4.9 | 27.3 |
| 7          | -5.0                         | -3.0  | 45.9  | Yield | 16.5  | 37.9 | 14.7 | 36.4 |
| 8          | -5.4                         | -4.7  | Yield | Yield | 22.0  | 42.8 | 19.8 | 41.1 |
| 9          | -5.7                         | -5.5  | Yield | Yield | 25.5  | 45.8 | 22.1 | 43.1 |
| 10*        | --                           | --    | --    | --    | --    | --   | --   | --   |
| 11**       | -10.9                        | -10.7 | Yield | Yield | Yield | 34.0 | 36.9 | 49.4 |

\*Failure

\*\*Past peak loads



TABLE B-11  
TEST RESULTS FOR BEAM 2-5

| Load Stage | Torque in.kips | Bending Moment in.kips | ∅    | Twist Radians per in. x 10 <sup>6</sup> | Deflection inches x 10 <sup>2</sup> |        |       |
|------------|----------------|------------------------|------|---|-------------------------------------|--------|-------|
|            |                |                        |      |   | West                                | Center | East  |
| 0          | 0              | 0                      | --   | 0                                       | 0                                   | 0      | 0     |
| 1          | 9              | 38                     | 0.24 | 8                                       | 1.5                                 | 1.5    | 1.5   |
| 2          | 20             | 78                     | 0.26 | 33                                      | 5.0                                 | 6.0    | 5.0   |
| 3          | 30             | 119                    | 0.25 | 58                                      | 8.5                                 | 10.5   | 8.5   |
| 4          | 40             | 159                    | 0.25 | 105                                     | 13.0                                | 14.5   | 12.5  |
| 5          | 51             | 200                    | 0.25 | 158                                     | 17.0                                | 19.5   | 17.0  |
| 6          | 61             | 240                    | 0.25 | 208                                     | 21.0                                | 23.5   | 20.5  |
| 7          | 70             | 281                    | 0.25 | 275                                     | 24.5                                | 27.5   | 24.0  |
| 8          | 80             | 321                    | 0.25 | 335                                     | 28.0                                | 32.0   | 28.0  |
| 9*         | 90             | 362                    | 0.25 | --                                      | --                                  | --     | --    |
| 10**       | 79             | 362                    | 0.22 | 1923                                    | 99.0                                | 123.0  | 104.5 |

| Load Stage | Reinforcement Stresses (ksi) |       |       |       |       |      |       |       |
|------------|------------------------------|-------|-------|-------|-------|------|-------|-------|
|            | Gauge                        |       |       |       |       |      |       |       |
| 1          | 2                            | 3     | 4     | 5     | 6     | 7    | 8     |       |
| 0          | 0                            | 0     | 0     | 0     | 0     | 0    | 0     | 0     |
| 1          | - 1.4                        | - 1.0 | + 1.6 | + 1.5 | +0.4  | +0.1 | +0.1  | +0.3  |
| 2          | - 5.0                        | - 2.8 | +10.6 | +13.7 | +3.1  | 0    | -0.3  | +1.7  |
| 3          | - 8.1                        | - 5.1 | 19.9  | 20.1  | 4.8   | +1.6 | -1.1  | 3.6   |
| 4          | -10.5                        | - 7.6 | 27.8  | 26.2  | 6.4   | +2.6 | -1.8  | 6.4   |
| 5          | -13.2                        | -10.1 | 34.1  | 33.1  | 6.9   | 3.3  | -2.2  | 10.5  |
| 6          | -15.9                        | -11.9 | 40.0  | 39.6  | 7.6   | 4.0  | -2.3  | 14.2  |
| 7          | -18.6                        | -13.6 | 46.5  | 46.1  | 7.8   | 5.5  | -1.8  | 17.5  |
| 8          | -21.0                        | -15.3 | Yield | Yield | 8.3   | 7.0  | -1.0  | 21.0  |
| 9*         | --                           | --    | --    | --    | --    | --   | --    | --    |
| 10**       | -29.4                        | -42.8 | Yield | --    | Yield | 46.8 | +20.0 | Yield |

\*Failure

\*\*Past peak loads



TABLE B-12

## TEST RESULTS FOR BEAM 3-1

| Load Stage | Torque in.kips | Bending Moment in.kips | $\phi$ | Twist Radians per in. $\times 10^6$ |
|------------|----------------|------------------------|--------|-------------------------------------|
| 0          | 0              | --                     | --     | 0                                   |
| 1          | 22             | --                     | --     | 25                                  |
| 2          | 45             | --                     | --     | 65                                  |
| 3          | 50             | --                     | --     | 75                                  |
| 4          | 55             | --                     | --     | 85                                  |
| 5          | 59             | --                     | --     | 172                                 |
| 6          | 64             | --                     | --     | 258                                 |
| 7          | 73             | --                     | --     | 593                                 |
| 8          | 82             | --                     | --     | 733                                 |
| 9          | 92             | --                     | --     | 922                                 |
| 10         | 102            | --                     | --     | 1140                                |
| 11         | 106            | --                     | --     | 1270                                |
| 12         | 111            | --                     | --     | 1440                                |
| 13*        | 115            | --                     | --     | --                                  |

| Load Stage | Reinforcement Stresses (ksi) |       |       |      |      |      |      |       |
|------------|------------------------------|-------|-------|------|------|------|------|-------|
|            | Gauge                        |       |       |      |      |      |      |       |
| 1          | 2                            | 3     | 4     | 5    | 6    | 7    | 8    |       |
| 0          | 0                            | 0     | 0     | 0    | 0    | 0    | 0    | 0     |
| 1          | -0.1                         | -0.1  | +0.4  | +0.1 | +0.2 | +0.3 | +0.2 | +0.1  |
| 2          | +0.2                         | +0.5  | +1.1  | +0.8 | +0.5 | +1.2 | +0.5 | +1.2  |
| 3          | +0.6                         | +0.9  | 1.7   | 1.2  | 0.9  | 2.1  | 0.6  | 1.8   |
| 4          | 0.8                          | 1.0   | 2.1   | 1.5  | 1.0  | 2.9  | 0.7  | 2.3   |
| 5          | 9.2                          | 11.8  | 8.4   | 14.9 | 6.0  | 18.7 | 8.4  | 25.8  |
| 6          | 16.1                         | 14.7  | 11.6  | 18.9 | 7.7  | 27.2 | 10.2 | 31.4  |
| 7          | 30.0                         | 33.7  | 25.8  | 25.8 | 11.7 | 34.4 | 17.0 | 41.8  |
| 8          | 38.9                         | 42.2  | 30.8  | 31.1 | 14.0 | 39.1 | 20.0 | 47.8  |
| 9          | 49.7                         | 51.9  | 37.3  | 36.6 | 18.2 | 44.1 | 22.9 | 53.0  |
| 10         | Yield                        | Yield | 45.1  | 41.5 | 22.1 | 47.7 | 24.8 | Yield |
| 11         | Yield                        | Yield | 48.8  | 44.2 | 24.0 | 49.3 | 26.1 | Yield |
| 12         | Yield                        | Yield | Yield | 47.2 | 25.9 | 51.2 | 28.0 | Yield |
| 13*        | --                           | --    | --    | --   | --   | --   | --   | --    |

\*Failure



TABLE B-13  
TEST RESULTS FOR BEAM 3-2

| Load Stage | Torque in.kips | Bending Moment in.kips | $\emptyset$ | Twist Radians per in. $\times 10^6$ | Deflection inches $\times 10^2$ |        |      |
|------------|----------------|------------------------|-------------|-------------------------------------|---------------------------------|--------|------|
|            |                |                        |             |                                     | West                            | Center | East |
| 0          | 0              | 0                      | --          | 0                                   | 0                               | 0      | 0    |
| 1          | 0              | 24                     | 0           | 0                                   | +0.5                            | +0.5   | 0    |
| 2          | 24             | 24                     | 1.00        | 25                                  | +1.0                            | +0.5   | +0.5 |
| 3          | 45             | 24                     | 1.87        | 75                                  | +1.0                            | +0.5   | +1.0 |
| 4          | 54             | 24                     | 2.25        | 330                                 | +1.0                            | 0      | +0.5 |
| 5          | 68             | 24                     | 2.83        | 613                                 | -2.0                            | -2.5   | -2.5 |
| 6          | 85             | 24                     | 3.54        | 1020                                | -5.5                            | -8.0   | -7.0 |
| 7          | 85             | 51                     | 1.67        | 1035                                | -3.5                            | -4.5   | -5.0 |
| 8          | 85             | 78                     | 1.09        | 1048                                | 0                               | -1.5   | -2.0 |
| 9          | 85             | 111                    | 0.77        | 1050                                | +4.0                            | +3.5   | +2.0 |
| 10         | 103            | 111                    | 0.93        | 1238                                | +5.0                            | +4.5   | +3.0 |
| 11         | 112            | 111                    | 1.01        | 1733                                | +6.0                            | +4.5   | +3.0 |
| 12*        | 117            | 111                    | 1.05        | --                                  | --                              | --     | --   |

| Load Stage | Reinforcement Stresses (ksi) |        |       |       |      |      |       |
|------------|------------------------------|--------|-------|-------|------|------|-------|
|            | Gauge                        |        |       |       |      |      |       |
|            | 1                            | 2      | 3     | 4     | 5    | 6    | 7     |
| 0          | 0                            | 0      | 0     | 0     | 0    | 0    | 0     |
| 1          | - 0.5                        | - 0.5  | +0.4  | +0.5  | 0    | 0    | 0     |
| 2          | - 0.3                        | - 0.4  | +0.7  | +1.1  | +0.3 | +0.6 | +0.1  |
| 3          | + 0.1                        | + 0.2  | 1.7   | 4.5   | +0.9 | +1.7 | +0.2  |
| 4          | +17.7                        | +26.8  | 17.2  | 29.3  | 7.3  | -0.5 | +15.8 |
| 5          | 29.0                         | 49.5   | 28.6  | 37.7  | 13.5 | +0.6 | 21.0  |
| 6          | 50.4                         | Yield  | 38.7  | Yield | 22.8 | +4.9 | 29.0  |
| 7          | 47.4                         | (48.4) | 42.0  | Yield | 23.3 | 5.1  | 28.9  |
| 8          | 40.8                         | (42.2) | 45.2  | Yield | 23.2 | 4.8  | 28.8  |
| 9          | 33.4                         | (34.4) | Yield | Yield | 22.9 | 4.3  | 28.5  |
| 10         | 39.2                         | (44.2) | Yield | Yield | 29.7 | 6.9  | 35.5  |
| 11         | 48.6                         | Yield  | Yield | Yield | 39.9 | 10.6 | 46.7  |
| 12*        | --                           | --     | --    | --    | --   | --   | --    |

\*Failure



TABLE B-13 (continued)

## TEST RESULTS FOR BEAM 3-2

| Load<br>Stage | Reinforcement Stresses (ksi) |      |      |      |      |      |
|---------------|------------------------------|------|------|------|------|------|
|               | Gauge                        |      |      |      |      |      |
|               | 8                            | 9    | 10   | 11   | 12   | 13   |
| 0             | 0                            | 0    | 0    | 0    | 0    | 0    |
| 1             | 0                            | 0    | 0    | 0    | 0    | 0    |
| 2             | +0.8                         | 0    | +0.3 | -0.2 | +0.3 | -0.3 |
| 3             | +8.0                         | +0.3 | +1.1 | -0.8 | +0.7 | -0.3 |
| 4             | 45.4                         | +4.8 | 3.6  | +1.7 | 1.8  | +1.3 |
| 5             | Yield                        | 10.3 | 6.7  | +1.2 | 2.9  | +6.6 |
| 6             | Yield                        | 16.4 | 11.0 | 2.4  | 2.5  | 10.8 |
| 7             | Yield                        | 17.0 | 10.7 | 1.8  | 1.5  | 10.1 |
| 8             | Yield                        | 17.3 | 10.5 | 1.5  | 1.2  | 10.0 |
| 9             | Yield                        | 17.4 | 10.4 | 1.7  | 1.2  | 10.1 |
| 10            | Yield                        | 23.4 | 13.7 | 2.1  | 2.0  | 13.4 |
| 11            | Yield                        | 31.7 | 19.7 | 8.2  | 3.3  | 18.8 |
| 12*           | --                           | --   | --   | --   | --   | --   |

\*Failure



TABLE B-14

## TEST RESULTS FOR BEAM 3-3

| Load Stage | Torque in. kips | Bending Moment in. kips | ∅    | Twist Radians per in. x 10 <sup>6</sup> |        | Deflection inches x 10 <sup>2</sup> |      |   |
|------------|-----------------|-------------------------|------|---|--------|-------------------------------------|------|---|
|            |                 |                         |      | West                                    | Center | East                                |      |   |
| 0          | 0               | 0                       | --   | 0                                       | 0      | 0                                   | 0    | 0 |
| 1          | 0               | 51                      | 0    | 0                                       | +0.5   | +2.0                                | +1.5 |   |
| 2          | 0               | 72                      | 0    | 0                                       | +2.5   | 3.0                                 | 2.5  |   |
| 3          | 0               | 92                      | 0    | 0                                       | 5.0    | 6.0                                 | 5.5  |   |
| 4          | 0               | 111                     | 0    | 0                                       | 7.0    | 9.0                                 | 8.0  |   |
| 5          | 22              | 111                     | 0.20 | 28                                      | 8.5    | 9.5                                 | 8.5  |   |
| 6          | 43              | 111                     | 0.39 | 73                                      | 9.5    | 10.5                                | 10.0 |   |
| 7          | 52              | 111                     | 0.47 | 110                                     | 9.0    | 12.0                                | 10.0 |   |
| 8          | 56              | 111                     | 0.50 | 140                                     | 9.5    | 11.5                                | 10.5 |   |
| 9          | 61              | 111                     | 0.55 | 170                                     | 10.0   | 12.5                                | 10.5 |   |
| 10         | 70              | 111                     | 0.63 | 273                                     | 11.0   | 12.5                                | 11.5 |   |
| 11         | 79              | 111                     | 0.71 | 398                                     | 11.5   | 13.0                                | 11.5 |   |
| 12         | 88              | 111                     | 0.79 | 580                                     | 11.0   | 13.0                                | 10.5 |   |
| 13         | 97              | 111                     | 0.87 | 780                                     | 10.5   | 11.5                                | 9.5  |   |
| 14         | 106             | 111                     | 0.95 | 1023                                    | 9.5    | 10.0                                | 8.0  |   |
| 15         | 115             | 111                     | 1.04 | 1430                                    | --     | --                                  | --   |   |
| 16*        | 120             | 111                     | 1.08 | --                                      | --     | --                                  | --   |   |

| Load Stage | Reinforcement Stresses (ksi) |       |       |       |      |       |      |       |
|------------|------------------------------|-------|-------|-------|------|-------|------|-------|
|            | Gauge                        |       |       |       |      |       |      |       |
| 1          | 2                            | 3     | 4     | 5     | 6    | 7     | 8    |       |
| 0          | 0                            | 0     | 0     | 0     | 0    | 0     | 0    | 0     |
| 1          | - 2.0                        | - 1.7 | +2.0  | +1.9  | +0.6 | +0.1  | +0.4 | +0.2  |
| 2          | - 3.9                        | - 3.3 | +3.9  | +4.1  | +1.6 | +0.3  | +0.8 | +0.6  |
| 3          | - 6.3                        | - 5.6 | 10.8  | 10.5  | 5.0  | 1.3   | 1.2  | 2.3   |
| 4          | - 8.4                        | - 7.3 | 16.4  | 15.6  | 5.7  | 1.2   | 1.6  | 3.0   |
| 5          | - 9.5                        | - 7.5 | 19.1  | 20.2  | 5.7  | +0.5  | 1.9  | 2.3   |
| 6          | -10.4                        | - 6.7 | 22.4  | 25.2  | 6.4  | 0     | 2.0  | 3.6   |
| 7          | -10.2                        | - 6.3 | 24.5  | 28.2  | 7.5  | +1.8  | 1.8  | 6.7   |
| 8          | - 8.7                        | - 6.5 | 25.5  | 31.2  | 8.4  | 11.0  | 2.3  | 8.1   |
| 9          | - 7.9                        | - 6.1 | 26.3  | 33.0  | 9.5  | 14.1  | 2.8  | 9.7   |
| 10         | - 1.2                        | + 1.7 | 30.2  | 38.9  | 12.6 | 24.9  | 9.3  | 15.9  |
| 11         | + 8.4                        | +11.4 | 34.5  | 43.7  | 16.1 | 35.9  | 16.0 | 25.1  |
| 12         | +15.6                        | 15.6  | 39.3  | 48.0  | 19.4 | 43.9  | 18.9 | 34.2  |
| 13         | 23.1                         | 21.0  | Yield | Yield | 25.0 | 49.8  | 22.0 | 42.6  |
| 14         | 30.9                         | 27.9  | Yield | Yield | 29.5 | Yield | 28.4 | 52.2  |
| 15         | 44.1                         | 38.1  | Yield | Yield | 43.2 | Yield | 34.7 | Yield |
| 16*        | --                           | --    | --    | --    | --   | --    | --   | --    |

\*Failure



TABLE B-15

## TEST RESULTS FOR BEAM 3-4

| Load Stage | Torque in.kips | Bending Moment in.kips | $\phi$ | Twist Radians per in. $\times 10^6$ |
|------------|----------------|------------------------|--------|-------------------------------------|
| 0          | 0              | 0                      | --     | 0                                   |
| 1          | 0              | 24                     | 0      | 0                                   |
| 2          | 22             | 24                     | 0.92   | 20                                  |
| 3          | 45             | 24                     | 1.87   | 53                                  |
| 4          | 49             | 24                     | 2.04   | 63                                  |
| 5          | 54             | 24                     | 2.25   | 75                                  |
| 6          | 59             | 24                     | 2.46   | 95                                  |
| 7          | 63             | 24                     | 2.63   | 150                                 |
| 8          | 68             | 24                     | 2.83   | 215                                 |
| 9          | 77             | 24                     | 3.21   | 415                                 |
| 10         | 86             | 24                     | 3.58   | 698                                 |
| 11         | 86             | 40                     | 2.15   | 723                                 |
| 12         | 86             | 55                     | 1.57   | 728                                 |
| 13         | 96             | 55                     | 1.75   | 790                                 |
| 14         | 106            | 55                     | 1.93   | 938                                 |
| 15         | 110            | 55                     | 2.00   | 1005                                |
| 16         | 115            | 55                     | 2.09   | 1110                                |
| 17         | 115            | 65                     | 1.77   | 1133                                |
| 18         | 115            | 78                     | 1.48   | 1140                                |
| 19         | 115            | 92                     | 1.25   | 1150                                |
| 20         | 115            | 105                    | 1.10   | 1155                                |
| 21         | 115            | 119                    | 0.97   | 1168                                |
| 22         | 115            | 132                    | 0.87   | 1170                                |
| 23         | 115            | 153                    | 0.75   | --                                  |
| 24         | 115            | 173                    | 0.67   | 1253                                |
| 25* -      | 115            | 188                    | 0.61   | --                                  |

\*Failure



TABLE B-15 (continued)

## TEST RESULTS FOR BEAM 3-4

| Load<br>Stage | Reinforcement Stresses (ksi) |      |       |       |      |        |      |       |
|---------------|------------------------------|------|-------|-------|------|--------|------|-------|
|               | 1                            | 2    | 3     | 4     | 5    | 6      | 7    | 8     |
| 0             | 0                            | 0    | 0     | 0     | 0    | 0      | 0    | 0     |
| 1             | -0.8                         | -0.8 | +0.8  | +1.0  | +0.2 | +0.1   | +0.2 | +0.2  |
| 2             | -0.8                         | -0.8 | +1.1  | +1.0  | +0.3 | +0.3   | 0    | +0.2  |
| 3             | -0.5                         | -0.5 | 1.8   | 1.7   | 0.6  | 1.1    | -0.1 | 1.5   |
| 4             | -0.5                         | -0.4 | 2.1   | 2.0   | 0.8  | 1.4    | 0    | 2.3   |
| 5             | -0.4                         | -0.1 | 2.3   | 2.6   | 0.8  | 1.7    | -0.1 | 3.4   |
| 6             | +0.7                         | +1.7 | 5.0   | 18.3  | 1.8  | 11.1   | -1.1 | 16.7  |
| 7             | +10.0                        | +3.4 | 8.0   | 24.4  | 4.0  | 26.6   | +0.1 | 21.1  |
| 8             | 17.5                         | 11.6 | 11.6  | 25.9  | 6.6  | 36.3   | +1.1 | 22.4  |
| 9             | 29.3                         | 22.6 | 21.7  | 32.1  | 11.9 | 46.4   | 10.4 | 30.9  |
| 10            | 41.4                         | 34.9 | 35.6  | 37.4  | 17.0 | Yield  | 19.8 | 40.3  |
| 11            | 41.0                         | 34.6 | 38.0  | 39.2  | 17.2 | (54.0) | 20.1 | 40.2  |
| 12            | 38.3                         | 32.5 | 40.2  | 41.6  | 17.1 | (53.6) | 20.0 | 40.1  |
| 13            | 41.3                         | 34.9 | 43.1  | 44.9  | 18.8 | Yield  | 21.9 | 43.4  |
| 14            | 48.2                         | 40.6 | 48.1  | 48.8  | 21.6 | Yield  | 25.2 | 48.2  |
| 15            | 51.7                         | 43.6 | Yield | Yield | 22.9 | Yield  | 26.7 | 49.4  |
| 16            | Yield                        | 47.1 | Yield | Yield | 24.6 | Yield  | 28.5 | 51.8  |
| 17            | Yield                        | 46.9 | Yield | Yield | 24.6 | Yield  | 28.9 | 51.5  |
| 18            | (50.9)                       | 44.7 | Yield | Yield | 24.5 | (54.4) | 28.9 | 51.1  |
| 19            | (48.8)                       | 42.4 | Yield | Yield | 24.4 | (54.2) | 29.0 | 51.1  |
| 20            | (46.1)                       | 39.5 | Yield | Yield | 24.2 | (54.1) | 29.2 | 51.0  |
| 21            | (43.4)                       | 36.7 | Yield | Yield | 24.1 | (53.8) | 29.1 | 50.9  |
| 22            | (40.7)                       | 33.9 | Yield | Yield | 23.9 | (53.5) | 29.3 | 51.2  |
| 23            | (36.9)                       | 29.1 | Yield | Yield | 24.5 | (53.3) | 29.6 | 52.7  |
| 24            | (33.3)                       | 24.4 | Yield | Yield | 25.1 | (53.2) | 31.1 | Yield |
| 25*           | --                           | --   | --    | --    | --   | --     | --   | --    |

\*Failure



TABLE B-16

## TEST RESULTS FOR BEAM 3-5

| Load Stage | Torque in. kips | Bending Moment in. kips | ∅    | Twist                             |                       | Deflection |        |       |
|------------|-----------------|-------------------------|------|-----------------------------------|-----------------------|------------|--------|-------|
|            |                 |                         |      | Radians per in. x 10 <sup>6</sup> | in. x 10 <sup>6</sup> | West       | Center | East  |
| 0          | 0               | 0                       | --   | 0                                 | 0                     | 0          | 0      | 0     |
| 1          | 0               | 24                      | 0    | 0                                 | 0                     | +0.5       | 0      | +0.5  |
| 2          | 21              | 24                      | 0.88 | 25                                | 0                     | +0.5       | 0      | +0.5  |
| 3          | 43              | 24                      | 1.79 | 60                                | 0                     | +0.5       | 0      | +0.5  |
| 4          | 55              | 24                      | 2.29 | 228                               | 0                     | +1.0       | +1.0   | +0.5  |
| 5          | 73              | 24                      | 3.04 | 680                               | 0                     | -2.5       | -4.5   | -3.0  |
| 6          | 73              | 92                      | 0.79 | 708                               | 0                     | +4.0       | +2.5   | +3.5  |
| 7          | 73              | 159                     | 0.46 | 718                               | 0                     | +9.5       | +11.0  | +10.5 |
| 8          | 73              | 227                     | 0.32 | 738                               | 0                     | +19.5      | +21.0  | +29.0 |
| 9*         | 73              | 265                     | 0.28 | -                                 | 1                     | --         | --     | --    |

| Load Stage | Reinforcement Stresses |       |       |       |      |      |       |      |       |       |
|------------|------------------------|-------|-------|-------|------|------|-------|------|-------|-------|
|            | Gauge                  |       |       |       |      |      |       |      |       |       |
| 1          | 2                      | 3     | 4     | 5     | 6    | 7    | 8     | 9    | 10    |       |
| 0          | 0                      | 0     | 0     | 0     | 0    | 0    | 0     | 0    | 0     | 0     |
| 1          | -0.8                   | -0.3  | +0.4  | +0.5  | 0    | +0.2 | +0.2  | +0.2 | -0.3  | -0.3  |
| 2          | -0.8                   | -0.4  | +0.5  | +0.6  | 0    | 0.2  | 0     | +0.3 | -0.2  | -0.3  |
| 3          | -1.1                   | -0.5  | 1.0   | 1.1   | +0.1 | 1.8  | -0.3  | 1.4  | -0.1  | -0.4  |
| 4          | --                     | +4.5  | 10.0  | 22.3  | 9.4  | 21.3 | +2.5  | 34.7 | +0.5  | +0.3  |
| 5          | --                     | 40.1  | 26.1  | 33.6  | 19.1 | 51.0 | 13.8  | 54.2 | +23.6 | 43.5  |
| 6          | --                     | 29.3  | 36.6  | 43.7  | 20.0 | 51.7 | 13.8  | 53.8 | 17.5  | 31.7  |
| 7          | --                     | 15.0  | 47.7  | Yield | 20.2 | 51.5 | 13.3  | 53.4 | 9.0   | 18.0  |
| 8          | --                     | 2.3   | Yield | Yield | 20.1 | 51.6 | 12.4  | 54.1 | 0.8   | 5.9   |
| 9*         | --                     | --    | --    | --    | --   | --   | --    | --   | --    | --    |
|            | Gauge                  |       |       |       |      |      |       |      |       |       |
|            | 11                     | 12    | 13    | 14    | 15   | 16   | 17    | 18   | 19    | 20    |
| 0          | 0                      | 0     | 0     | 0     | 0    | 0    | 0     | 0    | 0     | 0     |
| 1          | +0.6                   | +0.6  | +0.1  | +0.2  | +0.3 | +0.2 | -0.2  | +0.6 | +0.1  | +0.2  |
| 2          | +0.6                   | 0.8   | 0.3   | 0.3   | 0.3  | 0.1  | -0.3  | 0.6  | 0     | 0.3   |
| 3          | 0.9                    | 1.5   | 0.8   | 0.9   | 0.4  | 0.7  | -0.5  | 5.7  | -0.3  | 8.1   |
| 4          | 1.9                    | 3.7   | 2.3   | 2.2   | 0.8  | 4.3  | +2.5  | --   | +4.2  | 46.9  |
| 5          | 27.6                   | 31.3  | 19.6  | 49.0  | 18.1 | 54.7 | 38.9  | --   | 13.8  | Yield |
| 6          | 38.4                   | 41.7  | 20.0  | 48.3  | 18.0 | 53.0 | 25.7  | --   | 16.8  | Yield |
| 7          | Yield                  | Yield | 20.0  | 46.8  | 17.1 | 51.7 | +10.5 | --   | 18.6  | Yield |
| 8          | Yield                  | Yield | 19.6  | 44.7  | 16.4 | 51.5 | -3.3  | --   | 19.9  | Yield |
| 9*         | --                     | --    | --    | --    | --   | --   | --    | --   | --    | --    |

\*Failure



TABLE B-17  
TEST RESULTS FOR BEAM 4-1

| Load Stage | Torque in.kips | Bending Moment in.kips | ∅    | Twist Radians per in. $\times 10^6$ | Deflection inches $\times 10^2$ |        |       |
|------------|----------------|------------------------|------|-------------------------------------|---------------------------------|--------|-------|
|            |                |                        |      |                                     | West                            | Center | East  |
| 0          | 0              | 0                      | 0    | 0                                   | 0                               | 0      | 0     |
| 1          | 14             | 24                     | 0.58 | 15                                  | -0.5                            | +0.5   | +0.5  |
| 2          | 31             | 51                     | 0.61 | 40                                  | +1.5                            | 2.0    | 2.0   |
| 3          | 39             | 65                     | 0.60 | 58                                  | 3.0                             | 3.0    | 3.5   |
| 4          | 47             | 78                     | 0.60 | 93                                  | 5.0                             | 5.5    | 5.0   |
| 5          | 64             | 105                    | 0.61 | 215                                 | 9.5                             | 11.0   | 10.0  |
| 6          | 80             | 132                    | 0.61 | 355                                 | 14.0                            | 16.5   | 14.5  |
| 7          | 97             | 159                    | 0.61 | 548                                 | 17.5                            | 20.0   | 18.5  |
| 8          | 113            | 186                    | 0.61 | 818                                 | 22.0                            | 24.5   | 22.0  |
| 9*         | 120            | 200                    | 0.60 | --                                  | --                              | --     | --    |
| 10**       | 96             | 200                    | 0.48 | 2028                                | 52.0                            | 61.0   | 109.0 |

| Load Stage | Reinforcement Stresses (ksi) |      |       |       |      |      |      |      |
|------------|------------------------------|------|-------|-------|------|------|------|------|
|            | Gauge                        |      |       |       |      |      |      |      |
| 1          | 2                            | 3    | 4     | 5     | 6    | 7    | 8    |      |
| 0          | 0                            | 0    | 0     | 0     | 0    | 0    | 0    | 0    |
| 1          | -0.5                         | -0.6 | +0.5  | +0.6  | +0.1 | +0.2 | +0.1 | +0.2 |
| 2          | -3.3                         | -3.0 | +3.9  | +3.8  | +0.9 | +0.5 | +0.4 | 0.6  |
| 3          | -4.9                         | -4.4 | 9.0   | 6.1   | 1.8  | 0.9  | 0.6  | 1.5  |
| 4          | -6.9                         | -6.1 | 14.6  | 11.4  | 3.3  | 1.5  | 0.8  | 5.5  |
| 5          | -9.8                         | -7.9 | 26.6  | 24.9  | 8.1  | 5.7  | 1.9  | 18.8 |
| 6          | -10.2                        | -5.2 | 36.1  | 38.0  | 11.9 | 9.6  | 7.5  | 26.3 |
| 7          | -8.1                         | -2.4 | 44.9  | Yield | 17.2 | 15.3 | 18.3 | 29.4 |
| 8          | 0                            | +2.6 | Yield | Yield | 23.1 | 22.2 | 25.6 | 33.4 |
| 9*         | --                           | --   | --    | --    | --   | --   | --   | --   |
| 10**       | -5.4                         | -6.2 | Yield | Yield | 34.5 | 33.6 | 24.1 | 24.6 |

\*Failure

\*\*Past peak loads



TABLE B-18  
TEST RESULTS FOR BEAM 4-2

| Load Stage | Torque in.kips | Bending Moment in.kips | $\emptyset$ | Twist Radians per in. $\times 10^6$ | Deflection inches $\times 10^2$ |        |      |
|------------|----------------|------------------------|-------------|-------------------------------------|---------------------------------|--------|------|
|            |                |                        |             |                                     | West                            | Center | East |
| 0          | 0              | 0                      | --          | 0                                   | 0                               | 0      | 0    |
| 1          | 14             | 24                     | 0.58        | 22                                  | 0                               | +0.5   | 0    |
| 2          | 31             | 51                     | 0.61        | 48                                  | +2.0                            | 2.5    | +2.0 |
| 3          | 39             | 65                     | 0.60        | 70                                  | 3.0                             | 4.0    | 3.0  |
| 4          | 48             | 78                     | 0.62        | 115                                 | 5.5                             | 7.0    | 5.5  |
| 5          | 63             | 105                    | 0.60        | 288                                 | 11.5                            | 13.0   | 11.0 |
| 6          | 72             | 119                    | 0.61        | 383                                 | 12.5                            | 15.0   | 12.5 |
| 7          | 80             | 132                    | 0.61        | 495                                 | 14.5                            | 17.0   | 14.5 |
| 8          | 89             | 146                    | 0.61        | 693                                 | 16.0                            | 18.0   | 15.0 |
| 9          | 97             | 159                    | 0.61        | 935                                 | 18.0                            | 20.0   | 17.0 |
| 10*        | 101            | 166                    | 0.61        | --                                  | --                              | --     | --   |
| 11**       | 88             | 166                    | 0.53        | 2092                                | 31.0                            | 30.0   | 29.5 |

| Load Stage | Reinforcement Stresses (ksi) |       |       |       |       |      |       |       |
|------------|------------------------------|-------|-------|-------|-------|------|-------|-------|
|            | Gauge                        |       |       |       |       |      |       |       |
| 1          | 2                            | 3     | 4     | 5     | 6     | 7    | 8     |       |
| 0          | 0                            | 0     | 0     | 0     | 0     | 0    | 0     | 0     |
| 1          | -0.7                         | -0.5  | +0.5  | +0.4  | -0.1  | 0    | 0     | 0     |
| 2          | -3.3                         | -2.3  | 3.5   | 3.0   | +0.1  | +0.9 | +0.1  | +0.6  |
| 3          | -5.0                         | -3.5  | 6.6   | 5.1   | 1.1   | +1.9 | +0.1  | +3.5  |
| 4          | -6.9                         | -4.7  | 16.0  | 9.6   | 3.1   | 4.2  | -0.3  | 10.2  |
| 5          | -8.5                         | -3.0  | 40.3  | 26.1  | 5.9   | 7.1  | +5.0  | 25.9  |
| 6          | -6.9                         | -1.1  | 47.2  | 31.4  | 7.2   | 8.4  | 8.2   | 33.6  |
| 7          | -1.2                         | +1.8  | Yield | 37.2  | 9.4   | 10.9 | 10.6  | 44.7  |
| 8          | +1.7                         | +6.6  | Yield | Yield | 12.8  | 31.9 | 16.2  | 51.3  |
| 9          | +4.6                         | +9.0  | Yield | Yield | 18.0  | 39.8 | 22.8  | Yield |
| 10*        | --                           | --    | --    | --    | --    | --   | --    | --    |
| 11**       | +7.8                         | +28.5 | Yield | --    | +42.3 | +6.8 | Yield | Yield |

\*Failure

\*\*Past peak loads



TABLE B-19

## TEST RESULTS FOR BEAM 4-3

| Load Stage | Torque in.kips | Bending Moment in.kips | Twist       |                               | Deflection |        |      |
|------------|----------------|------------------------|-------------|-------------------------------|------------|--------|------|
|            |                |                        | $\emptyset$ | Radians per in. $\times 10^6$ | West       | Center | East |
| 0          | 0              | 0                      | --          | 0                             | 0          | 0      | 0    |
| 1          | 14             | 24                     | 0.58        | 17                            | +1.0       | +0.5   | +0.5 |
| 2          | 31             | 51                     | 0.61        | 45                            | 2.5        | 2.5    | 2.0  |
| 3          | 39             | 65                     | 0.60        | 77                            | 3.5        | 4.5    | 3.5  |
| 4          | 47             | 78                     | 0.60        | 155                           | 7.0        | 8.0    | 6.5  |
| 5          | 56             | 92                     | 0.61        | 245                           | 10.0       | 11.0   | 9.5  |
| 6          | 65             | 105                    | 0.62        | 388                           | 12.0       | 11.5   | 11.0 |
| 7          | 73             | 119                    | 0.61        | 559                           | 13.5       | 15.0   | 12.5 |
| 8          | 81             | 132                    | 0.61        | 775                           | 14.5       | 16.0   | 14.0 |
| 9          | 89             | 146                    | 0.61        | 1094                          | 16.0       | 18.5   | 15.0 |
| 10*        | 93             | 153                    | 0.61        | --                            | --         | --     | --   |
| 11**       | 83             | 153                    | 0.54        | 1998                          | 20.5       | 19.5   | 17.5 |

| Load Stage | Reinforcement Stresses (ksi) |      |       |       |      |       |       |       |
|------------|------------------------------|------|-------|-------|------|-------|-------|-------|
|            | Gauge                        |      |       |       |      |       |       |       |
| 1          | 2                            | 3    | 4     | 5     | 6    | 7     | 8     |       |
| 0          | 0                            | 0    | 0     | 0     | 0    | 0     | 0     | 0     |
| 1          | -0.5                         | -0.3 | +0.6  | +0.8  | +0.3 | +0.4  | +0.4  | 0     |
| 2          | -3.2                         | -2.4 | 2.4   | 3.2   | 1.1  | 1.3   | +0.5  | +0.3  |
| 3          | -4.9                         | -3.3 | 4.3   | 9.6   | 2.2  | 5.0   | +0.3  | 6.0   |
| 4          | -5.5                         | +0.2 | 20.7  | 16.7  | 6.8  | 14.0  | -0.3  | 20.1  |
| 5          | -7.9                         | +3.0 | 28.0  | 25.3  | 8.1  | 25.4  | +0.1  | 25.7  |
| 6          | -6.8                         | 6.4  | 35.9  | 35.0  | 9.9  | 35.7  | +9.2  | 36.0  |
| 7          | +4.2                         | 10.1 | 45.1  | 40.4  | 11.1 | 42.4  | 14.1  | 42.8  |
| 8          | +12.5                        | 8.3  | Yield | 46.3  | 13.9 | 51.7  | 21.0  | 49.1  |
| 9          | +19.5                        | 6.7  | Yield | Yield | 21.7 | Yield | 32.0  | Yield |
| 10*        | --                           | --   | --    | --    | --   | --    | --    | --    |
| 11**       | +33.1                        | +4.2 | Yield | Yield | 51.3 | Yield | Yield | Yield |

\*Failure

\*\*Past peak loads



TABLE B-20  
TEST RESULTS FOR BEAM 4-4

| Load Stage | Torque in.kips | Bending Moment in.kips | Twist |                                   | Deflection inches x 10 <sup>2</sup> |        |      |
|------------|----------------|------------------------|-------|-----------------------------------|-------------------------------------|--------|------|
|            |                |                        | ∅     | Radians per in. x 10 <sup>6</sup> | West                                | Center | East |
| 0          | 0              | 0                      | --    | 0                                 | 0                                   | 0      | 0    |
| 1          | 15             | 24                     | 0.62  | 22                                | +0.5                                | +0.5   | +0.5 |
| 2          | 24             | 38                     | 0.63  | 37                                | 1.5                                 | 1.0    | 1.0  |
| 3          | 31             | 51                     | 0.61  | 47                                | 2.0                                 | 2.5    | 2.0  |
| 4          | 40             | 65                     | 0.62  | 80                                | 4.5                                 | 4.5    | 4.0  |
| 5          | 47             | 78                     | 0.60  | 158                               | 7.0                                 | 8.0    | 7.0  |
| 6          | 56             | 92                     | 0.61  | 307                               | 9.5                                 | 11.0   | 10.0 |
| 7          | 64             | 105                    | 0.61  | 487                               | 12.5                                | 14.0   | 12.0 |
| 8          | 73             | 119                    | 0.61  | 713                               | 14.5                                | 16.0   | 13.5 |
| 9          | 81             | 132                    | 0.61  | 1019                              | 16.0                                | 17.5   | 14.0 |
| 10*        | 85             | 146                    | 0.58  | --                                | --                                  | --     | --   |
| 11**       | 77             | 146                    | 0.53  | 1777                              | 21.5                                | 21.5   | 19.0 |

| Load Stage | Reinforcement Stresses (ksi) |       |       |      |       |      |      |        |
|------------|------------------------------|-------|-------|------|-------|------|------|--------|
|            | Gauge                        |       |       |      |       |      |      |        |
| 1          | 2                            | 3     | 4     | 5    | 6     | 7    | 8    |        |
| 0          | 0                            | 0     | 0     | 0    | 0     | 0    | 0    | 0      |
| 1          | -0.6                         | +0.7  | +0.9  | +0.8 | +0.3  | +0.2 | 0    | 0      |
| 2          | -1.7                         | -0.2  | 1.9   | 2.2  | +0.9  | +0.5 | -0.1 | 0      |
| 3          | -3.0                         | -1.4  | 3.0   | 4.5  | 2.4   | +0.7 | -0.4 | +0.1   |
| 4          | -5.5                         | -2.5  | 6.3   | 13.4 | 4.9   | -0.8 | -0.9 | +7.5   |
| 5          | -6.7                         | -3.1  | 23.1  | 20.5 | 8.2   | +7.6 | -1.6 | 23.7   |
| 6          | -6.0                         | -0.1  | 31.2  | --   | 18.3  | 11.7 | -0.9 | 32.7   |
| 7          | +2.7                         | +7.9  | 38.7  | --   | 24.7  | 18.8 | +5.5 | 41.5   |
| 8          | 11.0                         | 15.2  | 46.5  | --   | 31.2  | 27.9 | 9.6  | 50.1   |
| 9          | 16.6                         | 21.7  | Yield | --   | 40.8  | 37.1 | 15.3 | Yield  |
| 10*        | --                           | --    | --    | --   | --    | --   | --   | --     |
| 11**       | +17.7                        | +30.7 | Yield | --   | Yield | 45.8 | 29.9 | (48.5) |

\*Failure

\*\*Past peak loads



TABLE B-21

## TEST RESULTS FOR BEAM 4-5

| Load Stage | Torque in. kips | Bending Moment in. kips | ∅    | Twist Radians per in. $\times 10^6$ |      | Deflection inches $\times 10^2$ |      |  |
|------------|-----------------|-------------------------|------|-------------------------------------|------|---------------------------------|------|--|
|            |                 |                         |      | 6                                   | West | Center                          | East |  |
| 0          | 0               | 0                       | --   | 0                                   | 0    | 0                               | 0    |  |
| 1          | 12              | 24                      | 0.50 | 15                                  | +0.5 | 0                               | +1.5 |  |
| 2          | 22              | 45                      | 0.49 | 27                                  | 1.5  | +2.0                            | 2.0  |  |
| 3          | 32              | 65                      | 0.49 | 47                                  | 3.5  | 3.5                             | 4.0  |  |
| 4          | 41              | 85                      | 0.48 | 95                                  | 6.0  | 7.5                             | 7.5  |  |
| 5          | 52              | 105                     | 0.50 | 170                                 | 10.0 | 12.0                            | 11.0 |  |
| 6          | 62              | 126                     | 0.49 | 252                                 | 14.0 | 15.5                            | 14.5 |  |
| 7          | 72              | 146                     | 0.49 | 335                                 | 16.5 | 18.0                            | 16.5 |  |
| 8          | 83              | 166                     | 0.50 | 445                                 | 19.0 | 21.5                            | 19.5 |  |
| 9          | 93              | 186                     | 0.50 | 588                                 | 22.0 | 25.0                            | 21.5 |  |
| 10         | 100             | 200                     | 0.50 | 717                                 | 24.5 | 27.0                            | 24.0 |  |
| 11*        | 103             | 212                     | 0.49 | --                                  | --   | --                              | --   |  |
| 12**       | 82              | 212                     | 0.39 | 1923                                | 54.0 | 68.5                            | 62.0 |  |

| Load Stage | Reinforcement Stresses (ksi) |      |       |        |      |      |      |      |
|------------|------------------------------|------|-------|--------|------|------|------|------|
|            | 1                            | 2    | 3     | 4      | 5    | 6    | 7    | 8    |
| 0          | 0                            | 0    | 0     | 0      | 0    | 0    | 0    | 0    |
| 1          | -0.4                         | -0.5 | +0.7  | +0.9   | +0.3 | +0.2 | +0.2 | +0.2 |
| 2          | -1.8                         | -2.2 | 2.4   | 2.9    | 0.9  | 0.5  | +0.2 | +0.3 |
| 3          | -3.6                         | -4.2 | 5.4   | 6.6    | 3.6  | 1.5  | +0.2 | +0.5 |
| 4          | -6.1                         | -4.8 | 12.9  | 19.0   | 6.6  | 2.1  | 0    | -0.3 |
| 5          | -7.6                         | -4.4 | 21.9  | 26.2   | 7.9  | 5.7  | -0.3 | -0.3 |
| 6          | -8.6                         | -3.4 | 28.9  | 35.9   | 10.9 | 9.3  | -0.3 | -0.2 |
| 7          | -7.9                         | -1.5 | 35.9  | 44.0   | 13.7 | 12.1 | -0.1 | +1.2 |
| 8          | -6.9                         | -0.2 | 43.3  | Yield  | 17.1 | 15.1 | +4.4 | 3.9  |
| 9          | -5.4                         | +0.8 | Yield | Yield  | 21.4 | 19.3 | 10.2 | 6.0  |
| 10         | -2.6                         | +2.0 | Yield | (47.2) | 24.3 | 22.2 | 17.2 | 6.9  |
| 11*        | --                           | --   | --    | --     | --   | --   | --   | --   |
| 12**       | -6.6                         | +8.0 | Yield | Yield  | 28.5 | 39.1 | 26.1 | 6.6  |

\*Failure

\*\*Past peak loads



TABLE B-22  
TEST RESULTS FOR BEAM 4-6

| Load Stage | Torque in. kips | Bending Moment in. kips | ∅    | Twist Radians per in. x 10 <sup>6</sup> |      | Deflection inches x 10 <sup>2</sup> |      |   |
|------------|-----------------|-------------------------|------|---|------|-------------------------------------|------|---|
|            |                 |                         |      | 6                                       | West | Center                              | East |   |
| 0          | 0               | 0                       | --   | 0                                       | 0    | 0                                   | 0    | 0 |
| 1          | 12              | 24                      | 0.50 | 19                                      | +1.0 | +1.0                                | +0.5 |   |
| 2          | 19              | 38                      | 0.50 | 30                                      | 1.5  | 2.0                                 | 1.5  |   |
| 3          | 26              | 51                      | 0.51 | 45                                      | 2.5  | 3.0                                 | 2.0  |   |
| 4          | 32              | 65                      | 0.49 | 65                                      | 4.5  | 5.0                                 | 4.0  |   |
| 5          | 39              | 78                      | 0.50 | 128                                     | 8.0  | 8.5                                 | 7.0  |   |
| 6          | 46              | 92                      | 0.50 | 202                                     | 12.0 | 13.5                                | 11.0 |   |
| 7          | 53              | 105                     | 0.50 | 347                                     | 15.0 | 18.0                                | 15.0 |   |
| 8          | 56              | 112                     | 0.50 | 405                                     | 17.0 | 20.0                                | 16.0 |   |
| 9          | 59              | 119                     | 0.50 | 453                                     | 19.0 | 21.0                                | 17.5 |   |
| 10         | 63              | 126                     | 0.50 | 567                                     | 21.5 | 24.5                                | 19.5 |   |
| 11*        | 66              | 132                     | 0.50 | --                                      | --   | --                                  | --   |   |
| 12**       | 58              | 132                     | 0.44 | 2260                                    | 57.0 | 63.0                                | 47.0 |   |

| Load Stage | Reinforcement Stresses (ksi) |      |        |        |      |      |      |      |
|------------|------------------------------|------|--------|--------|------|------|------|------|
|            | Gauge                        |      |        |        |      |      |      |      |
| 1          | 2                            | 3    | 4      | 5      | 6    | 7    | 8    |      |
| 0          | 0                            | 0    | 0      | 0      | 0    | 0    | 0    | 0    |
| 1          | -0.6                         | -0.4 | +0.4   | +0.6   | +0.2 | 0    | +0.1 | +0.1 |
| 2          | -1.9                         | -1.3 | 1.5    | 1.8    | 0.8  | +0.1 | +0.1 | +0.2 |
| 3          | -3.3                         | -2.3 | 2.8    | 4.6    | 1.7  | 0.3  | 0    | +0.4 |
| 4          | -5.2                         | -3.5 | 5.3    | 9.6    | 4.2  | 1.6  | -0.2 | -0.2 |
| 5          | -8.1                         | -5.0 | 26.5   | 24.3   | 7.8  | 4.2  | -0.4 | -0.8 |
| 6          | -9.6                         | -1.2 | 38.2   | 33.1   | 10.4 | 7.3  | -0.5 | +0.2 |
| 7          | -8.9                         | +1.4 | Yield  | 44.5   | 13.6 | 19.3 | +2.8 | +5.9 |
| 8          | -7.2                         | +3.4 | Yield  | Yield  | 15.5 | 22.4 | 5.0  | 8.1  |
| 9          | -6.3                         | 4.5  | Yield  | Yield  | 16.6 | 24.7 | 6.2  | 9.7  |
| 10         | +1.8                         | 6.5  | Yield  | Yield  | 20.4 | 28.9 | 7.5  | 12.0 |
| 11*        | +34.0                        | 6.4  | --     | --     | 39.4 | --   | --   | --   |
| 12**       | +37.0                        | 3.3  | (46.3) | (43.8) | 43.5 | 43.2 | 31.6 | 32.1 |

\*Failure

\*\*Past peak loads



TABLE B-23  
TEST RESULTS FOR BEAM 5-1

| Load Stage | Torque in.kips | Bending Moment in.kips | Twist |                                   |                  | Deflection |        |       |
|------------|----------------|------------------------|-------|-----------------------------------|------------------|------------|--------|-------|
|            |                |                        | ∅     | Radians per in. x 10 <sup>6</sup> | Shear Force kips | West       | Center | East  |
| 0          | 0              | 0                      | --    | 0                                 | 0                | 0          | 0      | 0     |
| 1          | 28             | 15                     | 1.87  | 51                                | 0                | 0          | 0      | 0     |
| 2          | 42             | 21                     | 2.00  | 82                                | 0.09             | +0.5       | 0      | 0     |
| 3          | 54             | 27                     | 2.00  | 169                               | 0.19             | +1.0       | +0.5   | 0     |
| 4          | 66             | 34                     | 1.94  | 420                               | 0.28             | 0          | -1.5   | -1.0  |
| 5          | 80             | 40                     | 2.00  | 649                               | 0.38             | -2.0       | -3.0   | -3.5  |
| 6          | 92             | 46                     | 2.00  | 926                               | 0.47             | -4.5       | -5.5   | -5.5  |
| 7          | 105            | 53                     | 1.98  | 1182                              | 0.57             | -6.5       | -8.0   | -8.5  |
| 8          | 117            | 59                     | 1.98  | 1528                              | 0.66             | -10.0      | -12.0  | -11.0 |
| 9*         | 128            | 65                     | 1.97  | --                                | 0.76             | --         | --     | --    |
| 10**       | 103            | 65                     | 1.59  | 2602                              | 0.76             | -17.0      | -22.0  | -19.5 |

| Load Stage | Reinforcement Stresses (ksi) |       |      |      |      |      |      |      |
|------------|------------------------------|-------|------|------|------|------|------|------|
|            | Gauge                        |       |      |      |      |      |      |      |
| 1          | 2                            | 3     | 4    | 5    | 6    | 7    | 8    |      |
| 0          | 0                            | 0     | 0    | 0    | 0    | 0    | 0    | 0    |
| 1          | 0                            | +0.1  | +0.3 | +0.8 | +0.6 | +0.1 | 0    | +0.3 |
| 2          | -0.6                         | 0.6   | 1.4  | 3.1  | 1.1  | 0.4  | 0    | 4.2  |
| 3          | -0.3                         | 2.1   | 6.6  | 10.2 | 3.1  | 7.0  | +4.6 | 20.9 |
| 4          | +6.0                         | 20.2  | 11.7 | 15.1 | 7.9  | 22.6 | 9.2  | 32.0 |
| 5          | +16.5                        | 27.4  | 17.4 | 20.3 | 14.1 | 30.8 | 18.3 | 38.4 |
| 6          | 33.0                         | 39.7  | 23.4 | 25.2 | 19.9 | 36.3 | 23.6 | 43.7 |
| 7          | 46.0                         | 50.7  | 27.5 | 28.8 | 25.0 | 41.5 | 28.4 | 49.2 |
| 8          | Yield                        | Yield | 31.9 | 33.7 | 30.9 | 47.3 | 36.0 | 53.0 |
| 9*         | --                           | --    | --   | --   | --   | --   | 42.0 | --   |
| 10**       | Yield                        | Yield | 33.6 | 34.9 | 31.8 | 45.0 | 36.4 | 53.1 |

\*Failure

\*\*Past peak loads



TABLE B-24  
TEST RESULTS FOR BEAM 5-2

| Load Stage | Torque in.kips | Bending Moment in.kips | ∅    | Twist          |                                   | Deflection       |                               |        |
|------------|----------------|------------------------|------|----------------|-----------------------------------|------------------|-------------------------------|--------|
|            |                |                        |      | Radians per ft | Shear Force in. x 10 <sup>6</sup> | Shear Force kips | West inches x 10 <sup>2</sup> | Center |
| 0          | 0              | 0                      | --   | 0              | 0                                 | 0                | 0                             | 0      |
| 1          | 15             | 15                     | 1.00 | 18             | 0                                 | +0.5             | 0                             | 0      |
| 2          | 31             | 32                     | 0.97 | 44             | 0.25                              | 1.0              | +0.5                          | +0.5   |
| 3          | 48             | 49                     | 0.98 | 129            | 0.50                              | 3.0              | 2.5                           | 1.0    |
| 4          | 65             | 66                     | 0.99 | 377            | 0.76                              | 4.0              | 3.5                           | 2.5    |
| 5          | 82             | 83                     | 0.99 | 634            | 1.01                              | 3.5              | 3.0                           | 1.5    |
| 6          | 100            | 100                    | 1.00 | 895            | 1.26                              | 3.0              | 2.0                           | +0.5   |
| 7          | 116            | 117                    | 0.99 | 1215           | 1.51                              | 2.5              | 1.5                           | 0      |
| 8          | 124            | 126                    | 0.98 | 1372           | 1.64                              | 3.0              | +0.5                          | -1.5   |
| 9          | 134            | 134                    | 1.00 | 1746           | 1.76                              | +1.0             | -1.5                          | -3.0   |
| 10*        | 141            | 143                    | 0.99 | --             | 1.89                              | --               | --                            | --     |
| 11**       | 117            | 143                    | 0.82 | 3131           | 1.89                              | -3.0             | -17.5                         | -8.0   |

| Load Stage | Reinforcement Stresses (ksi) |       |       |        |       |      |       |       |
|------------|------------------------------|-------|-------|--------|-------|------|-------|-------|
|            | Gauge                        |       |       |        |       |      |       |       |
| 1          | 2                            | 3     | 4     | 5      | 6     | 7    | 8     |       |
| 0          | 0                            | 0     | 0     | 0      | 0     | 0    | 0     | 0     |
| 1          | 0                            | 0     | +0.1  | +0.2   | +0.3  | 0    | -0.1  | +0.1  |
| 2          | -1.3                         | -1.0  | 1.7   | 2.0    | 1.6   | +0.1 | -0.3  | +0.8  |
| 3          | -3.2                         | +0.2  | 7.4   | 11.0   | 4.7   | 1.3  | -0.7  | 5.7   |
| 4          | +4.9                         | 16.1  | 18.1  | 20.8   | 14.8  | 14.5 | +7.8  | 27.6  |
| 5          | 12.9                         | 23.5  | 24.9  | 24.3   | 21.0  | 21.6 | 13.7  | 37.4  |
| 6          | 25.3                         | 29.1  | 31.8  | 28.5   | 29.5  | 30.9 | 19.3  | 45.8  |
| 7          | 35.4                         | 36.8  | 37.8  | 32.9   | 37.5  | 38.4 | 25.2  | Yield |
| 8          | 40.8                         | 42.0  | 40.5  | 35.2   | 40.5  | 40.8 | 28.6  | Yield |
| 9          | 49.0                         | Yield | 45.6  | 40.2   | 43.2  | 45.3 | 36.3  | Yield |
| 10*        | --                           | --    | --    | Yield  | --    | --   | --    | --    |
| 11**       | 52.0                         | Yield | Yield | (44.8) | Yield | 47.7 | Yield | Yield |

\*Failure

\*\*Past peak loads



TABLE B-25  
TEST RESULTS FOR BEAM 5-3

| Load Stage | Torque in.kips | Bending Moment in.kips | Twist |                                 |                  | Deflection               |      |        |
|------------|----------------|------------------------|-------|---------------------------------|------------------|--------------------------|------|--------|
|            |                |                        | ∅     | Radians Per in.x10 <sup>6</sup> | Shear Force kips | inches x 10 <sup>2</sup> | West | Center |
| 0          | 0              | 0                      | --    | 0                               | 0                | 0                        | 0    | 0      |
| 1          | 15             | 27                     | 0.56  | 25                              | 0.19             | +0.5                     | +1.0 | +0.5   |
| 2          | 29             | 55                     | 0.53  | 51                              | 0.60             | 2.0                      | 2.0  | 1.5    |
| 3          | 42             | 83                     | 0.51  | 102                             | 1.01             | 4.5                      | 4.5  | 3.5    |
| 4          | 55             | 111                    | 0.50  | 215                             | 1.42             | 7.5                      | 8.0  | 6.5    |
| 5          | 69             | 139                    | 0.50  | 377                             | 1.83             | 10.0                     | 10.0 | 8.5    |
| 6          | 83             | 167                    | 0.50  | 559                             | 2.23             | 11.5                     | 12.0 | 10.0   |
| 7          | 97             | 194                    | 0.50  | 787                             | 2.64             | 14.0                     | 13.5 | 11.0   |
| 8          | 110            | 222                    | 0.50  | 995                             | 3.05             | 15.0                     | 14.0 | 12.0   |
| 9          | 125            | 250                    | 0.50  | 1390                            | 3.46             | 19.0                     | 17.5 | 13.5   |
| 10*        | 130            | 278                    | 0.47  | --                              | 3.87             | --                       | --   | --     |
| 11**       | 111            | 278                    | 0.40  | 2734                            | 3.87             | 50.5                     | 47.0 | 40.5   |

| Load Stage | Reinforcement Stresses (ksi) |      |       |       |       |      |      |       |
|------------|------------------------------|------|-------|-------|-------|------|------|-------|
|            | Gauge                        |      |       |       |       |      |      |       |
| 1          | 2                            | 3    | 4     | 5     | 6     | 7    | 8    |       |
| 0          | 0                            | 0    | 0     | 0     | 0     | 0    | 0    | 0     |
| 1          | -0.9                         | -0.6 | +0.9  | +0.8  | +0.3  | +0.3 | -0.1 | +0.3  |
| 2          | -3.5                         | -2.3 | 4.9   | 3.8   | 1.3   | 1.0  | -0.3 | 1.8   |
| 3          | -7.4                         | -3.7 | 12.6  | 12.0  | 3.1   | 3.1  | -0.6 | 12.7  |
| 4          | -9.8                         | -2.7 | 21.1  | 19.5  | 4.9   | 12.5 | +1.2 | 20.2  |
| 5          | -9.6                         | +1.5 | 27.5  | 26.8  | 7.3   | 19.9 | 10.5 | 31.3  |
| 6          | -7.4                         | +7.0 | 32.7  | 33.2  | 9.9   | 29.2 | 23.7 | 38.4  |
| 7          | -6.0                         | 11.3 | 37.8  | 39.0  | 16.4  | 36.3 | 30.1 | 46.6  |
| 8          | -3.2                         | 15.6 | 43.8  | 44.7  | 23.1  | 41.9 | 36.3 | Yield |
| 9          | -0.5                         | 24.5 | Yield | Yield | 32.5  | 48.9 | 43.0 | Yield |
| 10*        | --                           | --   | --    | --    | --    | --   | --   | --    |
| 11**       | -14.4                        | 22.0 | Yield | Yield | Yield | 42.2 | 52.5 | Yield |

\*Failure

\*\*Past peak loads



TABLE B-26

## TEST RESULTS FOR BEAM 5-4

| Load Stage | Torque in.kips | Bending Moment in.kips | Twist |                                |                  | Deflection               |      |        |
|------------|----------------|------------------------|-------|--------------------------------|------------------|--------------------------|------|--------|
|            |                |                        | ∅     | Radians<br>in.x10 <sup>6</sup> | Shear Force kips | inches x 10 <sup>2</sup> | West | Center |
| 0          | 0              | 0                      | --    | 0                              | 0                | 0                        | 0    | 0      |
| 1          | 10             | 36                     | 0.28  | 15                             | 0.31             | +1.0                     | +1.0 | +0.5   |
| 2          | 18             | 72                     | 0.25  | 22                             | 0.85             | 3.0                      | 2.5  | 2.0    |
| 3          | 28             | 109                    | 0.26  | 54                             | 1.39             | 5.5                      | 5.5  | 4.5    |
| 4          | 36             | 145                    | 0.25  | 90                             | 1.92             | 9.0                      | 8.5  | 7.5    |
| 5          | 45             | 182                    | 0.25  | 128                            | 2.46             | 13.0                     | 12.5 | 10.0   |
| 6          | 54             | 218                    | 0.25  | 185                            | 2.99             | 16.5                     | 16.5 | 14.0   |
| 7          | 64             | 254                    | 0.25  | 246                            | 3.53             | 20.0                     | 19.0 | 17.0   |
| 8          | 72             | 291                    | 0.25  | 315                            | 4.06             | 23.0                     | 22.5 | 19.5   |
| 9          | 81             | 327                    | 0.25  | 403                            | 4.60             | 26.5                     | 26.0 | 22.5   |
| 10         | 91             | 364                    | 0.25  | 1815                           | 5.13             | 77.5                     | 75.0 | 62.5   |
| 11*        | 98             | 389                    | 0.25  | --                             | 5.51             | --                       | --   | --     |

| Load Stage | Reinforcement Stresses (ksi) |       |       |       |       |      |      |      |
|------------|------------------------------|-------|-------|-------|-------|------|------|------|
|            | Gauge                        |       |       |       |       |      |      |      |
| 1          | 2                            | 3     | 4     | 5     | 6     | 7    | 8    |      |
| 0          | 0                            | 0     | 0     | 0     | 0     | 0    | 0    | 0    |
| 1          | -1.3                         | -0.9  | +1.3  | +1.3  | +0.2  | 0    | +0.5 | +0.3 |
| 2          | -4.2                         | -3.5  | 6.3   | 4.2   | 1.1   | +0.1 | 0.7  | 0.5  |
| 3          | -7.9                         | -6.3  | 13.5  | 9.4   | 3.1   | +0.3 | 0.9  | 1.7  |
| 4          | -11.4                        | -8.1  | 19.6  | 20.9  | 4.6   | 0    | 1.1  | 3.3  |
| 5          | -15.1                        | -9.3  | 25.9  | 29.8  | 6.0   | 0    | 1.1  | 4.5  |
| 6          | -18.6                        | -10.3 | 32.0  | 37.5  | 7.0   | +0.2 | 1.6  | 6.6  |
| 7          | -22.0                        | -11.1 | 38.4  | 45.0  | 7.8   | 1.7  | 3.4  | 8.9  |
| 8          | -25.5                        | -10.6 | 44.7  | Yield | 9.0   | 6.3  | 5.1  | 10.9 |
| 9          | -29.3                        | -10.2 | Yield | Yield | 11.7  | 9.1  | 7.7  | 13.5 |
| 10         | -49.2                        | -12.0 | --    | Yield | Yield | 18.7 | 27.5 | 22.8 |
| 11*        | --                           | --    | --    | --    | --    | --   | --   | --   |

\*Failure



TABLE B-27

## TEST RESULTS FOR BEAM 6-1

| Load Stage | Torque in.kips | Bending Moment in.kips | Twist |                 |                                    | Deflection |        |       |
|------------|----------------|------------------------|-------|-----------------|------------------------------------|------------|--------|-------|
|            |                |                        | Ø     | Radians Per in. | Shear Force in. $\times 10^6$ kips | West       | Center | East  |
| 0          | 0              | 0                      | --    | 0               | 0                                  | 0          | 0      | 0     |
| 1          | 29             | 14                     | 2.07  | 21              | 0.30                               | 0          | 0      | 0     |
| 2          | 41             | 20                     | 2.05  | 42              | 0.49                               | +0.5       | 0      | 0     |
| 3          | 53             | 27                     | 1.96  | 149             | 0.68                               | +0.5       | 0      | +0.5  |
| 4          | 67             | 33                     | 2.03  | 419             | 0.87                               | 0          | -0.5   | -0.5  |
| 5          | 79             | 40                     | 1.97  | 600             | 1.06                               | -1.5       | -1.5   | -1.5  |
| 6          | 92             | 46                     | 2.00  | 832             | 1.24                               | -2.0       | -3.0   | -2.0  |
| 7          | 105            | 52                     | 2.02  | 1077            | 1.43                               | -3.5       | -4.0   | -3.0  |
| 8          | 118            | 59                     | 2.00  | 1397            | 1.62                               | -4.5       | -6.5   | -4.0  |
| 9*         | 129            | 65                     | 1.99  | --              | 1.81                               | --         | --     | --    |
| 10**       | 107            | 65                     | 1.65  | 3035            | 1.81                               | -16.0      | -21.0  | -14.0 |

| Load Stage | Reinforcement Stresses (ksi) |       |      |      |      |      |      |       |
|------------|------------------------------|-------|------|------|------|------|------|-------|
|            | Gauge                        |       |      |      |      |      |      |       |
| 1          | 2                            | 3     | 4    | 5    | 6    | 7    | 8    |       |
| 0          | 0                            | 0     | 0    | 0    | 0    | 0    | 0    | 0     |
| 1          | -0.1                         | +0.1  | +0.6 | +0.6 | +0.6 | -0.6 | +0.1 | +0.9  |
| 2          | -0.6                         | 0.3   | 1.5  | 1.7  | 1.0  | -0.1 | 0.3  | 2.3   |
| 3          | -1.0                         | 12.6  | 4.6  | 9.3  | 1.6  | +2.6 | 2.4  | 17.2  |
| 4          | +5.1                         | 33.3  | 14.1 | 16.5 | 13.2 | 30.3 | 14.9 | 33.9  |
| 5          | 16.9                         | 42.3  | 17.4 | 18.8 | 18.4 | 35.1 | 16.0 | 40.4  |
| 6          | 28.2                         | Yield | 20.4 | 22.7 | 26.1 | 39.4 | 16.8 | 43.9  |
| 7          | 36.9                         | Yield | 23.6 | 25.8 | 32.4 | 42.9 | 18.0 | 49.4  |
| 8          | 47.0                         | Yield | 27.9 | 28.2 | 40.8 | 45.8 | 24.6 | Yield |
| 9*         | --                           | --    | --   | --   | --   | --   | --   | --    |
| 10**       | 51.3                         | --    | 31.2 | 28.8 | 49.0 | 36.6 | 34.5 | Yield |

\*Failure

\*\*Past peak loads



TABLE B-28  
TEST RESULTS FOR BEAM 6-2

| Load Stage | Torque in.kips | Bending Moment in.kips | Ø    | Twist Radians per in. $\times 10^6$ | Shear Force kips | Deflection inches $\times 10^2$ |
|------------|----------------|------------------------|------|-------------------------------------|------------------|---------------------------------|
|            |                |                        |      |                                     |                  | West Center East                |
| 0          | 0              | 0                      | --   | 0                                   | 0                | 0 0 0                           |
| 1          | 15             | 14                     | 1.07 | 19                                  | 0.30             | 0 0 0                           |
| 2          | 30             | 29                     | 1.03 | 43                                  | 0.74             | +1.0 +0.5 +0.5                  |
| 3          | 51             | 44                     | 1.16 | 128                                 | 1.18             | 2.0 1.5 1.0                     |
| 4          | 59             | 59                     | 1.00 | 203                                 | 1.62             | 2.5 2.0 1.0                     |
| 5          | 74             | 74                     | 1.00 | 477                                 | 2.06             | 4.5 2.0 +0.5                    |
| 6          | 89             | 89                     | 1.00 | 747                                 | 2.50             | 3.5 1.5 0                       |
| 7          | 103            | 104                    | 0.99 | 987                                 | 2.94             | 2.5 +1.0 -0.5                   |
| 8          | 119            | 119                    | 1.00 | 1326                                | 3.39             | 1.5 -0.5 -2.0                   |
| 9          | 134            | 134                    | 1.00 | 1821                                | 3.83             | +0.5 -2.5 -3.0                  |
| 10*        | 145            | 149                    | 0.97 | --                                  | 4.27             | -- -- --                        |
| 11**       | 133            | 149                    | 0.89 | 2837                                | 4.27             | -4.0 -8.5 -9.5                  |

| Load Stage | Reinforcement Stresses (ksi) |       |      |      |      |       |      |      |
|------------|------------------------------|-------|------|------|------|-------|------|------|
|            | Gauge                        |       |      |      |      |       |      |      |
| 1          | 2                            | 3     | 4    | 5    | 6    | 7     | 8    |      |
| 0          | 0                            | 0     | 0    | 0    | 0    | 0     | 0    | 0    |
| 1          | 0                            | +0.1  | +0.1 | +0.1 | +0.1 | +0.2  | +0.1 | +0.2 |
| 2          | -0.9                         | -0.1  | 1.1  | 0.9  | 0.3  | 0.6   | 0    | 0.7  |
| 3          | -3.0                         | +0.6  | 5.8  | 9.3  | 1.9  | 3.3   | +2.7 | 1.8  |
| 4          | -2.6                         | +1.9  | 9.6  | 18.6 | 3.4  | 10.2  | 10.1 | 0.3  |
| 5          | +3.1                         | 18.4  | 17.0 | 25.6 | 10.2 | 24.9  | 19.7 | 20.8 |
| 6          | 8.8                          | 34.0  | 22.8 | 30.1 | 18.3 | 35.5  | 21.9 | 32.7 |
| 7          | 18.6                         | 44.1  | 27.1 | 33.7 | 26.0 | 42.1  | 24.3 | 42.4 |
| 8          | 31.1                         | Yield | 33.7 | 39.2 | 33.9 | 48.0  | 27.5 | 47.4 |
| 9          | 49.3                         | Yield | 41.4 | 43.7 | 43.6 | 52.6  | 33.3 | 49.2 |
| 10*        | --                           | --    | --   | --   | --   | --    | --   | --   |
| 11**       | Yield                        | Yield | 47.4 | 43.0 | 47.9 | Yield | 37.4 | 47.5 |

\*Failure

\*\*Past peak loads



TABLE B-29

## TEST RESULTS FOR BEAM 6-3

| Load Stage | Torque in. kips | Bending Moment in. kips | $\emptyset$ | Twist Radians per in. $\times 10^6$ | Shear Force kips | Deflection inches $\times 10^2$ |        |      |
|------------|-----------------|-------------------------|-------------|-------------------------------------|------------------|---------------------------------|--------|------|
|            |                 |                         |             |                                     |                  | West                            | Center | East |
| 0          | 0               | 0                       | --          | 0                                   | 0                | 0                               | 0      | 0    |
| 1          | 15              | 27                      | 0.56        | 24                                  | 0.68             | +0.5                            | 0      | +0.5 |
| 2          | 27              | 55                      | 0.49        | 43                                  | 1.50             | 1.5                             | +1.0   | 1.0  |
| 3          | 41              | 82                      | 0.50        | 83                                  | 2.31             | 4.0                             | 2.5    | 1.5  |
| 4          | 56              | 110                     | 0.51        | 187                                 | 3.13             | 6.0                             | 4.0    | 2.0  |
| 5          | 69              | 138                     | 0.50        | 315                                 | 3.95             | 8.0                             | 5.5    | 2.5  |
| 6          | 83              | 166                     | 0.50        | 498                                 | 4.77             | 11.0                            | 7.0    | 3.0  |
| 7          | 97              | 194                     | 0.50        | 725                                 | 5.59             | 12.0                            | 7.5    | 3.5  |
| 8          | 111             | 222                     | 0.50        | 995                                 | 6.41             | 13.5                            | 8.5    | 3.0  |
| 9          | 125             | 249                     | 0.50        | 1459                                | 7.23             | 15.5                            | 9.5    | 2.0  |
| 10*        | 132             | 264                     | 0.50        | --                                  | 7.67             | --                              | --     | --   |
| 11**       | 113             | 264                     | 0.43        | 2400                                | 7.67             | 25.0                            | 18.5   | 6.5  |

| Load Stage | Reinforcement Stresses (ksi) |      |       |       |        |      |      |        |
|------------|------------------------------|------|-------|-------|--------|------|------|--------|
|            | Gauge                        |      |       |       |        |      |      |        |
| 1          | 2                            | 3    | 4     | 5     | 6      | 7    | 8    |        |
| 0          | 0                            | 0    | 0     | 0     | 0      | 0    | 0    | 0      |
| 1          | -0.7                         | -0.5 | +0.7  | +0.6  | +0.3   | 0    | +0.2 | +0.1   |
| 2          | -3.4                         | -2.1 | 2.1   | 1.9   | 0.1    | -0.3 | 0    | 0.3    |
| 3          | -6.0                         | -2.2 | 5.9   | 6.8   | 0.7    | 0    | +0.8 | 8.1    |
| 4          | -9.4                         | +4.4 | 12.3  | 17.0  | 4.2    | +0.7 | 4.8  | 33.6   |
| 5          | -9.7                         | 15.4 | 17.3  | 22.6  | 6.5    | 5.4  | 7.4  | 46.6   |
| 6          | -8.4                         | 19.2 | 23.0  | 29.3  | 10.3   | 14.1 | 18.1 | 51.6   |
| 7          | -7.2                         | 22.8 | 29.4  | 34.1  | 16.6   | 21.4 | 24.1 | Yield  |
| 8          | -6.5                         | 29.3 | 35.8  | 40.9  | 24.1   | 26.6 | 30.0 | Yield  |
| 9          | +0.3                         | 43.2 | Yield | Yield | 40.3   | 31.5 | 37.1 | Yield  |
| 10*        | --                           | --   | --    | --    | Yield  | --   | --   | --     |
| 11**       | +5.7                         | 46.0 | Yield | Yield | (51.2) | 38.1 | 47.9 | (48.3) |

\*Failure

\*\*Past peak loads



TABLE B-30

## TEST RESULTS FOR BEAM 6-4

| Load Stage | Torque in.kips | Bending Moment in.kips | $\emptyset$ | Twist Radians per in. $\times 10^6$ | Shear Force kips | Deflection inches $\times 10^2$ |        |      |
|------------|----------------|------------------------|-------------|-------------------------------------|------------------|---------------------------------|--------|------|
|            |                |                        |             |                                     |                  | West                            | Center | East |
| 0          | 0              | 0                      | --          | 0                                   | 0                | 0                               | 0      | 0    |
| 1          | 8              | 35                     | 0.23        | 14                                  | 0.93             | +0.5                            | +0.5   | +0.5 |
| 2          | 18             | 72                     | 0.25        | 21                                  | 2.00             | 2.0                             | 1.5    | 1.5  |
| 3          | 27             | 108                    | 0.25        | 51                                  | 3.07             | 4.5                             | 3.0    | 2.0  |
| 4          | 36             | 144                    | 0.25        | 80                                  | 4.14             | 7.0                             | 5.5    | 3.5  |
| 5          | 45             | 181                    | 0.25        | 120                                 | 5.21             | 10.5                            | 7.5    | 5.0  |
| 6          | 54             | 217                    | 0.25        | 182                                 | 6.28             | 13.5                            | 9.5    | 5.5  |
| 7          | 63             | 254                    | 0.25        | 248                                 | 7.35             | 16.5                            | 11.5   | 6.0  |
| 8          | 72             | 290                    | 0.25        | 312                                 | 8.42             | 19.5                            | 13.5   | 7.5  |
| 9          | 81             | 326                    | 0.25        | 408                                 | 9.49             | 23.0                            | 16.5   | 8.5  |
| 10         | 90             | 363                    | 0.25        | 718                                 | 10.56            | 46.0                            | 33.0   | 16.0 |
| 11         | 95             | 380                    | 0.25        | 1006                                | 11.07            | 59.0                            | 42.0   | 20.0 |
| 12*        | 107            | 427                    | 0.25        | --                                  | 12.45            | --                              | --     | --   |
| 13**       | 89             | 427                    | 0.21        | ***                                 | 12.45            | 157.5                           | 113.5  | 54.0 |

| Load Stage | Reinforcement Stresses (ksi) |       |       |       |       |      |      |       |
|------------|------------------------------|-------|-------|-------|-------|------|------|-------|
|            | Gauge                        |       |       |       |       |      |      |       |
| 1          | 2                            | 3     | 4     | 5     | 6     | 7    | 8    |       |
| 0          | 0                            | 0     | 0     | 0     | 0     | 0    | 0    | 0     |
| 1          | -1.6                         | -0.8  | +1.0  | +0.6  | +0.4  | 0    | 0    | +0.1  |
| 2          | -5.2                         | -2.4  | 4.1   | 2.4   | 1.5   | 0    | 0    | 0.6   |
| 3          | -10.2                        | -3.8  | 9.3   | 7.3   | 3.0   | 0    | +0.1 | 4.4   |
| 4          | -15.6                        | -4.7  | 16.1  | 14.7  | 5.3   | -0.4 | -0.1 | 13.0  |
| 5          | -20.4                        | -5.3  | 21.9  | 20.1  | 6.7   | -0.6 | +0.3 | 21.4  |
| 6          | -25.6                        | -3.3  | 27.5  | 25.8  | 7.9   | -0.3 | 2.0  | 28.7  |
| 7          | -29.7                        | -1.8  | 33.0  | 31.3  | 9.9   | +0.9 | 6.5  | 34.8  |
| 8          | -33.6                        | +0.9  | 38.5  | 36.9  | 11.7  | 2.1  | 10.3 | 39.6  |
| 9          | -38.4                        | +11.7 | Yield | 42.8  | 14.6  | 4.5  | 12.1 | 44.1  |
| 10         | -52.7                        | +10.8 | Yield | Yield | 25.0  | 5.2  | 14.5 | 51.0  |
| 11         | Yield                        | +12.3 | Yield | Yield | 39.3  | 7.4  | 13.8 | Yield |
| 12*        | --                           | --    | --    | --    | --    | --   | --   | --    |
| 13**       | Yield                        | -17.8 | --    | --    | Yield | 22.6 | 53.5 | --    |

\*Failure

\*\*Past peak loads

\*\*\*Twist exceeded capacity of twistmeters



TABLE B-31

## TEST RESULTS FOR BEAM 7-1

| Load Stage | Torque in.kips | Bending Moment in.kips | $\emptyset$ | Twist Radians per in. $\times 10^6$ | Shear Force kips | Deflection inches $\times 10^2$ |        |       |
|------------|----------------|------------------------|-------------|-------------------------------------|------------------|---------------------------------|--------|-------|
|            |                |                        |             |                                     |                  | West                            | Center | East  |
| 0          | 0              | 0                      | --          | 0                                   | 0                | 0                               | 0      | 0     |
| 1          | 28             | 14                     | 2.00        | 40                                  | 0.30             | -0.5                            | -0.5   | -0.5  |
| 2          | 36             | 18                     | 2.00        | 61                                  | 0.43             | -0.5                            | -0.5   | -0.5  |
| 3          | 44             | 22                     | 2.00        | 98                                  | 0.55             | 0                               | -0.5   | -0.5  |
| 4          | 52             | 26                     | 2.00        | 203                                 | 0.68             | 0                               | -0.5   | -0.5  |
| 5          | 61             | 31                     | 1.97        | 472                                 | 0.80             | -2.0                            | -1.5   | -3.5  |
| 6          | 70             | 35                     | 2.00        | 586                                 | 0.93             | -2.0                            | -2.5   | -2.5  |
| 7          | 82             | 41                     | 2.00        | 808                                 | 1.12             | -5.0                            | -5.0   | -4.5  |
| 8          | 95             | 47                     | 2.02        | 1069                                | 1.31             | -6.5                            | -6.5   | -6.5  |
| 9          | 103            | 52                     | 1.98        | 1240                                | 1.43             | -6.0                            | -7.5   | -6.5  |
| 10*        | 112            | 56                     | 2.00        | 1592                                | 1.56             | --                              | --     | --    |
| 11**       | 97             | 56                     | 1.73        | 2237                                | 1.56             | -14.0                           | -16.0  | -14.0 |

| Load Stage | Reinforcement Stresses (ksi) |        |      |      |      |        |      |       |
|------------|------------------------------|--------|------|------|------|--------|------|-------|
|            | Gauge                        |        |      |      |      |        |      |       |
| 1          | 2                            | 3      | 4    | 5    | 6    | 7      | 8    |       |
| 0          | 0                            | 0      | 0    | 0    | 0    | 0      | 0    | 0     |
| 1          | 0                            | -0.2   | +0.3 | +0.6 | +0.4 | +0.2   | +0.4 | +0.9  |
| 2          | -0.1                         | -0.3   | 0.9  | 1.0  | 0.6  | 0.6    | 0.6  | 1.5   |
| 3          | -0.3                         | -0.5   | 1.4  | 2.5  | 0.4  | 1.1    | 1.0  | 3.1   |
| 4          | +3.6                         | +2.5   | 3.8  | 11.6 | 1.4  | 4.3    | 11.5 | 3.6   |
| 5          | +24.9                        | 29.9   | 11.1 | 16.0 | 3.8  | 30.7   | 12.0 | 28.6  |
| 6          | 33.1                         | 35.8   | 12.6 | 18.9 | 5.6  | 35.9   | 12.9 | 35.1  |
| 7          | 46.2                         | 46.4   | 15.0 | 21.9 | 10.3 | 41.9   | 16.7 | 47.7  |
| 8          | Yield                        | Yield  | 17.0 | 25.0 | 18.5 | 49.9   | 23.7 | Yield |
| 9          | Yield                        | Yield  | 18.1 | 26.4 | 24.3 | Yield  | 28.3 | Yield |
| 10*        | Yield                        | Yield  | 19.7 | 27.3 | 30.9 | Yield  | 34.2 | Yield |
| 11**       | (50.1)                       | (43.7) | 16.8 | 23.2 | 34.7 | (47.6) | 33.0 | Yield |

\*Failure

\*\*Past peak loads



TABLE B-32

## TEST RESULTS FOR BEAM 7-2

| Load Stage | Torque in.kips | Bending Moment in.kips | ∅    | Twist Radians per in. $\times 10^6$ | Shear Force kips | Deflection inches $\times 10^2$ |
|------------|----------------|------------------------|------|-------------------------------------|------------------|---------------------------------|
|            |                |                        |      |                                     |                  | West Center East                |
| 0          | 0              | 0                      | --   | 0                                   | 0                | 0 0 0                           |
| 1          | 13             | 14                     | 0.93 | 16                                  | 0.30             | +0.5 0 0                        |
| 2          | 26             | 26                     | 1.00 | 40                                  | 0.68             | 1.0 -0.5 +0.5                   |
| 3          | 39             | 39                     | 1.00 | 61                                  | 1.06             | 1.0 +0.5 0                      |
| 4          | 52             | 52                     | 1.00 | 131                                 | 1.43             | 2.0 +0.5 +1.0                   |
| 5          | 65             | 64                     | 1.02 | 301                                 | 1.81             | 2.5 +1.0 0                      |
| 6          | 77             | 77                     | 1.00 | 490                                 | 2.19             | 1.5 0 0                         |
| 7          | 90             | 90                     | 1.00 | 683                                 | 2.57             | +1.0 -1.0 -1.0                  |
| 8          | 102            | 102                    | 1.00 | 955                                 | 2.94             | 0 -2.5 -2.0                     |
| 9*         | 115            | 115                    | 1.00 | --                                  | 3.32             | -- -- --                        |
| 10**       | 96             | 115                    | 0.84 | 2466                                | 3.32             | -7.0 -12.5 -17.0                |

| Load Stage | Reinforcement Stresses (ksi) |       |      |      |      |      |      |       |
|------------|------------------------------|-------|------|------|------|------|------|-------|
|            | Gauge                        |       |      |      |      |      |      |       |
| 1          | 2                            | 3     | 4    | 5    | 6    | 7    | 8    |       |
| 0          | 0                            | 0     | 0    | 0    | 0    | 0    | 0    | 0     |
| 1          | 0                            | 0     | +0.1 | +0.2 | +0.3 | +0.1 | +0.1 | +0.3  |
| 2          | +2.1                         | -0.2  | 1.2  | 0.9  | 0.7  | 0.2  | 0.3  | 0.6   |
| 3          | -1.1                         | -0.2  | 2.3  | 1.8  | 1.2  | 0.8  | 0.8  | 1.9   |
| 4          | -0.3                         | +0.8  | 7.8  | 6.1  | 2.1  | 9.3  | 2.9  | 15.0  |
| 5          | +0.6                         | 24.9  | 13.0 | 11.1 | 7.0  | 26.4 | 4.9  | 30.3  |
| 6          | 11.2                         | 41.9  | 16.3 | 14.7 | 11.7 | 37.0 | 12.6 | 36.7  |
| 7          | 25.9                         | 52.7  | 20.0 | 16.7 | 16.9 | 43.8 | 12.9 | 42.3  |
| 8          | 36.3                         | Yield | 23.5 | 19.6 | 28.0 | 51.0 | 19.1 | 47.9  |
| 9*         | 43.6                         | Yield | 25.1 | 24.4 | --   | --   | --   | --    |
| 10**       | 46.2                         | Yield | 15.3 | 27.4 | 48.0 | 50.9 | 48.7 | Yield |

\*Failure

\*\*Past peak loads



TABLE B-33

## TEST RESULTS FOR BEAM 7-3

| Load Stage | Torque in.kips | Bending Moment in.kips | $\emptyset$ | Twist Radians per in. $\times 10^6$ | Shear Force kips | Deflection inches $\times 10^2$ |       |
|------------|----------------|------------------------|-------------|-------------------------------------|------------------|---------------------------------|-------|
|            |                |                        |             |                                     | West             | Center                          | East  |
| 0          | 0              | 0                      | --          | 0                                   | 0                | 0                               | 0     |
| 1          | 16             | 29                     | 0.55        | 18                                  | 0.74             | 0                               | +1.0  |
| 2          | 29             | 56                     | 0.52        | 45                                  | 1.56             | +1.0                            | 1.5   |
| 3          | 42             | 83                     | 0.51        | 86                                  | 2.38             | 2.5                             | 2.0   |
| 4          | 55             | 111                    | 0.50        | 173                                 | 3.20             | 4.5                             | 3.5   |
| 5          | 69             | 138                    | 0.50        | 304                                 | 4.01             | 5.5                             | 4.0   |
| 6          | 82             | 166                    | 0.49        | 520                                 | 4.83             | 6.5                             | 4.5   |
| 7          | 97             | 193                    | 0.50        | 701                                 | 5.65             | 8.0                             | 4.5   |
| 8          | 110            | 220                    | 0.50        | 933                                 | 6.47             | 8.5                             | 4.5   |
| 9          | 124            | 248                    | 0.50        | 1504                                | 7.29             | 8.5                             | 2.0   |
| 10*        | 132            | 275                    | 0.48        | --                                  | 8.11             | --                              | --    |
| 11**       | 105            | 275                    | 0.38        | 2477                                | 8.11             | 10.0                            | 2.0   |
|            |                |                        |             |                                     |                  |                                 | -15.5 |

| Load Stage | Reinforcement Stresses (ksi) |       |      |      |      |        |       |       |
|------------|------------------------------|-------|------|------|------|--------|-------|-------|
|            | Gauge                        |       |      |      |      |        |       |       |
| 1          | 2                            | 3     | 4    | 5    | 6    | 7      | 8     |       |
| 0          | 0                            | 0     | 0    | 0    | 0    | 0      | 0     | 0     |
| 1          | -0.7                         | -0.4  | +0.6 | +0.5 | +0.1 | 0      | +0.3  | +0.6  |
| 2          | -2.4                         | -1.1  | 2.2  | 1.5  | 0.6  | +0.1   | 0.6   | 1.8   |
| 3          | -4.5                         | -0.6  | 5.4  | 7.5  | 0.8  | 1.8    | 1.3   | 4.1   |
| 4          | -4.3                         | +1.3  | 10.5 | 14.5 | 2.1  | 11.1   | 10.8  | 4.7   |
| 5          | -3.6                         | 15.0  | 14.2 | 21.3 | 5.1  | 21.6   | 16.3  | 10.8  |
| 6          | -1.6                         | 30.5  | 18.0 | 25.6 | 10.1 | 31.5   | 19.2  | 43.3  |
| 7          | -0.6                         | 39.0  | 21.1 | 27.4 | 14.4 | 39.8   | 23.3  | Yield |
| 8          | +2.3                         | 51.6  | 24.6 | 30.0 | 22.4 | 49.5   | 29.8  | Yield |
| 9          | 8.5                          | Yield | 27.9 | 34.1 | 35.0 | Yield  | 46.3  | Yield |
| 10*        | --                           | --    | --   | 41.8 | --   | --     | --    | --    |
| 11**       | 7.5                          | Yield | 30.0 | 38.5 | 39.4 | (45.3) | Yield | --    |

\*Failure

\*\*Past peak loads



TABLE B-34

## TEST RESULTS FOR BEAM 7-4

| Load Stage | Torque in. kips | Bending Moment in. kips | $\emptyset$ | Twist Radians per in. $\times 10^6$ |               | Shear Force kips | Deflection inches $\times 10^2$ |        |      |
|------------|-----------------|-------------------------|-------------|-------------------------------------|---------------|------------------|---------------------------------|--------|------|
|            |                 |                         |             | 0                                   | $\times 10^6$ |                  | West                            | Center | East |
| 0          | 0               | 0                       | --          | 0                                   |               | 0                | 0                               | 0      | 0    |
| 1          | 13              | 50                      | 0.26        | 19                                  |               | 1.37             | +0.5                            | +1.0   | +0.5 |
| 2          | 25              | 100                     | 0.25        | 40                                  |               | 2.88             | 3.5                             | 2.5    | 1.5  |
| 3          | 38              | 151                     | 0.25        | 88                                  |               | 4.39             | 6.0                             | 5.0    | 2.5  |
| 4          | 50              | 201                     | 0.25        | 162                                 |               | 5.90             | 9.0                             | 7.0    | 4.0  |
| 5          | 62              | 252                     | 0.25        | 240                                 |               | 7.41             | 13.0                            | 10.0   | 4.0  |
| 6          | 76              | 303                     | 0.25        | 360                                 |               | 8.93             | 15.5                            | 11.5   | 6.5  |
| 7          | 88              | 353                     | 0.25        | 496                                 |               | 10.44            | 18.5                            | 13.5   | 7.5  |
| 8          | 101             | 404                     | 0.25        | 661                                 |               | 11.95            | 21.5                            | 15.5   | 8.5  |
| 9          | 114             | 455                     | 0.25        | 870                                 |               | 13.46            | 24.5                            | 17.5   | 8.5  |
| 10*        | 125             | 505                     | 0.25        | --                                  |               | 14.97            | --                              | --     | --   |
| 11**       | 103             | 505                     | 0.20        | 2653                                |               | 14.97            | 54.0                            | 39.5   | 19.0 |

| Load Stage | Reinforcement Stresses (ksi) |      |       |       |      |      |       |       |
|------------|------------------------------|------|-------|-------|------|------|-------|-------|
|            | Gauge                        |      |       |       |      |      |       |       |
| 1          | 2                            | 3    | 4     | 5     | 6    | 7    | 8     |       |
| 0          | 0                            | 0    | 0     | 0     | 0    | 0    | 0     | 0     |
| 1          | -2.1                         | -1.2 | +1.9  | +1.5  | +0.4 | 0    | +0.7  | +0.6  |
| 2          | -5.7                         | -3.0 | 5.1   | 3.1   | 0.3  | -0.3 | 1.2   | 2.0   |
| 3          | -9.9                         | -3.8 | 9.3   | 10.1  | 0.7  | -1.1 | 2.1   | 5.6   |
| 4          | -13.8                        | -3.1 | 14.0  | 18.5  | 2.8  | -2.3 | 4.1   | 11.2  |
| 5          | -17.1                        | +3.4 | 18.3  | 26.5  | 5.1  | -1.1 | 9.1   | 17.2  |
| 6          | -20.4                        | 11.8 | 22.7  | 33.6  | 8.0  | +4.0 | 14.9  | 24.4  |
| 7          | -23.5                        | 18.6 | 27.7  | 39.0  | 12.5 | 10.6 | 23.7  | Yield |
| 8          | -26.7                        | 27.4 | 32.7  | Yield | 17.4 | 21.0 | 29.9  | Yield |
| 9          | -28.9                        | 36.0 | 37.5  | Yield | 23.5 | 32.3 | 37.0  | Yield |
| 10*        | --                           | 39.7 | --    | --    | --   | --   | --    | --    |
| 11**       | -26.3                        | 20.1 | Yield | Yield | 51.7 | 33.1 | Yield | --    |

\*Failure

\*\*Past peak loads



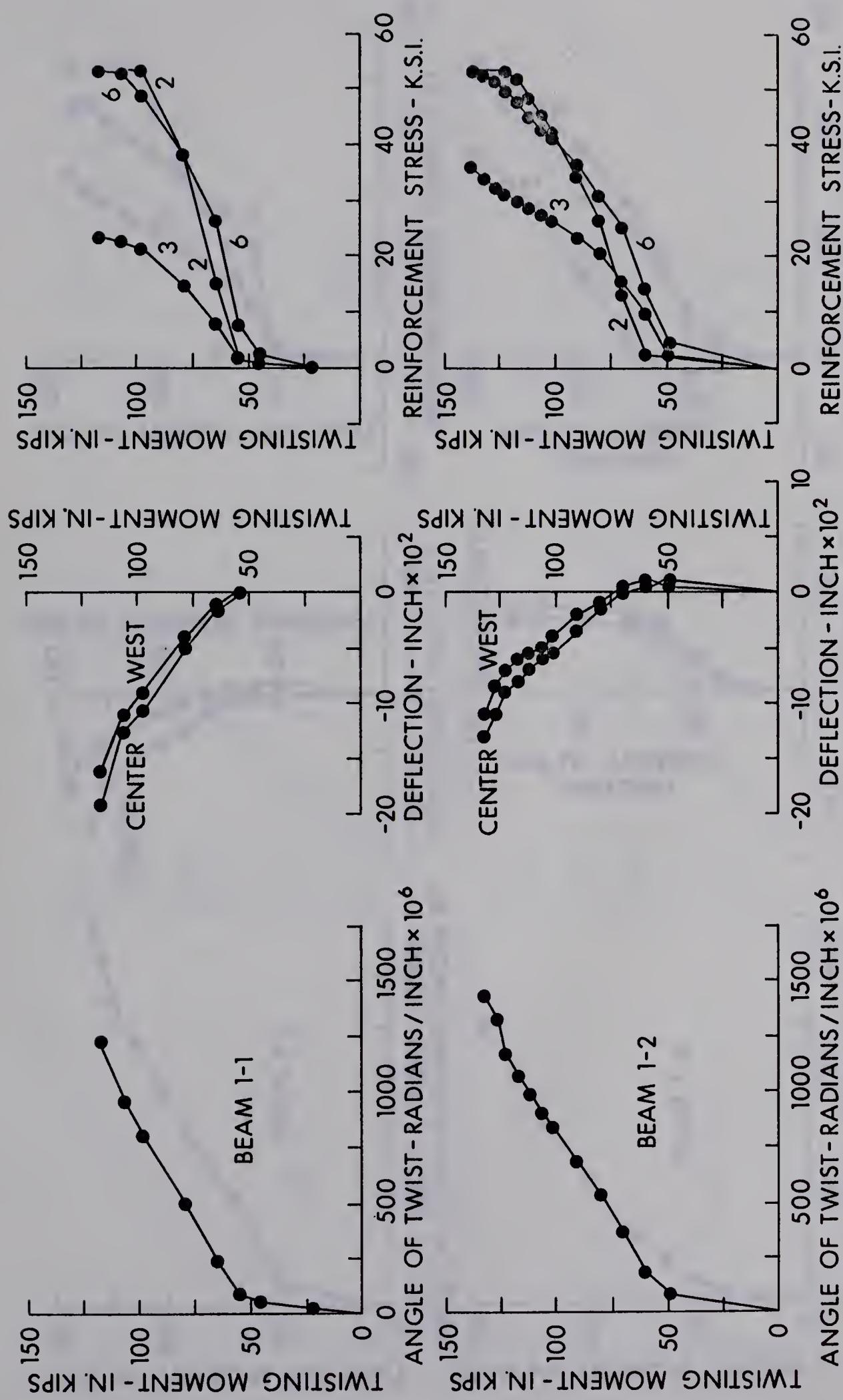


FIGURE B-1 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 1-1 AND 1-2



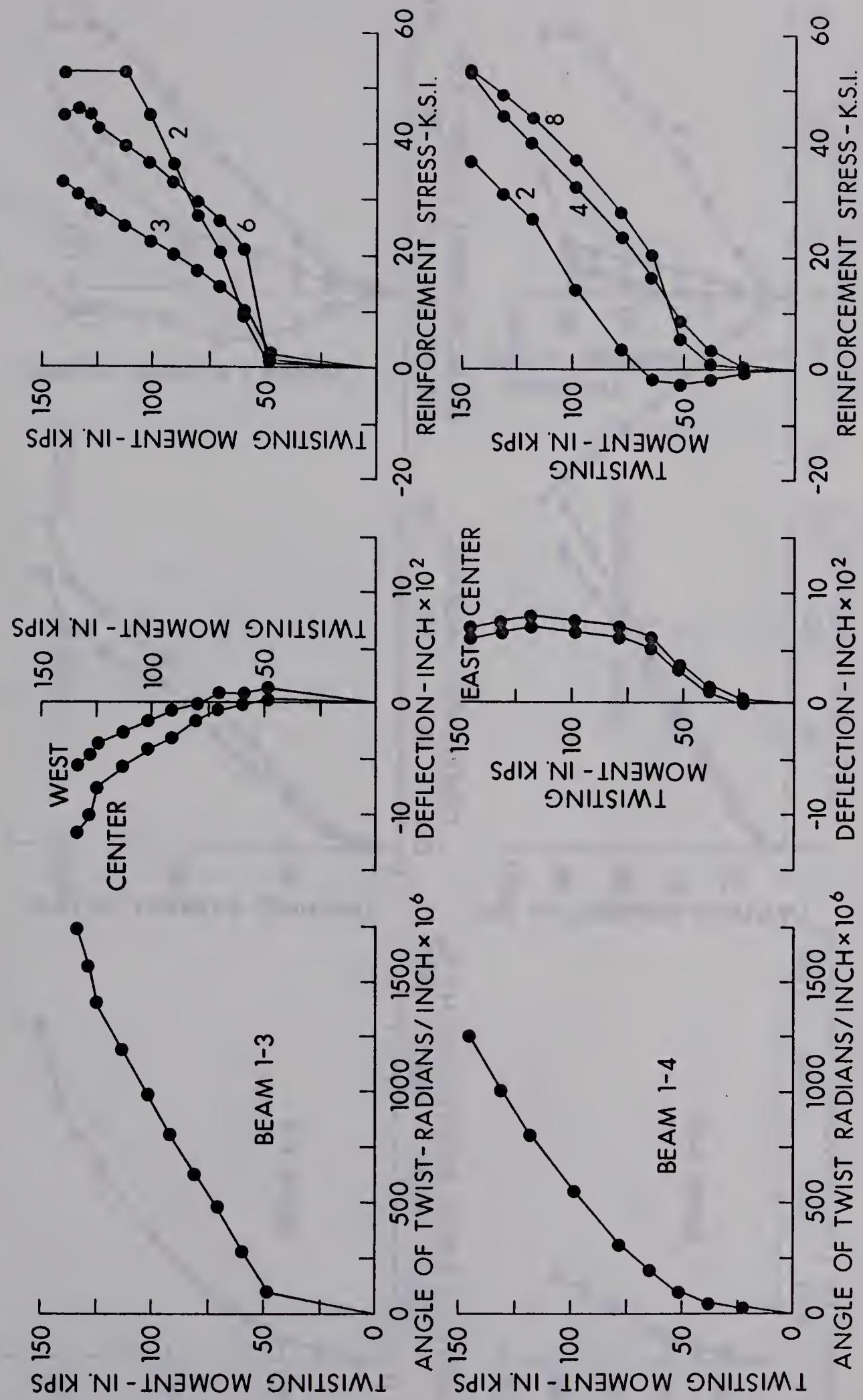


FIGURE B-2 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 1-3 AND 1-4



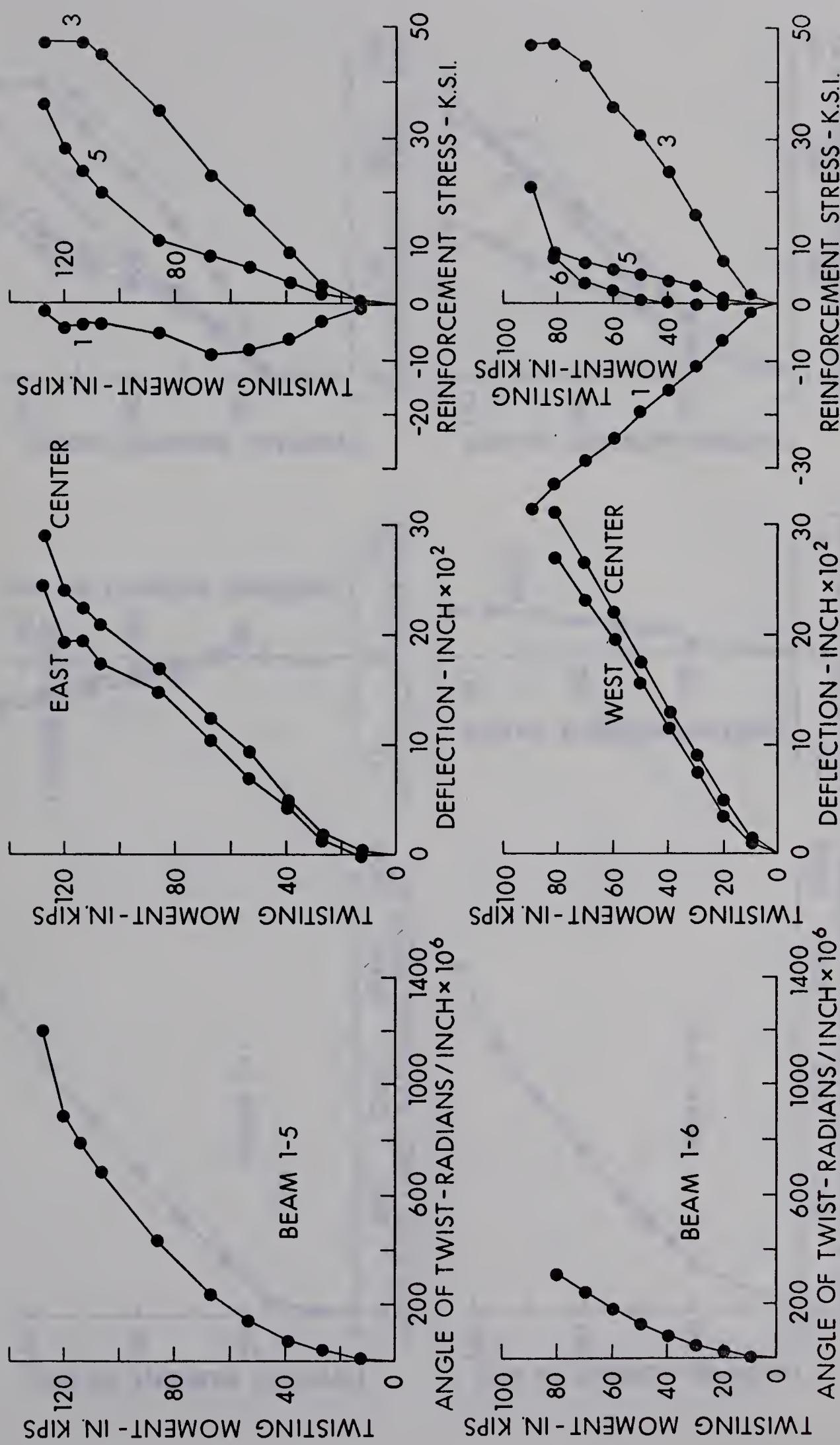


FIGURE B-3 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 1 - 5 AND 1 - 6



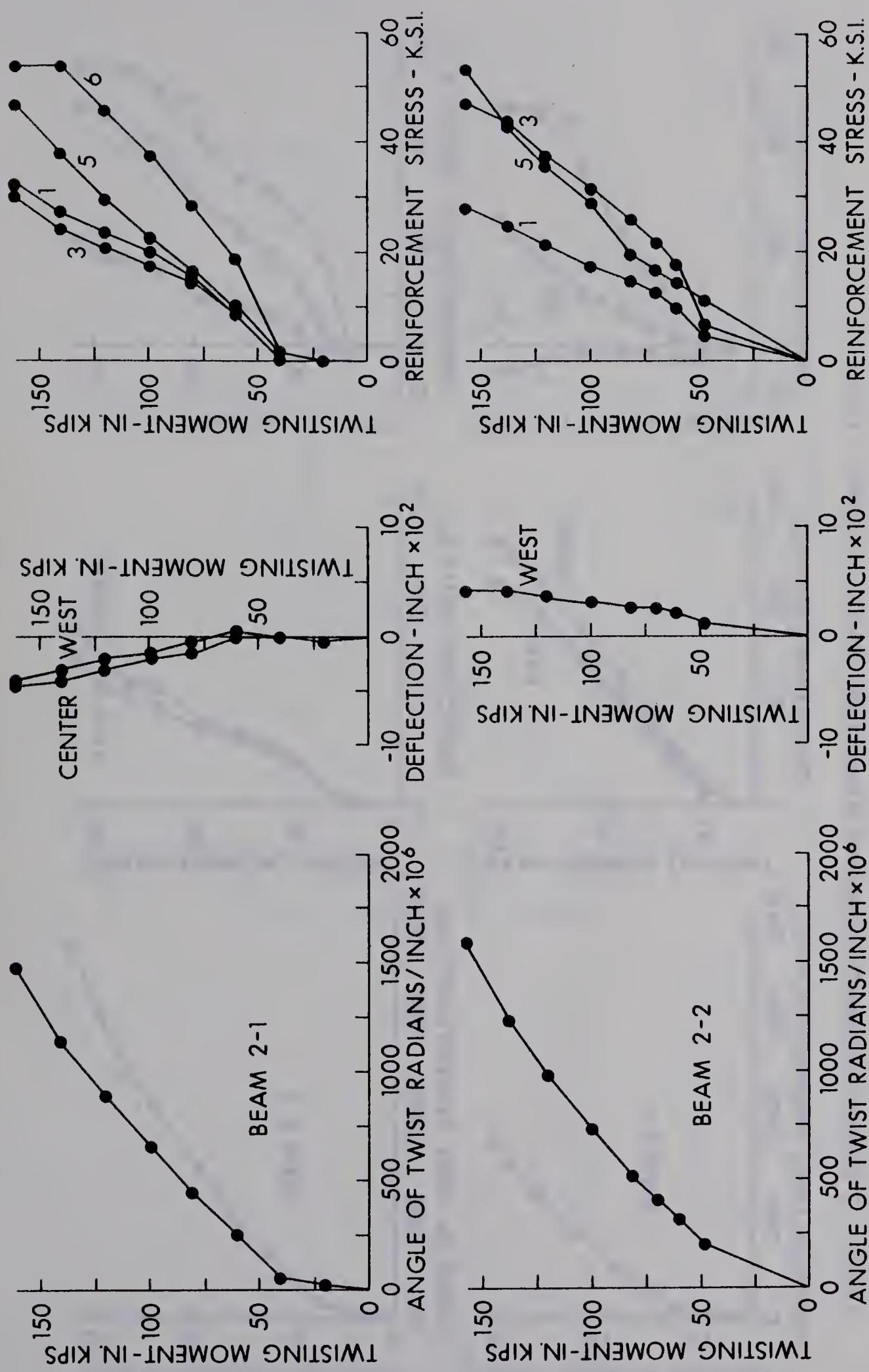


FIGURE B-4 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 2-1 AND 2-2



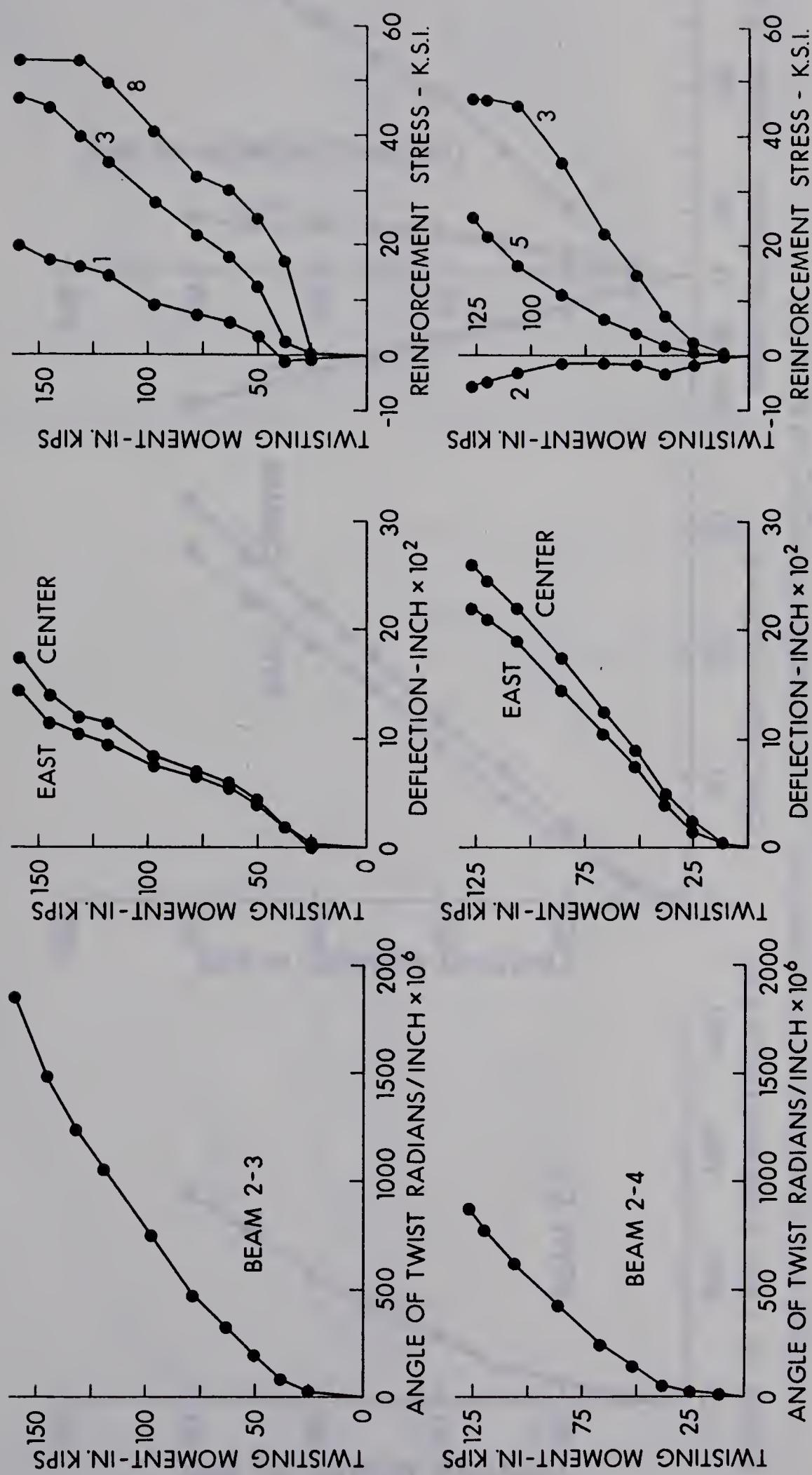


FIGURE B - 5 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 2 - 3 AND 2 - 4



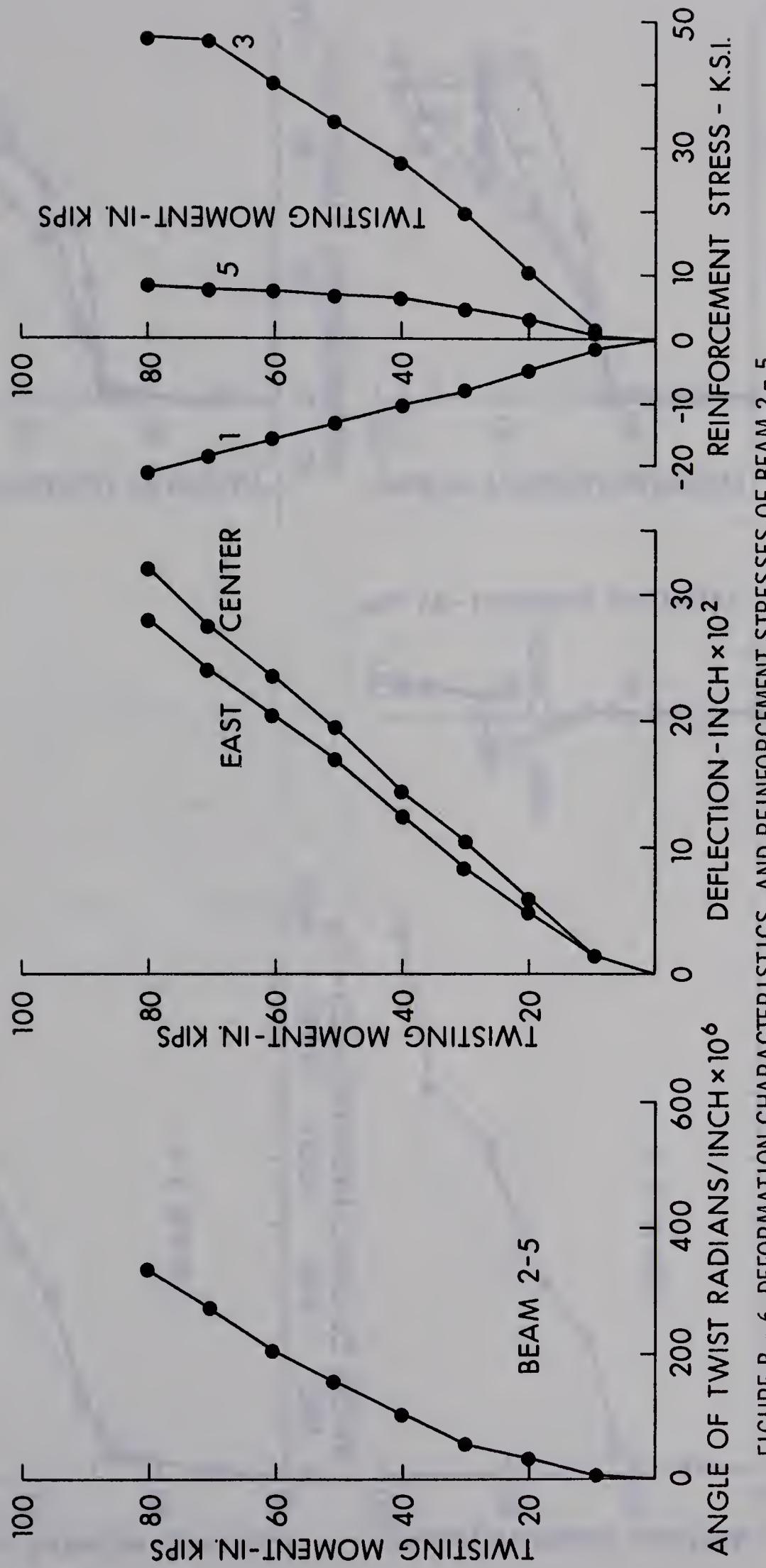


FIGURE B - 6 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAM 2 - 5



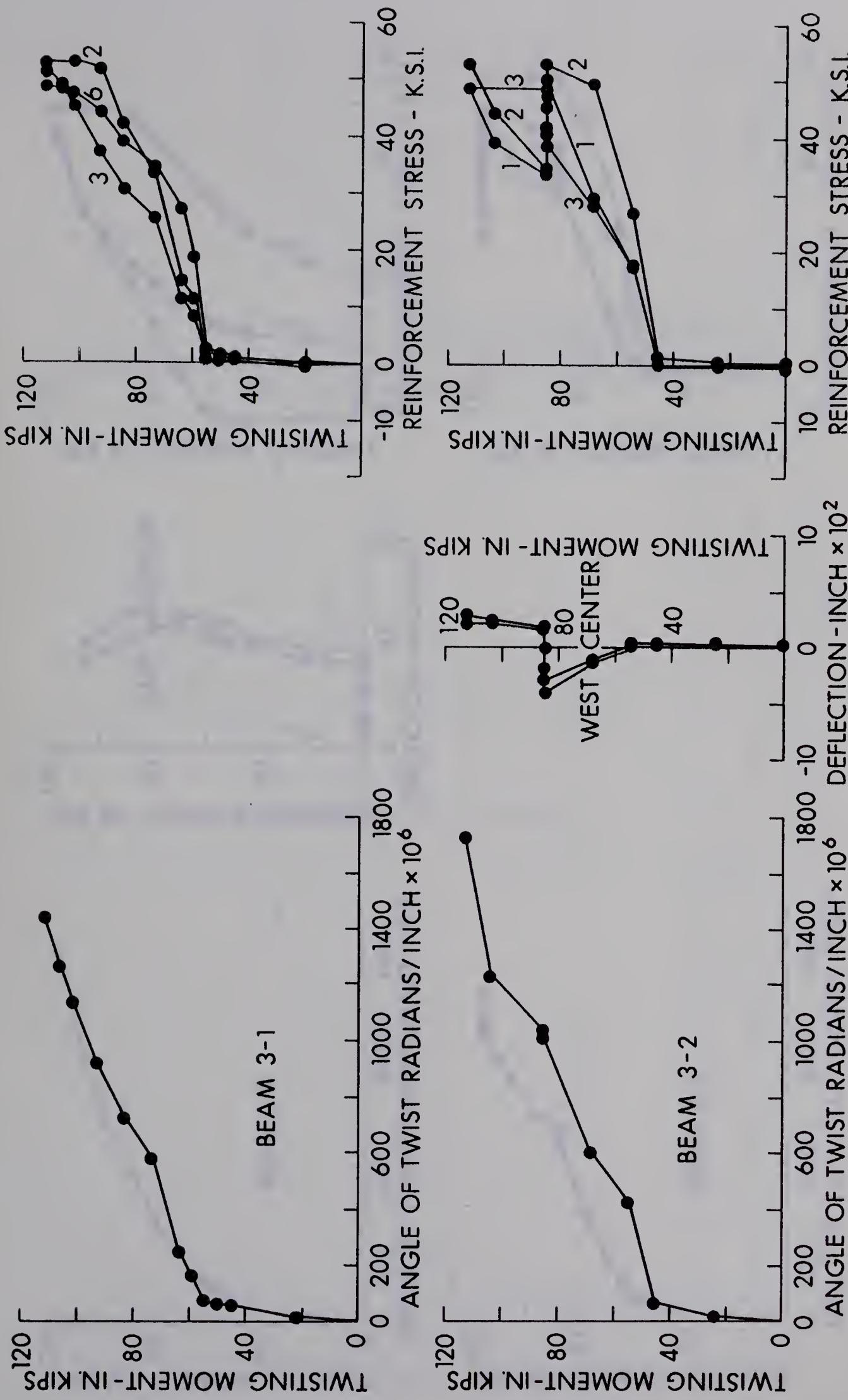
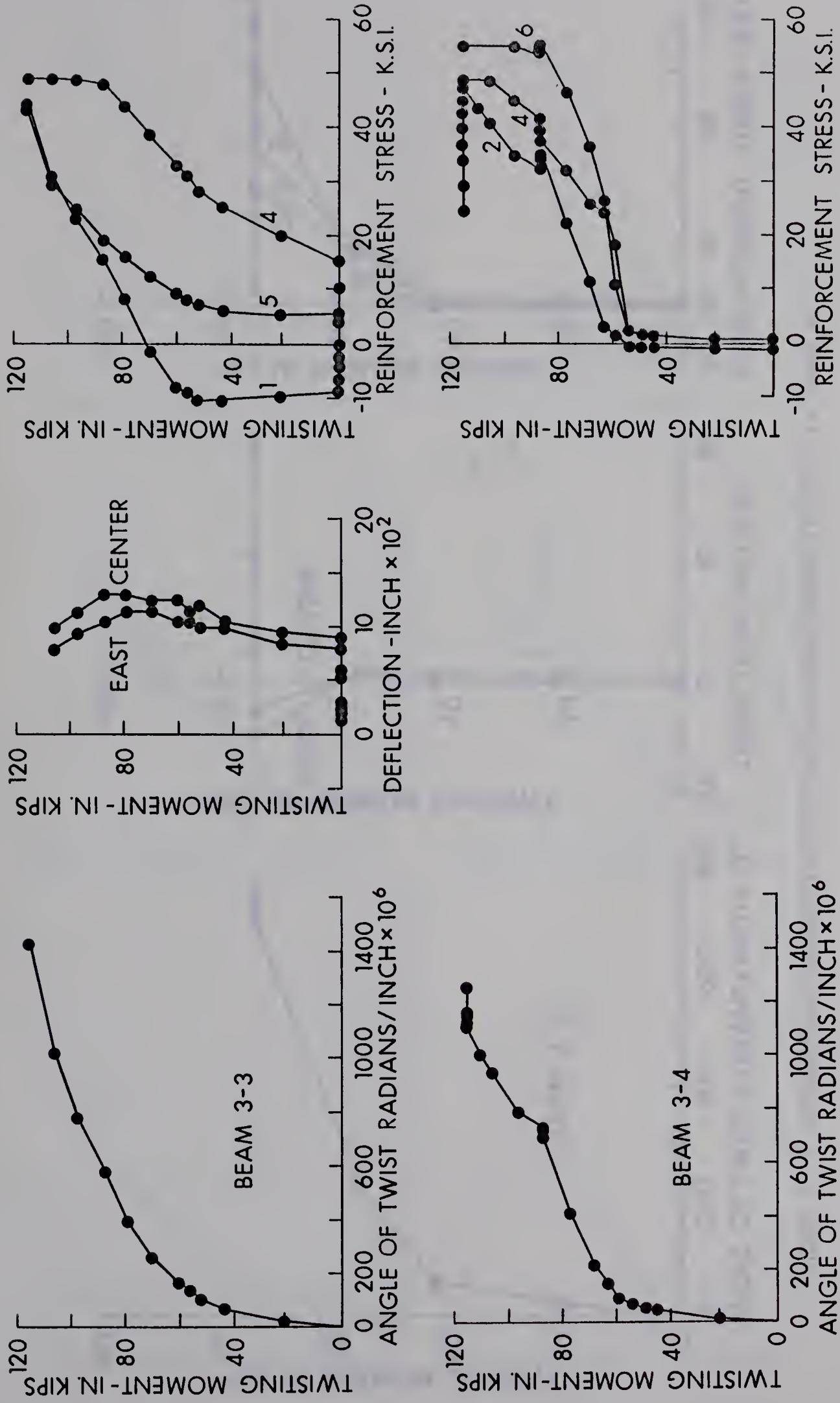


FIGURE B-7 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 3 - 1 AND 3 - 2.



FIGURE B-8 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 3 - 3 AND 3 - 4





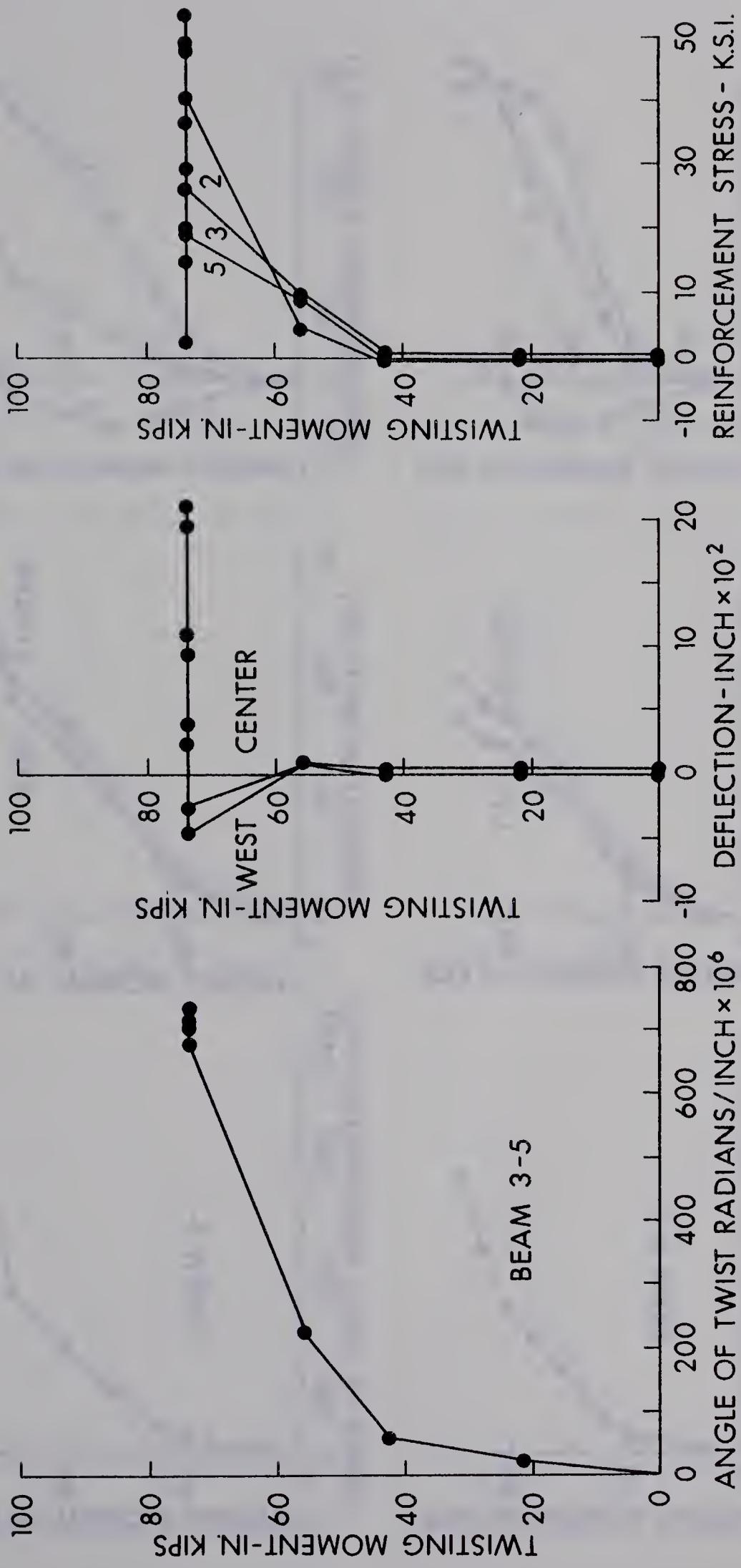


FIGURE B - 9 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAM 3 - 5



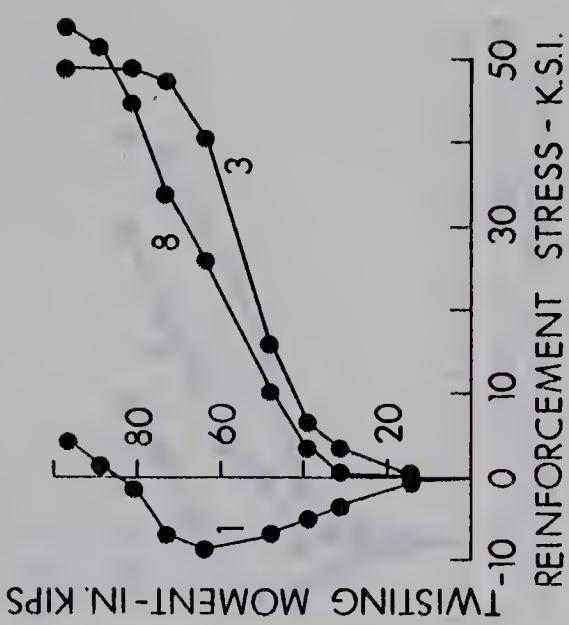
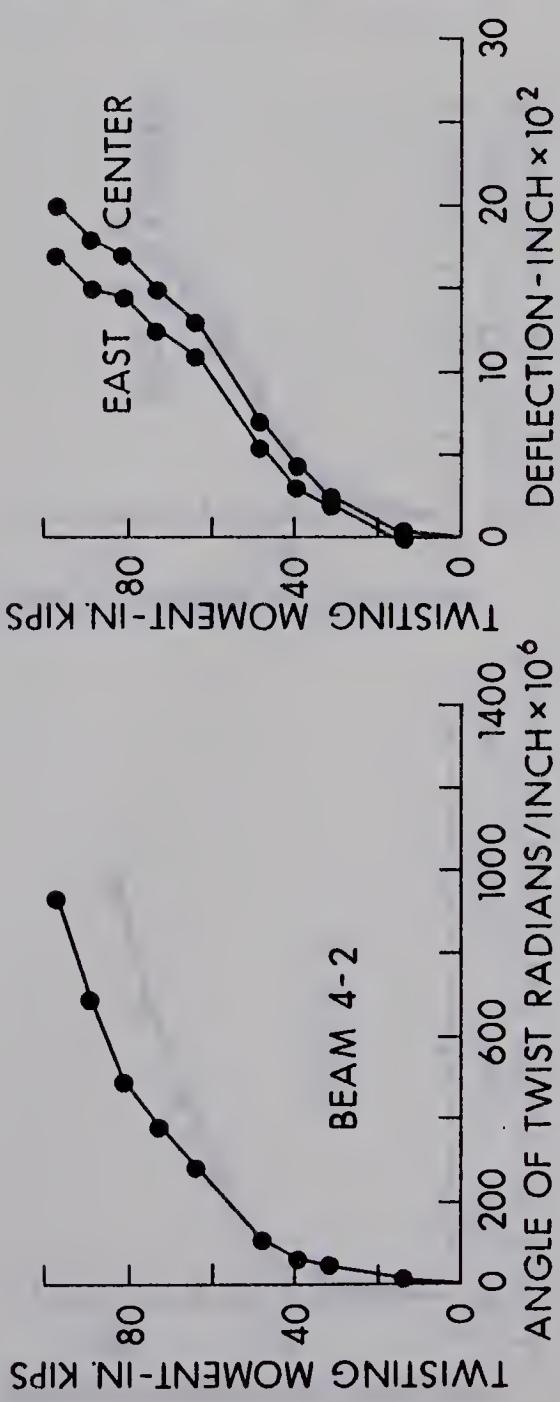
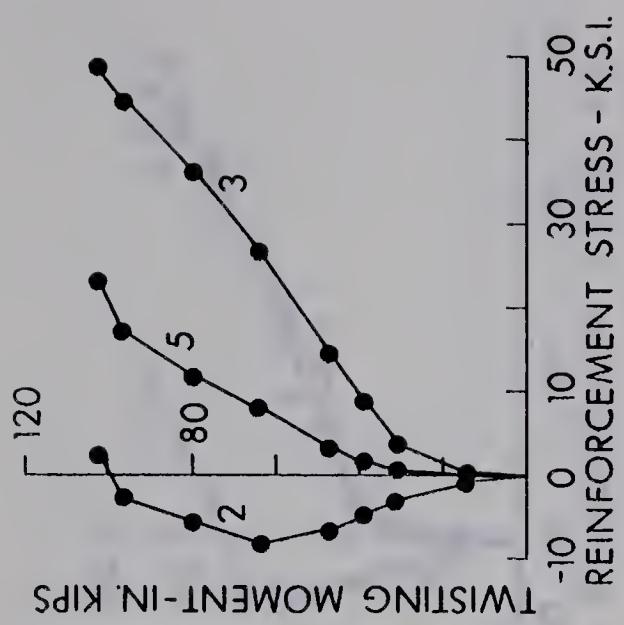
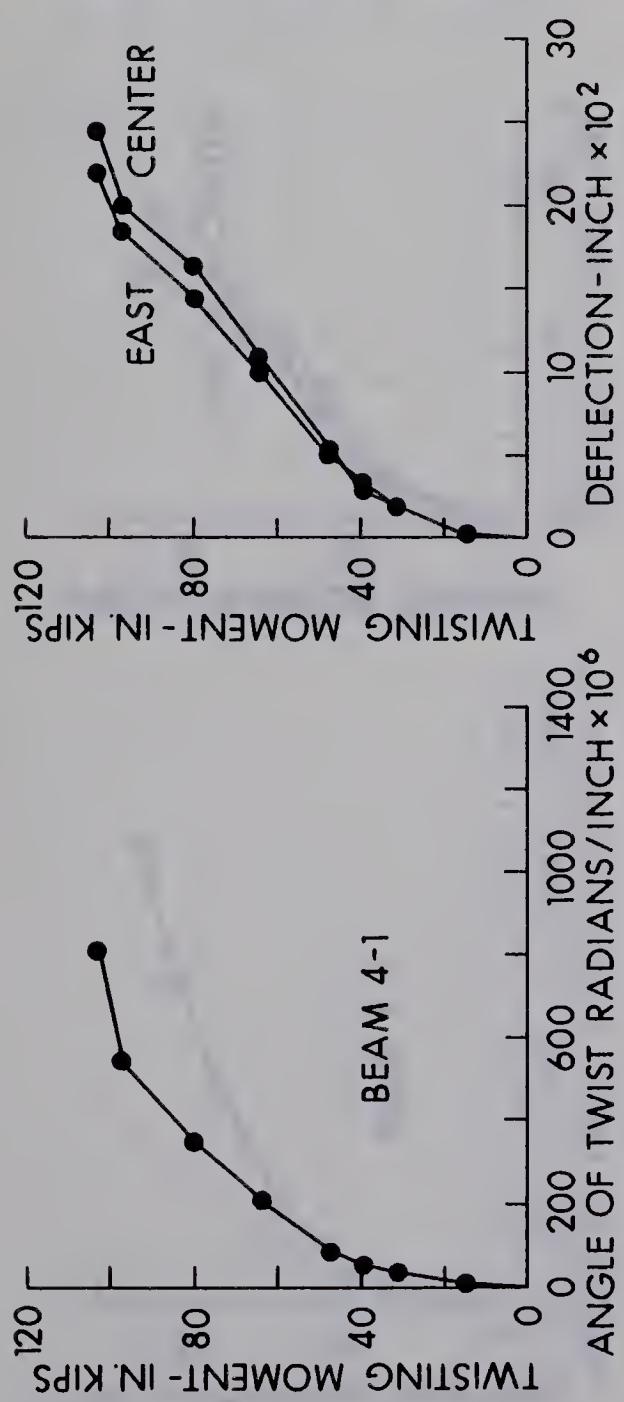


FIGURE B - 10 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 4 - 1 AND 4 - 2



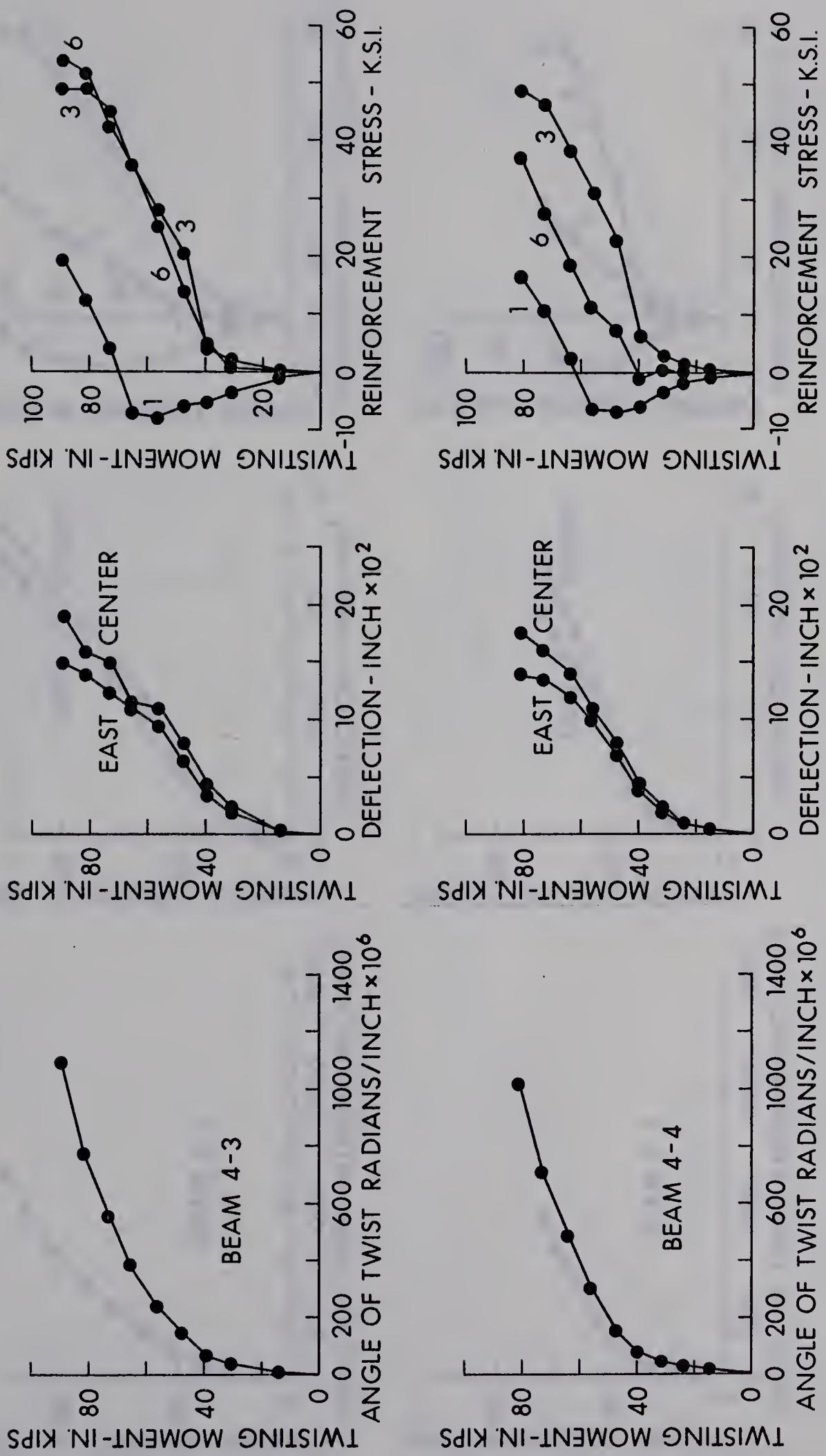


FIGURE B - 11 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 4 - 3 AND 4 - 4



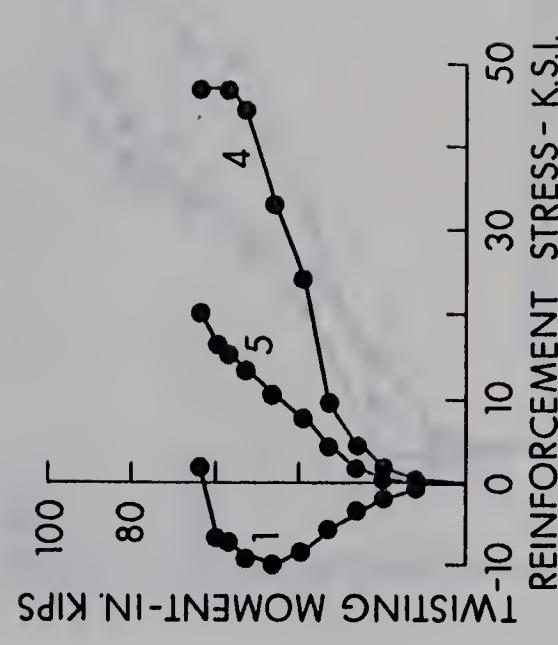
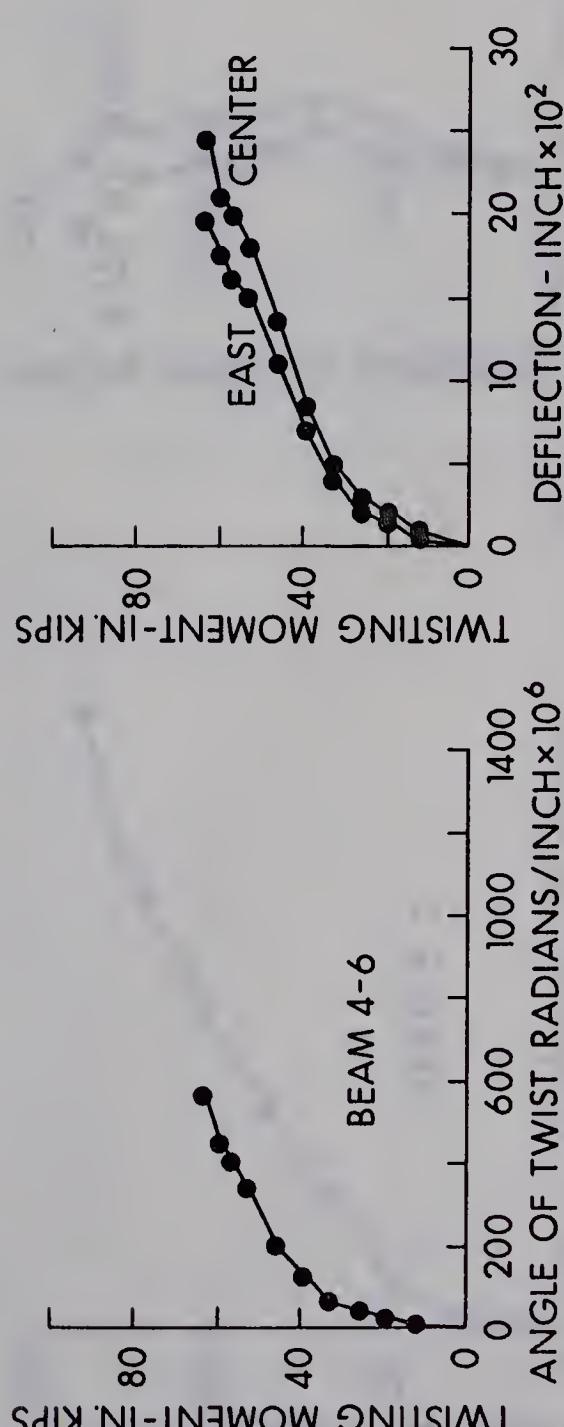
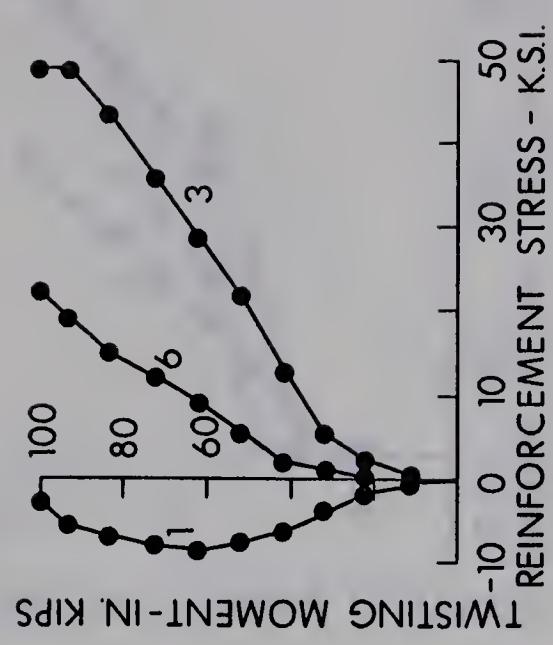
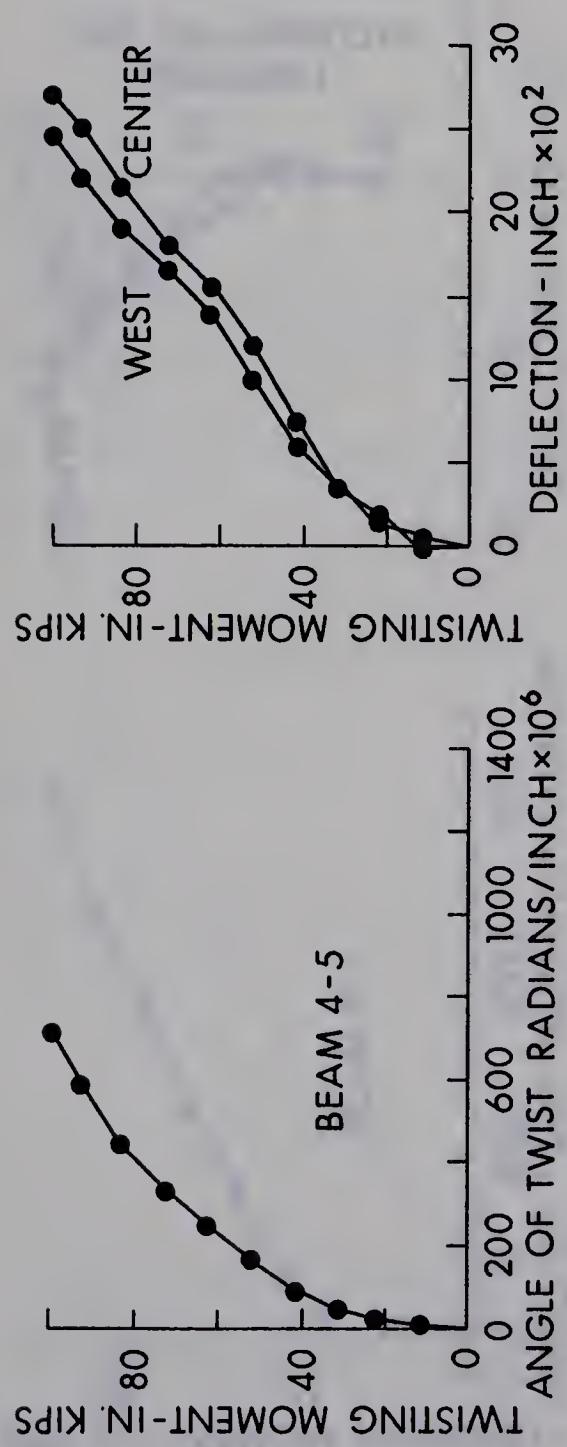


FIGURE B-12 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 4-5 AND 4-6



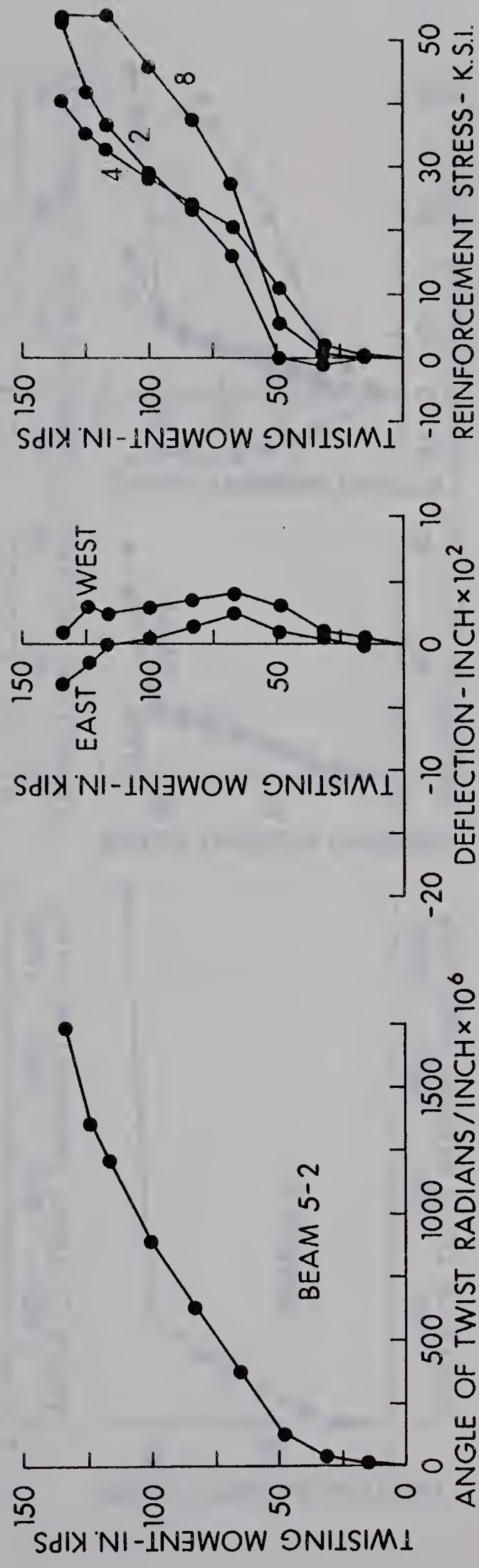
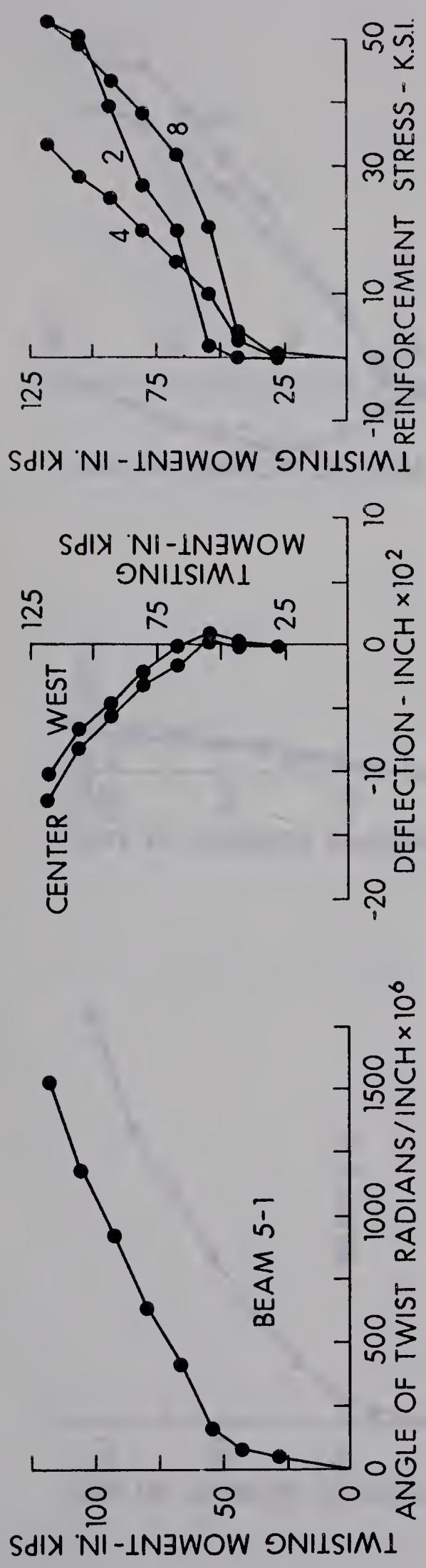
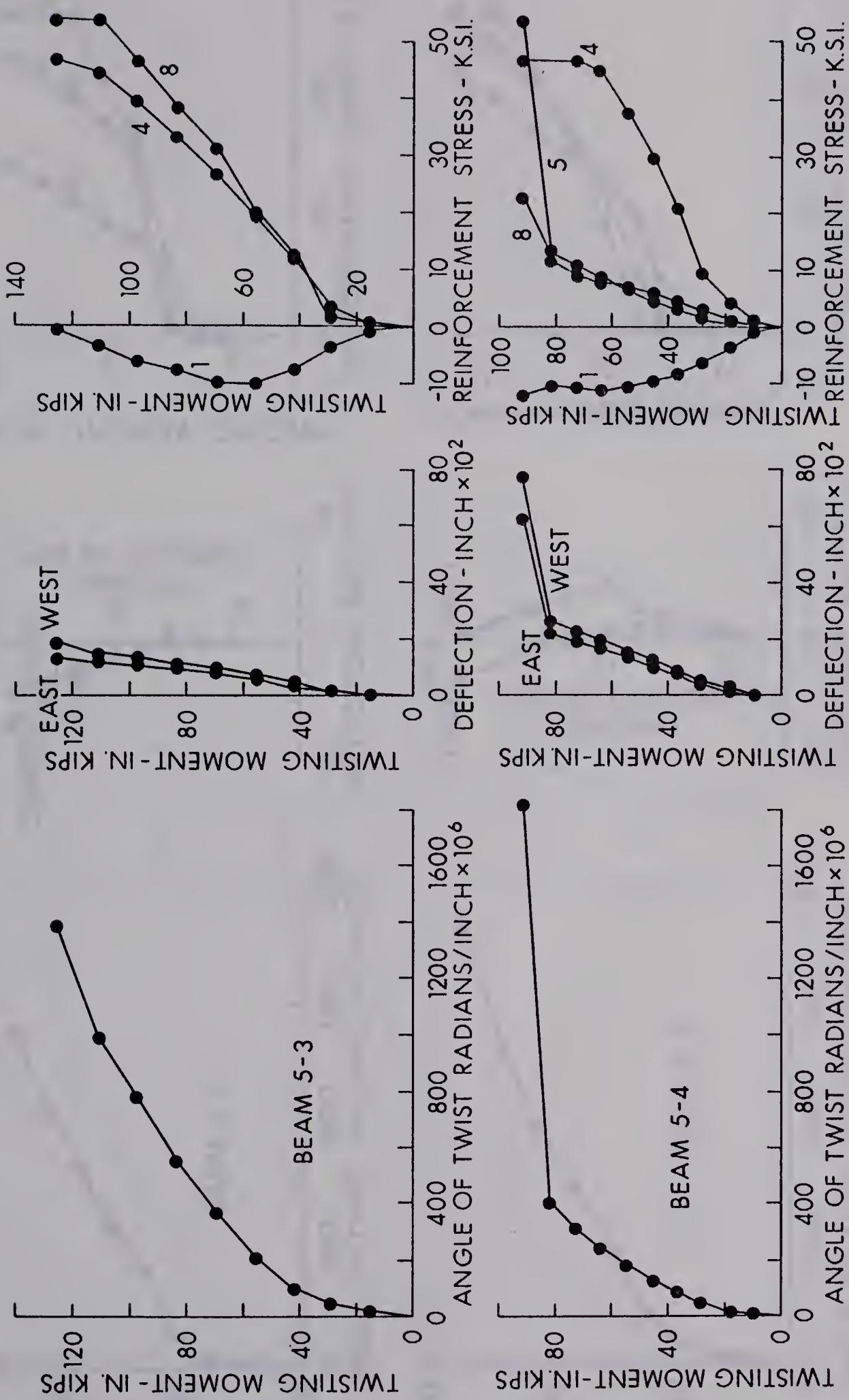


FIGURE B - 13 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 5 - 1 AND 5 - 2







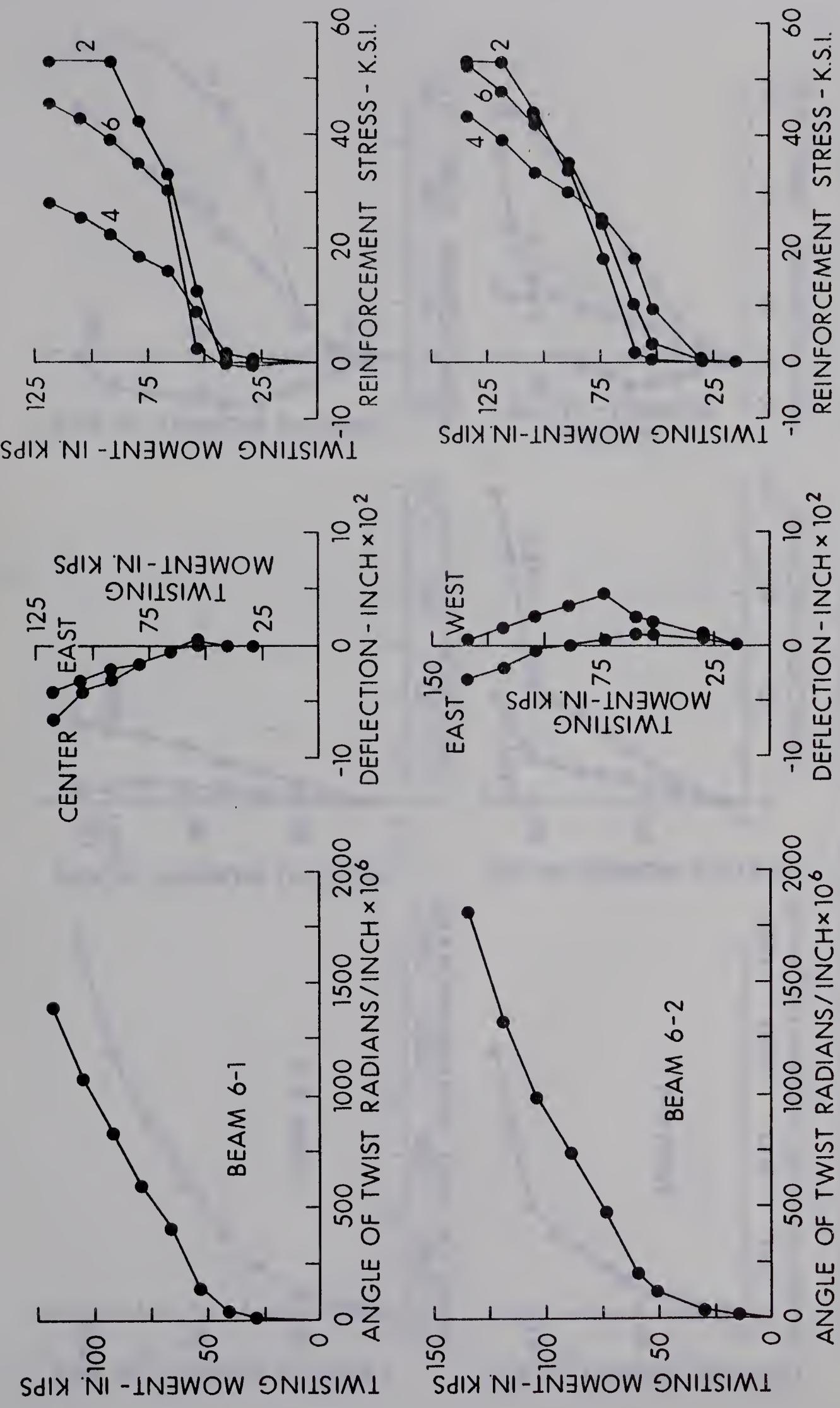


FIGURE B - 15 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 6 - 1 AND 6 - 2



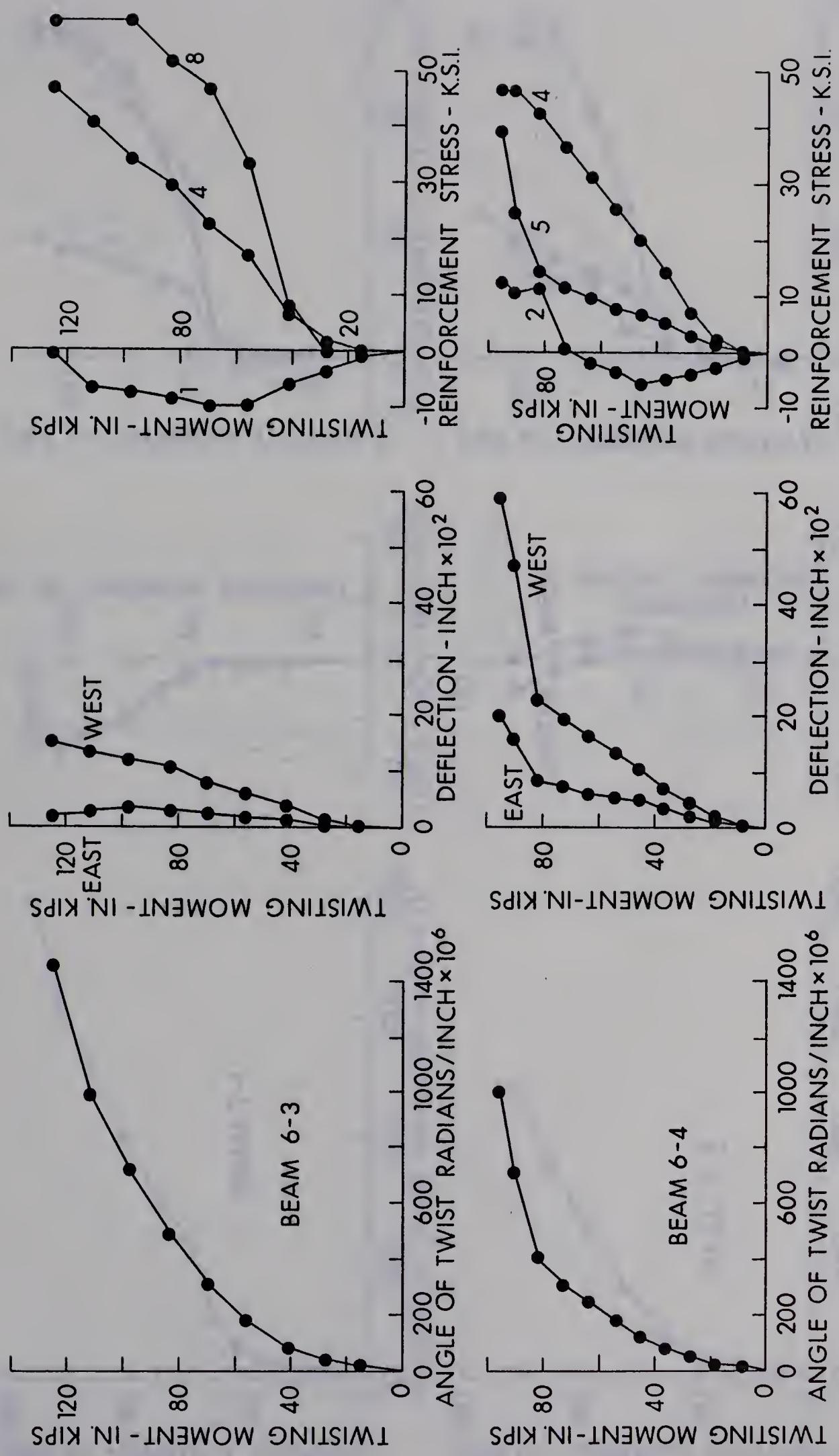


FIGURE B-16 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 6-3 AND 6-4



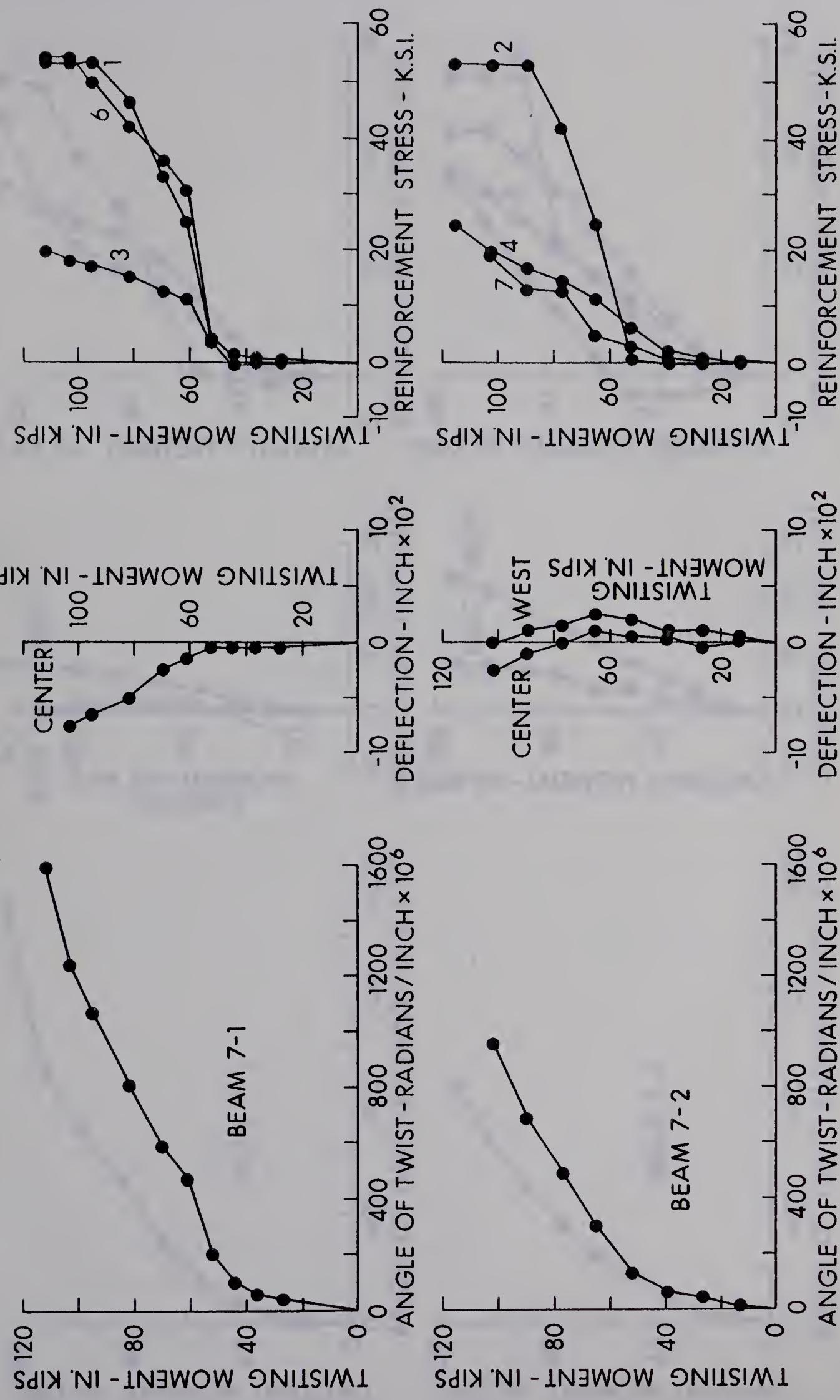


FIGURE B-17 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 7-1 AND 7-2



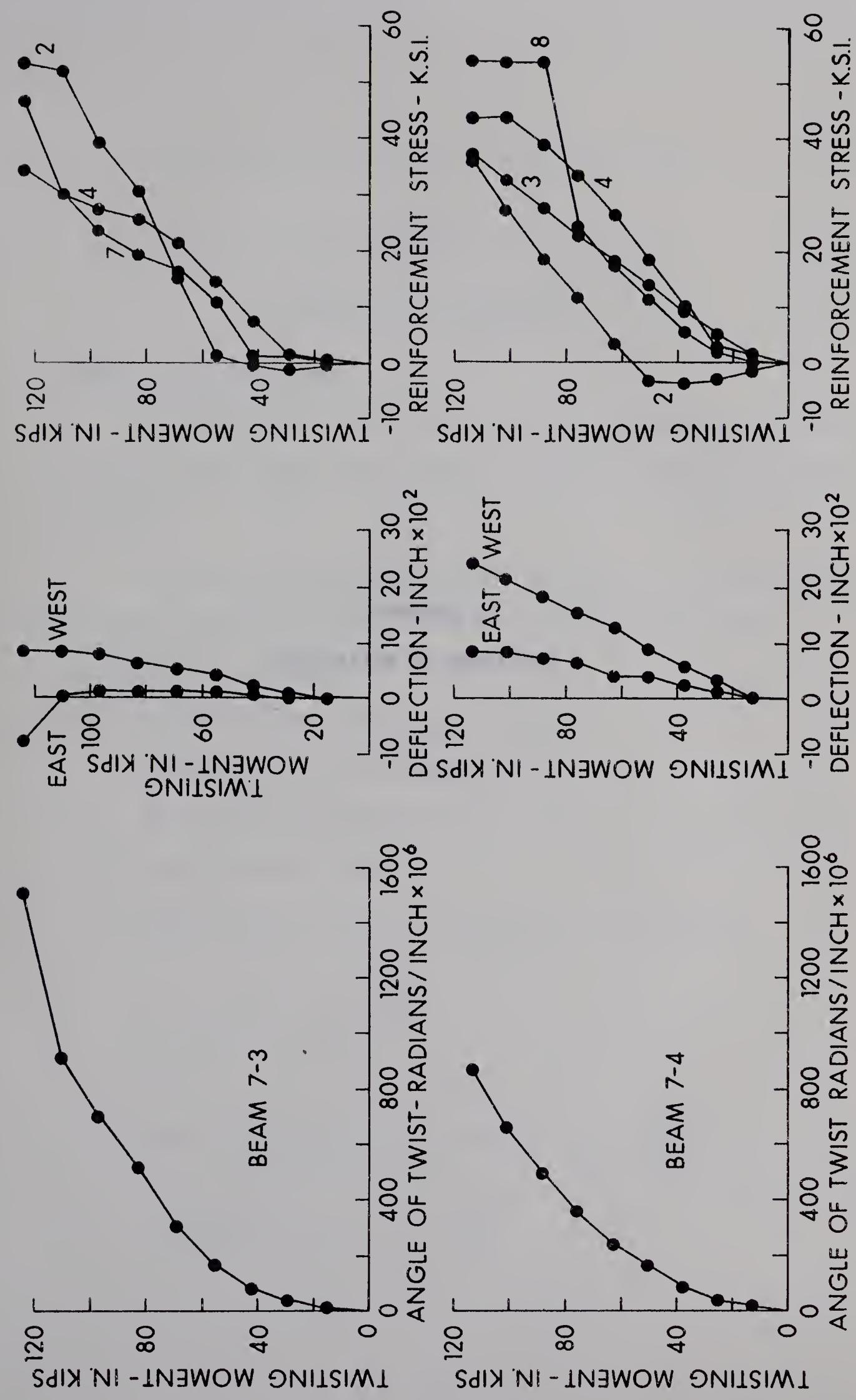


FIGURE B-18 DEFORMATION CHARACTERISTICS AND REINFORCEMENT STRESSES OF BEAMS 7-3 AND 7-4



## APPENDIX C

## DERIVATION OF EQUATIONS



## APPENDIX C

### DERIVATION OF EQUATIONS

#### C-1 Mode 1 Failure Surface

The assumptions referred to below are presented in SECTION 5-3 of CHAPTER V. The symbols are defined in SECTION 5-4(a) and FIGURE 5-1 of CHAPTER V.

Moments are taken about an axis parallel to the neutral axis and located a distance  $k_1 x_1 / 2$  from the top face of the beam.

The external moments are:

$$\text{Component of bending moment} = M_{b1} b / L_1$$

$$\text{Component of twisting moment} = M_{t1} c_1 / L_1$$

$$\text{Moment due to transverse shear} = 0$$

The internal moments are:

Moment due to force in longitudinal reinforcement

$$= A_{s1} f_{y1} \frac{b}{L_1} (h_{01} - \frac{k_1 x_1}{2})$$

Moment due to force in horizontal legs of ties

$$= \frac{k_{01} c_1^2 a_v f_{yt}}{S L_1} (h - a_{1t} - \frac{k_1 x_1}{2})$$



Moment due to force in vertical legs of ties

$$= \frac{a_v f_{yt} (1 - k_{01}) c_1}{s} \left[ \frac{(1 - k_{01})b}{4} - a_{2t} \right] \frac{c_1}{L_1}$$

Equating external and internal moments gives:

$$\frac{M_{b1} b}{L_1} + \frac{M_{t1} c_1}{L_1} = \frac{A_{s1} f_{yl} b}{L_1} (h_{01} - \frac{k_1 x_1}{2}) + \frac{k_{01} c_1^2 a_v f_{yt}}{s L_1} (h - a_{1t} - \frac{k_1 x_1}{2})$$

$$+ \frac{a_v f_{yt} (1 - k_{01}) c_1^2}{s L_1} \left[ \frac{(1 - k_{01})b}{4} - a_{2t} \right]$$

$$\frac{M_{b1} b}{L_1} + \frac{M_{t1} c_1}{L_1} = A_{s1} f_{yl} b (h_{01} - \frac{k_1 x_1}{2})$$

$$+ \frac{a_v f_{yt} c_1^2}{s} \left[ k_{01} (h - a_{1t} - \frac{k_1 x_1}{2}) + (1 - k_{01}) \left[ \frac{(1 - k_{01})b}{4} - a_{2t} \right] \right]$$

$$\text{Let } \emptyset = \frac{M_{t1}}{M_{b1}} \text{ and } p_1 = \frac{f_{yt} a_v h}{f_{yl} A_{s1} s}$$

$$\frac{M_{t1}}{M_{b1}} \left[ \frac{c_1}{b} + \frac{1}{\emptyset} \right] = A_{s1} f_{yl} (h_{01} - \frac{k_1 x_1}{2})$$

$$+ \frac{p_1 A_{s1} f_{yl} c_1^2}{b h} \left[ k_{01} (h - a_{1t} - \frac{k_1 x_1}{2}) + (1 - k_{01}) \frac{b}{4} (1 - k_{01} - \frac{4 a_{2t}}{b}) \right]$$



$$M_{t1} \left[ \frac{c_1}{b} + \frac{1}{\emptyset} \right] = A_{s1} f_{y1} \left[ (h_{01} - \frac{k_1 x_1}{2}) + \frac{p_1 c_1^2}{bh} \left[ k_{01}(h - a_{1t} - \frac{k_1 x_1}{2}) \right. \right. \\ \left. \left. + \frac{b}{4} (1 - k_{01})(1 - k_{01} - \frac{4a_{2t}}{b}) \right] \right]$$

$$\text{Let } z_1 = h_{01} - k_1 x_1 / 2$$

$$\text{and } y_1 = k_{01}(h - a_{1t} - \frac{k_1 x_1}{2}) + \frac{b}{4}(1 - k_{01})(1 - k_{01} - \frac{4a_{2t}}{b})$$

$$M_{t1} = A_{s1} f_{y1} \left[ \frac{z_1 + \frac{p_1 y_1 c_1^2}{bh}}{\frac{c_1}{b} + \frac{1}{\emptyset}} \right] \quad (5-1)$$

The sum of the forces acting perpendicular to a plane that contains the neutral axis and is perpendicular to the top face of the beam must equal zero. These forces are:

$$\text{Force in longitudinal reinforcement} = \frac{A_{s1} f_{y1} b}{L_1}$$

$$\text{Force in horizontal legs of ties} = \frac{a_v f_{yt} k_{01} c_1^2}{S L_1}$$

$$\text{Force in concrete compression zone} = 0.85 k_1 x_1 L_1 f'_c$$

Summing of forces gives:

$$\frac{A_{s1} f_{y1} b}{L_1} + \frac{a_v f_{yt} k_{01} c_1^2}{S L_1} - 0.85 k_1 x_1 L_1 f'_c = 0$$



$$x_1 = \frac{A_{s1} f_y_1 (b + p_1 k_{01} c_1^2 / h)}{0.85 k_1 f'_c L_1^2} \quad (5-2)$$

Using assumption two and neglecting the depth of the compression zone:

$$k_{01} = \frac{b}{2h + b}$$

In order to solve Equations (5-1) and (5-2) the value of  $c_1$  must be determined. The value of this parameter that should be used is the one which gives the minimum value of twisting moment. The value of  $x_1$ , which depends on  $c_1$ , has a relatively small effect on  $z_1$  and  $y_1$ . Therefore, in the following operation it is assumed that  $z_1$  and  $y_1$  are not functions of  $x_1$ .

Differentiating Equation (5-1) with respect to  $c_1$ , equating the first derivative to zero and solving for  $c_1$  yields:

$$c_1 = -\frac{b}{\phi} \pm b \left[ \frac{1}{\phi^2} + \frac{z_1 h}{p_1 y_1 b} \right]^{1/2} \quad (5-3)$$

The positive sign for the second term is used since negative values of  $c_1$  have no physical significance. Similarly, the minimum value that  $c_1$  can have is zero. The maximum value that  $c_1$  can have is obtained by neglecting the depth of the compression zone and using assumption two. The resulting equation is:



$$c_{m1} = 2h + b \quad (5-4)$$

### C-2 Mode 2 Failure Surface

The assumptions referred to below are presented in SECTION 5-3 of CHAPTER V. The symbols are defined in SECTION 5-4(b) and FIGURE 5-2 of CHAPTER V.

Moments are taken about an axis parallel to the neutral axis and located a distance  $k_1 x_2 / 2$  from the vertical face of the beam that is adjacent to the compression zone.

The external moments are:

Component of bending moment = 0

$$\text{Component of twisting moment} = \frac{M_{t2} c_2}{L_2}$$

$$\text{Moment due to transverse shear} = V \left[ \frac{b}{2} - \frac{k_1 x_2}{2} \right] \frac{c_2}{L_2}$$

The internal moments are:

Moment due to force in longitudinal reinforcement

$$= A_{s2} f_{y2} \frac{h}{L_2} \left( b_{02} - \frac{k_1 x_2}{2} \right)$$

Moment due to force in vertical legs of ties

$$= \frac{a_v f_{yt} k_{02} c_2^2}{S L_2} \left( b - a_{2t} - \frac{k_1 x_2}{2} \right)$$



Moment due to force in horizontal legs of ties

$$= \frac{a_v f_{yt} (1 - k_{02}) c_2}{s} \left[ \frac{(1 - k_{02}) h}{4} - a_{1t} \right] \frac{c_2}{L_2}$$

Equating external and internal moments yields:

$$\frac{M_{t2} c_2}{L_2} + \frac{V c_2}{L_2} \left( \frac{b}{2} - \frac{k_1 x_2}{2} \right) = \frac{A_{s2} f_{y2} h}{L_2} \left( b_{02} - \frac{k_1 x_2}{2} \right)$$

$$+ \frac{a_v f_{yt} k_{02} c_2^2}{s L_2} \left( b - a_{2t} - \frac{k_1 x_2}{2} \right)$$

$$+ \frac{a_v f_{yt} (1 - k_{02}) c_2^2}{s L_2} \left[ \frac{(1 - k_{02}) h}{4} - a_{1t} \right]$$

$$M_{t2} c_2 + V c_2 \left( \frac{b}{2} - \frac{k_1 x_2}{2} \right) = A_{s2} f_{y2} h \left( b_{02} - \frac{k_1 x_2}{2} \right)$$

$$+ \frac{a_v f_{yt} c_2^2}{s} \left[ k_{02} \left( b - a_{2t} - \frac{k_1 x_2}{2} \right) + (1 - k_{02}) \frac{h}{4} \left( 1 - k_{02} - \frac{4a_{1t}}{h} \right) \right]$$

$$\text{Let } \delta = \frac{Vb}{2M_{t2}} ; p_2 = \frac{f_{yt} a_v b}{f_{y2} A_{s2} s} ; z_2 = b_{02} - \frac{k_1 x_2}{2} ;$$

$$y_2 = k_{02} \left( b - a_{2t} - \frac{k_1 x_2}{2} \right) + (1 - k_{02}) \frac{h}{4} \left( 1 - k_{02} - \frac{4a_{1t}}{h} \right)$$

$$M_{t2} \left( 1 + \delta - \frac{\delta k_1 x_2}{b} \right) = \frac{A_{s2} f_{y2} h}{c_2} \left[ z_2 + \frac{p_2 y_2 c_2^2}{bh} \right]$$



$$M_{t2} = \frac{A_s f_y 2 h}{c_2} \left[ \frac{z_2 + \frac{p_2 y_2 c_2^2}{bh}}{1 + \delta - \frac{\delta k_1 x_2}{b}} \right] \quad (5-5)$$

The sum of the forces acting perpendicular to a plane that contains the neutral axis and is perpendicular to the vertical face of the beam on which the compression zone is located must equal zero. These forces are:

$$\text{Force in longitudinal reinforcement} = \frac{A_s f_y 2 h}{L_2}$$

$$\text{Force in vertical legs of ties} = \frac{a_v f_y t k_{02} c_2^2}{S L_2}$$

$$\text{Force in concrete compression zone} = 0.85 k_1 x_2 L_2 f'_c$$

$$\text{Transverse shear force} = V c_2 / L_2$$

Summing of forces yields:

$$\frac{A_s f_y 2 h}{L_2} + \frac{a_v f_y t k_{02} c_2^2}{S L_2} - 0.85 k_1 x_2 L_2 f'_c - \frac{V c_2}{L_2} = 0$$

$$x_2 = \frac{A_s f_y 2 \left[ h + \frac{p_2 k_{02} c_2^2}{b} \right]}{0.85 k_1 f'_c L_2^2} - V c_2 \quad (5-6)$$

Using assumption two and neglecting the depth of the compression zone:

$$k_{02} = \frac{h}{2b + h}$$



In order to solve Equations (5-5) and (5-6) the value of  $c_2$  must be determined. The value of this parameter that should be used is the one which yields the minimum value of twisting moment. The value of  $x_2$ , which depends on  $c_2$ , has a relatively small effect on  $z_2$  and  $y_2$ . Therefore, in the following operation it is assumed that  $z_2$  and  $y_2$  are not functions of  $x_2$ .

Differentiating Equation (5-5) with respect to  $c_2$ , equating the first derivative to zero and solving for  $c_2$  yields:

$$c_2 = \pm \left[ \frac{h z_2 b}{p_2 y_2} \right]^{1/2} \quad (5-7)$$

The positive sign is used since negative values of  $c_2$  have no physical significance. The maximum value that  $c_2$  can have is obtained by neglecting the depth of the compression zone and using assumption two. The resulting equation is:

$$c_{m2} = 2b + h \quad (5-8)$$

The non-dimensional quantity  $\delta$  that was used in deriving Equation (5-5) was defined as:

$$\delta = \frac{Vb}{2M_{t2}}$$



This equation is rearranged to obtain Equation (5-9).

$$V = \frac{2M_{t2}\delta}{b} \quad (5-9)$$

### C-3 Mode 3 Failure Surface

The assumptions referred to below are presented in SECTION 5-3 of CHAPTER V. The symbols are defined in SECTION 5-4(c) and FIGURE 5-3 of CHAPTER V.

Moments are taken about an axis parallel to the neutral axis and located a distance  $k_1 x_3 / 2$  from the bottom face of the beam.

The external moments are:

$$\text{Component of bending moment} = M_{b3} \frac{b}{L_3}$$

$$\text{Component of twisting moment} = M_{t3} \frac{c_3}{L_3}$$

Moment due to transverse shear = 0

The internal moments are:

Moment due to force in longitudinal reinforcement

$$= \frac{A_{s3} f_y b}{L_3} \left( h_{03} - \frac{k_1 x_3}{2} \right)$$

Moment due to force in horizontal legs of ties

$$= \frac{a_v f_y t k_{03} c_3^2}{S L_3} \left( h - a_{3t} - \frac{k_1 x_3}{2} \right)$$



Moment due to force in vertical legs of ties

$$= \frac{a_v f_{yt} (1 - k_{03}) c_3}{s} \left[ \frac{(1 - k_{03}) b}{4} - a_{2t} \right] \frac{c_3}{L_3}$$

Equating external and internal moments gives:

$$M_{t3} \frac{c_3}{L_3} - \frac{M_{b3} b}{L_3} = \frac{A_{s3} f_{y3} b}{L_3} (h_{03} - \frac{k_1 x_3}{2}) + \frac{a_v f_{yt} k_{03} c_3^2}{s L_3} (h - a_{3t} - \frac{k_1 x_3}{2})$$

$$+ \frac{a_v f_{yt} (1 - k_{03}) c_3^2}{s L_3} \left[ \frac{(1 - k_{03}) b}{4} - a_{2t} \right]$$

$$M_{t3} c_3 - M_{b3} b = A_{s3} f_{y3} b (h_{03} - \frac{k_1 x_3}{2})$$

$$+ \frac{a_v f_{yt} c_3^2}{s} \left[ k_{03} (h - a_{3t} - \frac{k_1 x_3}{2}) + (1 - k_{03}) \left[ \frac{(1 - k_{03}) b}{4} - a_{2t} \right] \right]$$

$$\text{Let } \emptyset = \frac{M_{t3}}{M_{b3}} \quad \text{and} \quad p_3 = \frac{a_v f_{yt} h}{A_{s3} f_{y3} s}$$

$$M_{t3} \left[ \frac{c_3}{b} - \frac{1}{\emptyset} \right] = A_{s3} f_{y3} (h_{03} - \frac{k_1 x_3}{2})$$

$$+ \frac{p_3 A_{s3} f_{y3} c_3^2}{b h} \left[ k_{03} (h - a_{3t} - \frac{k_1 x_3}{2}) + (1 - k_{03}) \frac{b}{4} (1 - k_{03} - \frac{4a_{2t}}{b}) \right]$$

$$M_{t3} \left[ \frac{c_3}{b} - \frac{1}{\emptyset} \right] = A_{s3} f_{y3} \left[ (h_{03} - \frac{k_1 x_3}{2}) + \frac{p_3 c_3^2}{b h} \left[ k_{03} (h - a_{3t} - \frac{k_1 x_3}{2}) \right. \right. \\ \left. \left. + \frac{b}{4} (1 - k_{03}) (1 - k_{03} - \frac{4a_{2t}}{b}) \right] \right]$$



$$\text{Let } z_3 = h_{03} - \frac{k_1 x_3}{2}$$

$$\text{and } y_3 = k_{03}(h - a_{3t} - \frac{k_1 x_3}{2}) + \frac{b}{4} (1 - k_{03})(1 - k_{03} - \frac{4a_{2t}}{b})$$

$$M_{t3} = A_{s3} f_{y3} \left[ \frac{\frac{z_3 + \frac{p_3 y_3 c_3}{bh}}{c_3}^2}{\frac{1}{b} - \frac{1}{\phi}} \right] \quad (5-10)$$

The sum of the forces acting perpendicular to a plane that contains the neutral axis and is perpendicular to the bottom face of the beam must equal zero. These forces are:

$$\text{Force in longitudinal reinforcement} = \frac{A_{s3} f_{y3} b}{L_3}$$

$$\text{Force in horizontal legs of ties} = \frac{a_v f_{yt} k_{03} c_3^2}{s L_3}$$

$$\text{Force in concrete compression zone} = 0.85 k_1 x_3 L_3 f'_c$$

Summing of forces yields:

$$\frac{A_{s3} f_{y3} b}{L_3} + \frac{a_v f_{yt} k_{03} c_3^2}{s L_3} - 0.85 k_1 x_3 L_3 f'_c = 0$$

$$x_3 = \frac{A_{s3} f_{y3} \left[ b + \frac{p_3 k_{03} c_3^2}{h} \right]}{0.85 k_1 f'_c L_3^2} \quad (5-11)$$



Using assumption two and neglecting the depth of the compression zone:

$$k_{03} = \frac{b}{2h + b}$$

Before Equations (5-10) and (5-11) can be solved, the value of  $c_3$  must be determined. The value of this parameter that should be used is the one that yields the minimum value of twisting moment. The value of  $x_3$ , which depends on  $c_3$ , has a relatively small effect on  $z_3$  and  $y_3$ . Therefore, in the following operation it is assumed that  $z_3$  and  $y_3$  are not functions of  $x_3$ .

Differentiating Equation (5-10) with respect to  $c_3$ , equating the first derivative to zero and solving for  $c_3$  yields:

$$c_3 = \frac{b}{\phi} \pm b \left[ \left( \frac{1}{\phi^2} + \frac{z_3 h}{p_3 y_3 b} \right) \right]^{1/2} \quad (5-12)$$

Since the value of the terms within the brackets is always greater than  $\frac{1}{\phi}$  and since negative values of  $c_3$  have no physical significance, the positive sign is used. The maximum value that  $c_3$  can have is obtained by neglecting the depth of the compression zone and using assumption two. The resulting equation is:

$$c_{m3} = 2h + b \quad (5-13)$$













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